BOSTES | BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

2016 HSC Mathematics
Marking Guidelines

Section I

Multiple-choice Answer Key

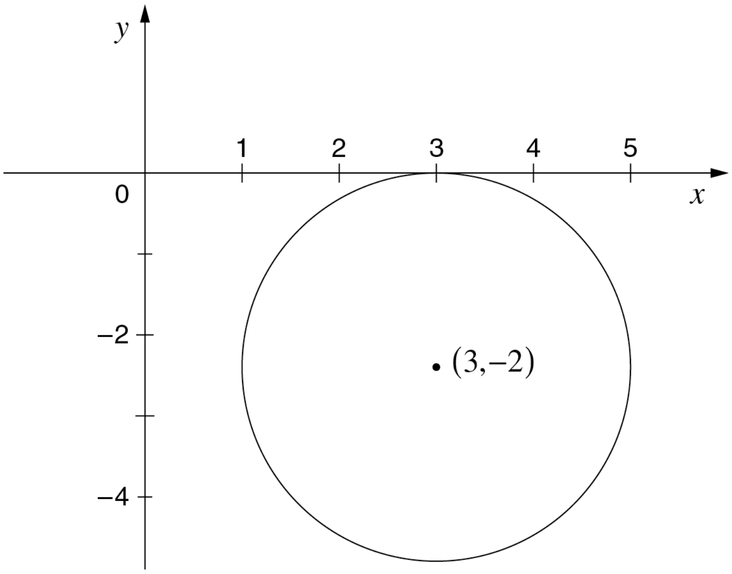
Question	Answer
1	B
2	C
3	B
4	A
5	B
6	A
7	A
8	D
9	C
10	D

Section II

Question 11 (a)

Criteria	Marks
• Provides correct sketch	2
• Identifies radius, or equivalent merit	1

Sample answer:



Question 11 (b)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use quotient rule, or equivalent merit	1

Sample answer:

$$\begin{aligned}\frac{d}{dx}\left(\frac{x+2}{3x-4}\right) &= \frac{(3x-4)(1)-(x+2)(3)}{(3x-4)^2}\\ &= \frac{3x-4-3x-6}{(3x-4)^2}\\ &= \frac{-10}{(3x-4)^2}\end{aligned}$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Establishes that $x \le 5$, or equivalent merit	1

Sample answer:

$$|x-2| \le 3$$

$$\therefore \quad -3 \le x-2 \le 3$$

$$-1 \le x \le 5$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Correct primitive, or equivalent merit	1

Sample answer:

$$\int_0^1 (2x+1)^3\,dx = \left[\frac{1}{2}\times\frac{1}{4}(2x+1)^4\right]_0^1$$

$$= \frac{1}{8}(3)^4 - \frac{1}{8}(1)^4$$

$$= \frac{81}{8} - \frac{1}{8}$$

$$= \frac{80}{8}$$

$$= 10$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	3
• Obtains $x^2 - 2x - 8 = 0$ and solves for x, or equivalent merit	2
• Attempts to eliminate x or y, or equivalent merit	1

Sample answer:

$$\left.\begin{aligned} y &= -5 - 4x \\ y &= 3 - 2x - x^2 \end{aligned}\right\} \begin{aligned} &① \\ &② \end{aligned}$$

Subs ① into ②

$-5 - 4x = 3 - 2x - x^2$
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$\therefore\ x = 4 \quad \text{or} \quad -2$

Subst $x = 4$ into ① $\Rightarrow$ $y = -5 - 4(4) = -21$

Subst $x = -2$ into ① $\Rightarrow$ $y = -5 - 4(-2) = 3$

$\therefore$ points of intersection are $(4, -21)$ and $(-2,3)$.

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Obtains $\sec^2 \dfrac{\pi}{8}$, or equivalent merit	1

Sample answer:

$$y = \tan x$$

$$\frac{dy}{dx} = \sec^2 x$$

$$\text{When } x = \frac{\pi}{8}, \frac{dy}{dx} = \sec^2\left(\frac{\pi}{8}\right) = \frac{1}{\cos^2 \frac{\pi}{8}} \approx 1.17$$

$\therefore$ Gradient of tangent is 1.17.

Question 11 (g)

Criteria	Marks
• Provides correct solution	2
• Obtains one correct answer, or equivalent merit	1

Sample answer:

$$\sin\left(\frac{x}{2}\right) = \frac{1}{2} \qquad \text{for} \qquad 0 \le x \le 2\pi$$

$$\text{Note} \qquad 0 \le \frac{x}{2} \le \pi$$

$$\therefore \left(\frac{x}{2}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Question 12 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains correct slope, or equivalent merit	1

Sample answer:

$$\frac{y-4}{x-2} = \frac{1-4}{6-2}$$

$$\frac{y-4}{x-2} = \frac{-3}{4}$$

$$4(y-4) = -3(x-2)$$

$$4y - 16 = -3x + 6$$

$$3x + 4y - 22 = 0$$

Question 12 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to calculate perpendicular distance of a point from a line, or equivalent merit	1

Sample answer:

$$BC : 3x + 4y - 22 = 0$$

$$\therefore AD = \frac{|3(1) + 4(0) - 22|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{|-19|}{\sqrt{25}}$$

$$= \frac{19}{5}$$

Question 12 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Finds distance from B to C, or equivalent merit	1

Sample answer:

$$BC = \sqrt{(2-6)^2 + (4-1)^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

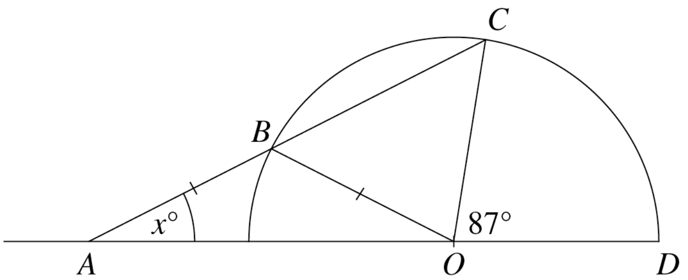
$$\therefore \text{Area } \triangle ABC = \frac{1}{2} \times 5 \times \frac{19}{5}$$

$$= \frac{19}{2} \text{ square units}$$

Question 12 (b) (i)

Criteria	Marks
• Provides correct explanation	1

Sample answer:



NOT TO SCALE

$\angle BOA = x°$ ($\angle$s opposite equal sides in $\triangle ABO$)

$\therefore \angle CBO = 2x°$ (exterior $\angle$ of $\triangle ABO$ equal to sum of two opposite interior $\angle$s)

Question 12 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the fact that triangle *OBC* is isosceles, or equivalent merit	1

Sample answer:

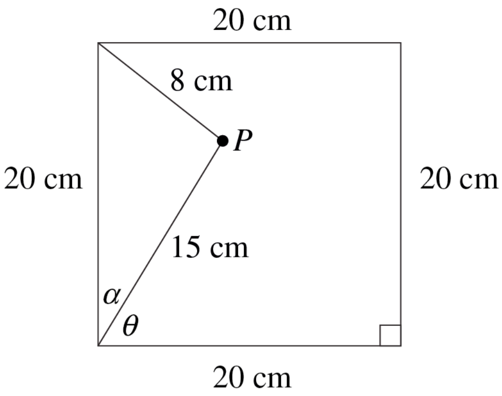
$OC = OB$ (radii)
$\angle BCO = \angle CBO$ ($\angle$s opposite equal sides in $\triangle CBO$)
$= 2x°$
Then $\angle COD = \angle CAO + \angle BCO$ (exterior $\angle$ of $\triangle CAO$ equals sum of two opposite interior $\angle$s)

$\therefore\ 87 = x + 2x$
$87 = 3x$
$x = 29$

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Substitutes correctly into cosine rule, and finds angle or equivalent merit	2
• Identifies the complementary angle to θ, or equivalent merit	1

Sample answer:



$$\cos\alpha = \frac{15^2 + 20^2 - 8^2}{2 \times 20 \times 15}$$
$$= \frac{561}{600}$$
$$\alpha \approx 20.77°$$
$$\theta \approx 69° \text{ to nearest degree.}$$

Question 12 (d) (i)

Criteria	Marks
• Provides correct derivative	1

Sample answer:

$$y = xe^{3x}$$

$$\frac{dy}{dx} = 3xe^{3x} + e^{3x}$$
$$= e^{3x}(1 + 3x)$$

Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use part (i) or equivalent merit	1

Sample answer:

Find exact value of $\int_0^2 e^{3x}(3+9x)\,dx$

$$\int_0^2 e^{3x}(3+9x)\,dx = \int_0^2 3e^{3x}(1+3x)\,dx$$

$$= 3\int_0^2 e^{3x}(1+3x)\,dx$$

$$= 3\left[xe^{3x}\right]_0^2$$

$$= 3\left[\left(2e^6\right)-\left(0e^0\right)\right]$$

$$= 6e^6$$

Question 13 (a) (i)

Criteria	Marks
• Provides correct solution	4
• Finds the x-values at which the stationary points occur and verifies the maximum turning point, or equivalent merit	3
• Finds the x-values at which the stationary points occur, or equivalent merit	2
• Attempts to solve $\frac{dy}{dx} = 0$, or equivalent merit	1

Sample answer:

$y = 4x^3 - x^4$

Find stationary points and determine nature.

$$\frac{dy}{dx} = 12x^2 - 4x^3$$

Need $\frac{dy}{dx} = 0$

$$12x^2 - 4x^3 = 0$$

$$4x^2(3 - x) = 0$$

when $x = 0$ or $x = 3$.

when $x = 0$ $y = 0$
when $x = 3$ $y = 27$

Checking the gradients for

$x = 0$,

x	0^-	0	0^+
y'	$+ve$	0	$+ve$

$\therefore$ horizontal point of inflexion at $x = 0$, ie at (0, 0)

$x = 3$,

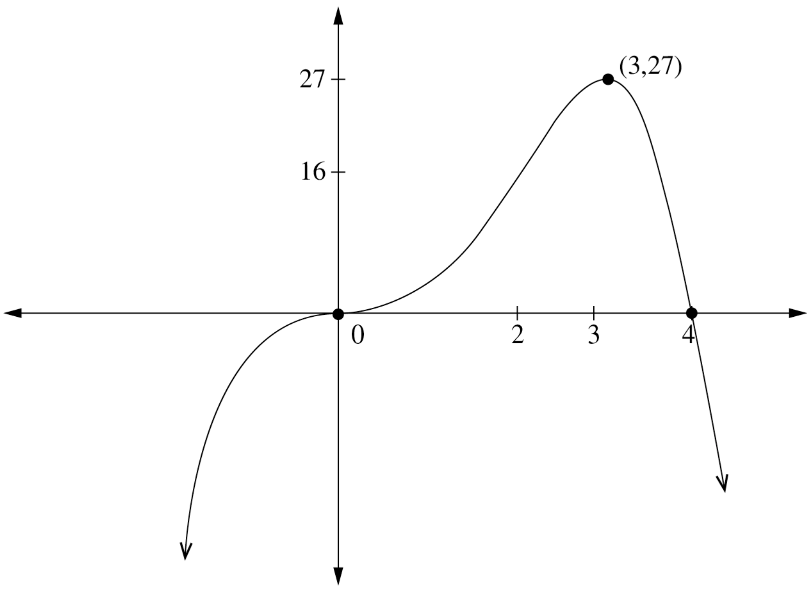
x	3^+	3	3^-
y'	$+ve$	0	$-ve$

$\therefore$ local maximum at $x = 3$, ie at (3, 27)

Question 13 (a) (ii)

Criteria	Marks
• Correct solution	2
• Locates the stationary points on the sketch of the curve, or equivalent merit	1

Sample answer:



Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to complete the square, or equivalent merit	1

Sample answer:

$$x^2 - 4x = 12y + 8$$

$$x^2 - 4x + 4 = 12y + 12$$

$$x^2 - 4x + 4 = 12(y + 1)$$

$$(x - 2)^2 = 4(3)(y + 1)$$

$\therefore$ focal length = 3

Question 13 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$\therefore$ focus (2,2)

Question 13 (c) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

A = 10

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains $5 = 10e^{-163k}$, or equivalent merit	1

Sample answer:

$$M(t) = 10e^{-kt}$$

$$M(163) = 5$$

$$\therefore 5 = 10e^{-163k}$$

$$\frac{1}{2} = e^{-163k}$$

$$-163k = \log_e\left(\frac{1}{2}\right)$$

$$k = \frac{\log_e\left(\frac{1}{2}\right)}{-163}$$

$$\approx 0.004252436$$

Question 13 (d)

Criteria	Marks
• Provides correct solution	3
• Finds the area under the cosine curve, or equivalent merit	2
• Finds the area under $y = x$, or equivalent merit	1

Sample answer:

$$\text{Area } = \int_0^1 \sqrt{2}\cos\left(\frac{\pi}{4}x\right)dx - \frac{1}{2}\times 1\times 1$$

$$= \sqrt{2}\times\frac{4}{\pi}\left[\sin\left(\frac{\pi}{4}x\right)\right]_0^1 - \frac{1}{2}$$

$$= \frac{4\sqrt{2}}{\pi}\left(\sin\frac{\pi}{4} - \sin 0\right) - \frac{1}{2}$$

$$= \frac{4\sqrt{2}}{\pi}\left(\frac{1}{\sqrt{2}} - 0\right) - \frac{1}{2}$$

$$= \frac{4}{\pi} - \frac{1}{2} \text{ square units}$$

$$\text{or } \frac{8-\pi}{2\pi}$$

Question 14 (a)

Criteria	Marks
• Provides correct solution	3
• Attempts to find a difference in areas or equivalent merit	2
• Applies Simpson's Rule to the existing heights or equivalent merit	1

Sample answer:

The increase in area can be approximated using Simpson's Rule.

Area increase (Simpson, 5 values)

$$= \frac{1}{3}\left[2\times(2-2) + 2\times(3-2.5) + 8\times(2.78-2.38)\right]$$

$$= \frac{1}{3}[0 + 1 + 3.2]$$

$$\approx 1.4 \text{ m}^2$$

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains a correct expression for A_1, or equivalent merit	1

Sample answer:

$A_0 = 100\,000$

$A_1 = 0.65 \times 100\,000 + 5000$

$A_2 = 0.65\, A_1 + 5000$

$= 0.65(0.65 \times 100\,000 + 5000) + 5000$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$A_2 = 0.65^2 \times 100\,000 + 0.65 \times 5000 + 5000$

$A_n = 0.65^n \times 100\,000 + 5000(0.65^{n-1} + 0.65^{n-2} + \square + 1)$

$$S_n = \frac{1\left(1 - 0.65^n\right)}{0.35}$$

$$A_n = 0.65^n \times 100\,000 + 5000\frac{\left(1 - 0.65^n\right)}{0.35}$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$A_{14} = 0.65^{14} \times 100\,000 + 5000\frac{\left(1 - 0.65^{14}\right)}{0.35}$$

$= 14\,491.7\square$

$\approx 14\,500$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

If w is the total width then

Area $= 720 = x \times w$

$$w = \frac{720}{x}$$

$$\text{Perimeter} = 5 \times x + w$$
$$= 5x + \frac{720}{x}$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds length at stationary point, or equivalent merit	2
• Finds $\frac{d\ell}{dx}$, or equivalent merit	1

Sample answer:

Stationary points occur when

$$0 = \frac{dl}{dx}$$
$$= 5 - \frac{720}{x^2}$$
$$\frac{720}{x^2} = 5$$
$$x^2 = 144$$
$$x = 12 \quad (x \text{ is length so ignore } -12)$$
$$\frac{d^2\ell}{dx^2} = \frac{1440}{x^3}$$

at $x = 12$ $\quad \frac{d^2\ell}{dx^2} = \frac{1440}{12^3} > 0$

so minimum at $x = 12$

$$\ell = 5 \times 12 + \frac{720}{12}$$
$$= 120$$

Question 14 (d)

Criteria	Marks
• Provides correct solution	2
• Sums the series, or equivalent merit	1

Sample answer:

$$1 + x + x^2 + x^3 + x^4 = \frac{1 - x^5}{1 - x} \qquad a = 1,\ r = x$$

$$= \frac{x^5 - 1}{x - 1} \qquad \text{for } x \neq 1$$

$$\lim_{x \to 1} \frac{x^5 - 1}{x - 1} = \lim_{x \to 1} \quad 1 + x + x^2 + x^3 + x^4$$

$$= 5$$

Question 14 (e)

Criteria	Marks
• Provides correct solution	2
• Expresses terms involving powers of 2, or equivalent merit	1

Sample answer:

$$\log 2 + \log 4 + \log 8 + \cdots + \log 512$$

$$= \log 2 + \log 2^2 + \log 2^3 + \cdots + \log 2^9$$

$$= \log 2 + 2\log 2 + 3\log 2 + \cdots 9\log 2$$

$$= \frac{9}{2}(\log 2 + 9\log 2)$$

$$= \frac{9}{2} \times 10\log 2$$

$$= 45\log 2$$

Question 15 (a)

Criteria	Marks
• Provides correct solution	4
• Evaluates the correct integral for the volume when C_2 is revolved, or equivalent merit	3
• Recognises the sum of two volumes is required and makes progress with both, or equivalent merit	2
• Attempts to find a volume using integration, or equivalent merit	1

Sample answer:

Volume of C_1 rotated:

$$V_1 = \frac{1}{2} \times \frac{4}{3}\pi 2^3$$

$$= \frac{16\pi}{3}$$

Volume of C_2 rotated:

$$V_2 = \pi \int_0^3 y^2\, dx$$

$$= \pi \int_0^3 4\left(1 - \frac{x^2}{9}\right) dx$$

$$= 4\pi \left[x - \frac{x^3}{27} \right]_0^3$$

$$= 4\pi \left(3 - \frac{27}{27} \right)$$

$$= 8\pi$$

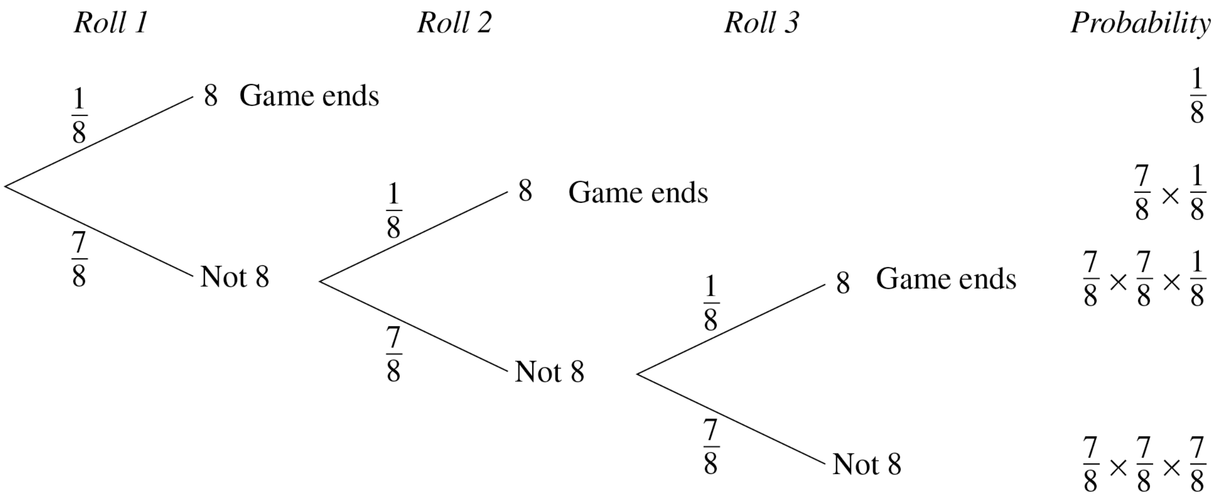
$$\therefore V = \frac{16\pi}{3} + 8\pi$$

$$= \frac{40\pi}{3}$$

Question 15 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Provides correct tree diagram, or equivalent merit	1

Sample answer:



$\therefore$ P (game ends before 4th roll) $= \frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$

Alternative solution:

P (ends after 1 roll) $= \frac{1}{8}$

P (ends after 2 rolls) $= \frac{7}{8} \times \frac{1}{8}$

P (ends after 3 rolls) $= \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$

$\therefore$ P (ends before 4th roll) $= \frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$

Question 15 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds an equation or inequality of probabilities that can be solved for n, or equivalent merit	2
• Finds, as a series, the probability that the game ends before the nth roll, or equivalent merit	1

Sample answer:

P (ends before nth roll)

$$= \frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8} + \left(\frac{7}{8}\right)^3 \times \frac{1}{8} + \cdots + \left(\frac{7}{8}\right)^{n-2} \times \frac{1}{8}$$

$$= \frac{1}{8}\left[1 + \frac{7}{8} + \left(\frac{7}{8}\right)^2 + \left(\frac{7}{8}\right)^3 + \cdots + \left(\frac{7}{8}\right)^{n-2}\right]$$

$$= \frac{1}{8} \times \frac{\left(\left(\frac{7}{8}\right)^{n-1} - 1\right)}{\left(\frac{7}{8}\right) - 1}$$

$$= 1 - \left(\frac{7}{8}\right)^{n-1}$$

Let $1 - \left(\frac{7}{8}\right)^{n-1} = \frac{3}{4}$

$$\left(\frac{7}{8}\right)^{n-1} = \frac{1}{4}$$

$$(n-1)\log\left(\frac{7}{8}\right) = \log\left(\frac{1}{4}\right)$$

$$n = 1 + \frac{\log\frac{1}{4}}{\log\frac{7}{8}}$$

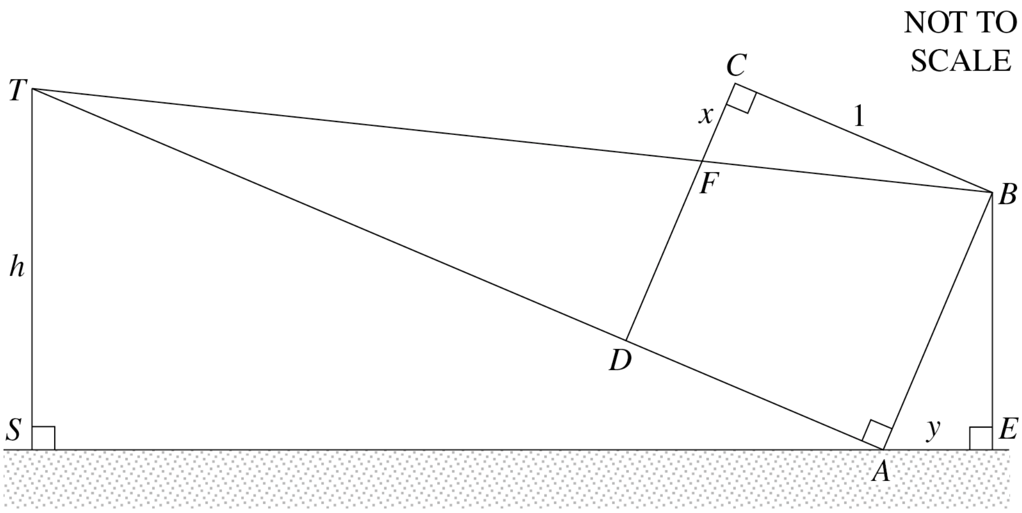
$$\approx 11.3817\ldots$$

$\therefore$ For probability of more than $\frac{3}{4}$, we require $n = 12$

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Identifies one pair of equal angles, giving reason(s)	1

Sample answer:



In $\triangle$s FCB, BAT

$\angle FCB = \angle BAT$ (both 90° angles in square $ABCD$)

Now $AB \parallel DC$ (opposite sides of a square)

$\therefore \angle CFB = \angle ABT$ (alternate $\angle$s, $AB \parallel DC$)

$\therefore \triangle FCB \,|||\, \triangle BAT$ (2 pairs of equal $\angle$s).

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows that $\angle TAS$ and $\angle BAE$ are complementary, or equivalent merit	1

Sample answer:

$\angle TSA = \angle BAD = \angle AEB = 90°$

$\angle TAS + \angle BAE = 90°$ ($\angle$sum, straight line SAE)

$\angle \mathrm{ABE} + \angle \mathrm{BAE} = 90°$ ($\angle$sum $\triangle ABE$)

$\therefore \angle TAS = \angle ABE$

In $\triangle$s TSA, AEB

$\angle TAS = \angle ABE$ (above)

$\angle TSA = \angle AEB = 90°$ (given)

$\therefore \triangle TSA \,|||\, \triangle AEB$ (2 pairs of equal $\angle$s).

Question 15 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains $AT = \frac{1}{x}$, or equivalent merit	1

Sample answer:

$$\frac{TS}{AE} = \frac{TA}{AB} \qquad \text{(matching sides in similar } \triangle\text{s)}$$

$$\frac{h}{y} = \frac{TA}{1}$$

$$\therefore h = y.TA$$

Also $$\frac{TA}{BC} = \frac{BA}{FC} \qquad \text{(matching sides in similar } \triangle\text{s)}$$

$$\frac{TA}{1} = \frac{1}{x}$$

$$\therefore TA = \frac{1}{x}$$

Hence $$h = y \times \frac{1}{x}$$

$$\therefore h = \frac{y}{x}$$

Question 16 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

When $t = 0$

$$v = 2 - \frac{4}{0+1} = -2$$

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds the value of t for which $v = 0$, or equivalent merit	1

Sample answer:

The particle is stationary when $v = 0$.

So $v = 0 \Rightarrow 0 = 2 - \dfrac{4}{t+1}$

$$\frac{4}{t+1} = 2$$
$$4 = 2(t+1)$$
$$4 = 2t + 2$$
$$2 = 2t$$
$$1 = t$$

So particle is stationary when $t = 1$.

acceleration $= \dfrac{dv}{dt}$

$$\frac{dv}{dt} = -4(t+1)^{-2} \times -1$$
$$= \frac{4}{(t+1)^2}$$

when $t = 1$ $\quad \dfrac{dv}{dt} = \dfrac{4}{(1+1)^2} = \dfrac{4}{4} = 1$

acceleration is 1 ms^{-2} when particle is stationary.

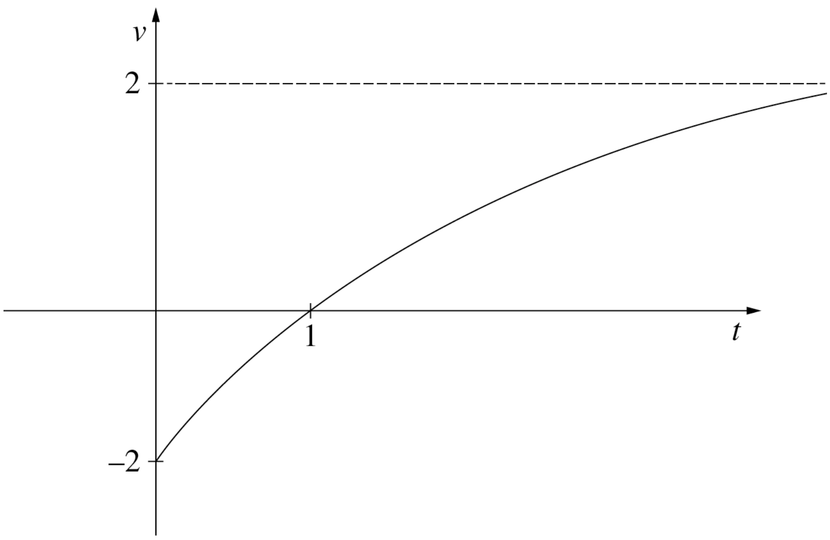
Question 16 (a) (iii)

Criteria	Marks
• Provides correct graph	2
• Describes the behaviour of v for large t, or equivalent merit	1

Sample answer:

As $t \to \infty, \dfrac{4}{t+1} \to 0$

$\Rightarrow v \to 2$



Question 16 (a) (iv)

Criteria	Marks
• Provides correct solution	3
• Correctly evaluates $\int_1^7 v\,dt$, or equivalent merit	2
• Recognises the particle changes direction at $t = 1$, or equivalent merit	1

Sample answer:

Particle changes direction when $t = 1$ travelling in negative direction for $0 \le t < 1$, so

$$\text{Distance travelled} = -\int_0^1 v\,dt + \int_1^7 v\,dt$$

$$= -\int_0^1 2 - \frac{4}{t+1}\,dt + \int_1^7 2 - \frac{4}{t+1}\,dt$$

$$= -\left[2t - 4\log(t+1)\right]_0^1 + \left[2t - 4\log(t+1)\right]_1^7$$

$$= -\left(2 - 4\log 2 - 0 + 4\log 1\right) + \left(14 - 4\log 8 - 2 + 4\log 2\right)$$

$$= -2 + 4\log 2 + 14 - 4\log 8 - 2 + 4\log 2$$

$$= 10 + 8\log 2 - 4\log 2^3$$

$$(= 10 + 8\log 2 - 12\log 2)$$

$$(= 10 - 4\log 2)$$

Question 16 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to find $\frac{dy}{dt}$, or equivalent merit	1

Sample answer:

$$y = 200\left(1+19e^{-0.5t}\right)^{-1}$$

$$\text{Rate of growth} = \frac{dy}{dt}$$

$$= -200\left(1+19e^{-0.5t}\right)^{-2} \times \left(\frac{-19e^{-0.5t}}{2}\right)$$

$$= \frac{1900e^{-0.5t}}{\left(1+19e^{-0.5t}\right)^{2}}$$

Question 16 (b) (ii)

Criteria	Marks
• Provides correct solution with justification	2
• Provides range, or equivalent merit	1

Sample answer:

All terms in $\frac{dy}{dt}$ are positive so y is increasing.

when $t = 0$ $y = \frac{200}{1+19} = 10$.

so $y \geq 10$ for $t \geq 0$.

As $t \to \infty$ $e^{-0.5t} \to 0$

so $y \to \frac{200}{1+0} = 200$

hence $10 \leq y < 200$.

Question 16 (b) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned}\frac{y}{400}(200-y) &= \frac{1}{2\left(1+19e^{-0.5t}\right)}\left(200-\frac{200}{1+19e^{-0.5t}}\right)\\ &= \frac{100}{\left(1+19e^{-0.5t}\right)}\left(\frac{1+19e^{-0.5t}-1}{1+19e^{-0.5t}}\right)\\ &= \frac{1900e^{-0.5t}}{\left(1+19e^{-0.5t}\right)^2}\\ &= \frac{dy}{dt}\end{aligned}$$

Question 16 (b) (iv)

Criteria	Marks
• Provides correct solution	2
• Recognises the importance of the vertex of the parabola in part (iii), or equivalent merit	1

Sample answer:

$$\begin{aligned}\frac{dy}{dt} &= \frac{y}{400}(200-y)\\ &= \frac{200y-y^2}{400}\end{aligned}$$

which is a quadratic in y with roots at $y = 0$ and $y = 200$. Since the coefficient of y^2 is negative, the quadratic has a max at $y = 100$.

$\therefore$ Population growing fastest when population is $y = 100$.

2016 HSC Mathematics Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	4.2	P5
2	1	3.1	H5
3	1	9.1, 9.5	P5
4	1	5.1	P4
5	1	12.5, 13.5	H5
6	1	13.3	H5
7	1	13.1	H5
8	1	1.2, 13.2	H5
9	1	1.2, 11.1	H8
10	1	12.2	H3

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	4.3	P5
11 (b)	2	8.9	P7
11 (c)	2	1.2	P3
11 (d)	2	11.1	H5
11 (e)	3	1.4, 6.3, 9.1	P4
11 (f)	2	1.1, 8.4, 13.5	P6, H5
11 (g)	2	13.1, 13.2	H5
12 (a) (i)	2	6.2	P4
12 (a) (ii)	2	6.5	P4
12 (a) (iii)	2	2.3, 6.5	P4
12 (b) (i)	1	2.3	P4
12 (b) (ii)	2	2.4	P4
12 (c)	3	5.5	P4
12 (d) (i)	1	8.8, 12.4	H3, H5
12 (d) (ii)	2	11.1	H3, H5
13 (a) (i)	4	10.2, 10.4	H6
13 (a) (ii)	2	10.5	H6
13 (b) (i)	2	1.3, 9.5	P5
13 (b) (ii)	1	9.5	P5
13 (c) (i)	1	14.2	H4
13 (c) (ii)	2	14.2	H4, H5

Question	Marks	Content	Syllabus outcomes
13 (d)	3	11.4, 13.6	H8
14 (a)	3	11.3	H5, H8
14 (b) (i)	2	7.5	H4, H5
14 (b) (ii)	1	7.5	H4, H5
14 (b) (iii)	1	7.5	H4, H5
14 (c) (i)	1	10.6	H4, H5
14 (c) (ii)	3	10.6	H4, H5
14 (d)	2	7.2, 8.2	P8, H5
14 (e)	2	7.1, 12.2	H3, H5
15 (a)	4	11.4	H8
15 (b) (i)	2	3.3	H5
15 (b) (ii)	3	3.3, 7.2, 12.2	H3, H5
15 (c) (i)	2	2.3, 2.5	H5
15 (c) (ii)	2	2.3, 2.5	H5
15 (c) (iii)	2	2.3, 2.5	H5
16 (a) (i)	1	14.3	H4
16 (a) (ii)	2	14.3	H4, H5
16 (a) (iii)	2	4.2, 10.5	H4, H5, H6
16 (a) (iv)	3	12.5, 14.3	H4, H5
16 (b) (i)	2	12.5, 14.1	H5
16 (b) (ii)	2	4.1, 12.5	H5
16 (b) (iii)	1	1.3, 14.2	H3
16 (b) (iv)	2	9.1, 12.5, 14.2	H5, H7