NSW EDUCATION STANDARDS AUTHORITY

2017 HSC Mathematics Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	D
3	C
4	A
5	B
6	D
7	B
8	A
9	C
10	A

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Recognises a conjugate surd, or equivalent merit	1

Sample answer:

$$\frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$= \frac{2\left(\sqrt{5}+1\right)}{\left(\sqrt{5}\right)^2 - 1^2}$$

$$= \frac{2\left(\sqrt{5}+1\right)}{5-1}$$

$$= \frac{2\left(\sqrt{5}+1\right)}{4}$$

Question 11 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\int (2x+1)^4 \, dx$$

$$= \frac{(2x+1)^5}{2(5)} + c$$

$$= \frac{(2x+1)^5}{10} + c$$

Question 11 (c)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use the quotient rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right)$$

$$= \frac{x(\cos x) - \sin x(1)}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$v = x$$

$$\frac{dv}{dx} = 1$$

Question 11 (d)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use the product rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(x^3 \times \ln x\right)$$

$$= x^3 \times \frac{1}{x} + 3x^2 \times \ln x$$

$$= x^2 + 3x^2 \times \ln x$$

$$u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$v = \ln x$$

$$\frac{dv}{dx} = \frac{1}{x}$$

Question 11 (e) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$A = \frac{1}{2}ab\sin c$$

$$= \frac{1}{2} \times 6 \times 6 \times \sin 30°$$

$$= \frac{1}{2} \times 36 \times \frac{1}{2}$$

$\therefore$ Area = 9 cm^2

Question 11 (e) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Shaded area = Area of sector AOB – Area of $\triangle OAB$

$$= \frac{\pi}{12}\left(6^2\right) - 9$$

$$= \frac{36\pi}{12} - 9$$

$\therefore$ Area = $(3\pi - 9)$ cm^2

Question 11 (f)

Criteria	Marks
• Provides correct equation	2
• Identifies the vertex, or equivalent merit	1

Sample answer:

$$v = (2,\ 1)$$

$$pt = (0,\ 4)$$

$$(x-h)^2 = 4a(y-k)$$

$$(x-2)^2 = 4a(y-1)$$

$$(0-2)^2 = 4a(4-1)$$

$$4 = 4a(3)$$

$$4 = 12a$$

$$a = \frac{1}{3}$$

$$\therefore\ (x-2)^2 = \frac{4}{3}(y-1)$$

Question 11 (g)

Criteria	Marks
• Provides correct solution	2
• Attempts to deal with the absolute value, or equivalent merit	1

Sample answer:

$$|3x-1| = 2$$

$$+(3x-1) = 2 \qquad -(3x-1) = 2$$

$$3x-1 = 2 \qquad -3x+1 = 2$$

$$3x = 3 \qquad -3x = 1$$

$$x = 1 \qquad x = -\frac{1}{3}$$

Question 11 (h)

Criteria	Marks
• Provides correct domain	2
• Attempts to obtain an inequality, or equivalent merit	1

Sample answer:

$f(x) = \sqrt{3 - x}$

$3 - x \geq 0$

$3 \geq x$

$\therefore \quad x \leq 3$

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the slope of the curve at the given point, or equivalent merit	1

Sample answer:

$$y = x^2 + 4x - 7$$

$$y' = 2x + 4$$

when $x = 1$ $\quad y' = 2 + 4 = 6$

$\therefore m = 6$

$$y - y_1 = m(x - x_1)$$

$$y - -2 = 6(x - +1)$$

$$y = 6x - 8$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains the primitive of an integrand for the volume, involving the square of the function, or equivalent merit	2
• Obtains an integral for the volume involving the square of the function, or equivalent merit	1

Sample answer:

$$V = \pi \int_{-2}^{2} \left(\sqrt{16 - 4x^2}\right)^2 dx$$

$$= 2\pi \int_{0}^{2} \left(16 - 4x^2\right) dx$$

$$= 2\pi \left[16x - \frac{4x^3}{3}\right]_0^2$$

$$= 2\pi \left[32 - \frac{32}{3} - 0 + 0\right]$$

$$\text{Volume} = \frac{128\pi}{3} \text{ units}^3$$

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Finds the three correct equations and attempts to solve, or equivalent merit	2
• Finds a valid equation linking *a* and *d*, or equivalent merit	1

Sample answer:

$T_5 = a + 4d = 200$

$S_5 = a + a + d + a + 2d + a + 3d = 1200$

$$\begin{aligned} a + 4d &= 200 \quad \text{———} ① \\ 4a + 6d &= 1200 \quad \text{———} ② \\ 4a + 16d &= 800 \quad \text{———} ① \times 4 \end{aligned}$$

Subtracting

$$\begin{aligned} 10d &= -400 \\ d &= -40 \end{aligned}$$

$$\begin{aligned} a + -160 &= 200 \\ a &= 360 \end{aligned}$$

$$\begin{aligned} T_{10} &= 360 + 9 \times -40 \\ &= 0 \end{aligned}$$

Question 12 (d) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Point A $(-4, 0)$ to the line $y = x - 2$

$$x - y - 2 = 0$$

$$a = 1 \quad b = -1 \quad c = -2$$

$$\text{Perpendicular distance} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|1 \times -4 + -1 \times 0 - 2|}{\sqrt{1^2 + (-1)^2}}$$

$$= \frac{|-4 - 2|}{\sqrt{2}} = \frac{6}{\sqrt{2}}$$

$$= \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{2}$$

$$= 3\sqrt{2}$$

Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds the length of *DC*, or equivalent merit	1

Sample answer:

$$A = \frac{1}{2}(a+b)h$$

$h = 3\sqrt{2}$ and one length (AB) $= 5\sqrt{2}$

Other length is distance $CD \quad = \sqrt{(3-0)^2 + (1--2)^2}$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

So $A \quad = \frac{1}{2}\left(5\sqrt{2} + 3\sqrt{2}\right) \times 3\sqrt{2}$

$$= \frac{1}{2} \times 8\sqrt{2} \times 3\sqrt{2}$$

$$= \frac{24 \times 2}{2}$$

$$\therefore \text{Area} = 24 \text{ units}^2$$

Question 12 (e) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$P \text{ (even number)} = \frac{2}{5}$$

Question 12 (e) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Probability (at least 1 even number)

= 1 – Probability (all odd numbers)

$= 1 - \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{98}{125}$

Question 12 (e) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$P \text{ (even, odd, odd)} = \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{18}{125}$

Question 12 (e) (iv)

Criteria	Marks
• Provides correct answer	1

Sample answer:

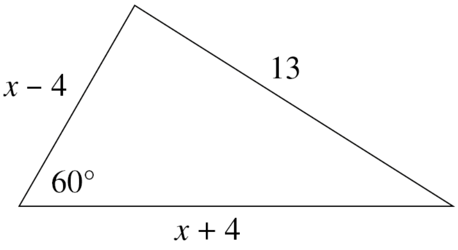
$P \text{ (exactly one even)} = 3 \times \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{54}{125}$

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Attempts to expand each term and evaluate the cosine term, or equivalent merit	2
• Correctly uses the cosine rule, or equivalent merit	1

Sample answer:



Using cosine rule:

$$13^2 = (x+4)^2 + (x-4)^2 - 2(x+4)(x-4)\cos 60°$$

$$169 = x^2 + 8x + 16 + x^2 - 8x + 16 - 2\left(x^2 - 16\right) \times \frac{1}{2}$$

$$= 2x^2 + 32 - x^2 + 16$$

$$= x^2 + 48$$

$$121 = x^2$$

$$\therefore \quad x = \pm\sqrt{121}$$

$$= \pm 11$$

But $x - 4 > 0$ since it is a length

So $x > 4$

$\therefore \; x = 11$

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	4
• Determines the x-coordinates of the two stationary points and determines the nature of one of them, or equivalent merit	3
• Obtains x-values of the stationary points, or equivalent merit	2
• Differentiates and sets $\frac{dy}{dx} = 0$, or equivalent merit	1

Sample answer:

$$y = 2x^3 + 3x^2 - 12x + 7$$

$$\frac{dy}{dx} = 6x^2 + 6x - 12$$

$$= 6(x^2 + x - 2)$$

$$\frac{d^2y}{dx^2} = 6(2x + 1)$$

For stationary point, $\frac{dy}{dx} = 0$

$$\therefore \quad x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-2)}}{2}$$

$$= \frac{-1 \pm 3}{2}$$

$$= 1 \quad \text{or} \quad -2$$

When $x = 1$, $y = 2(1)^3 + 3(1)^2 - 12(1) + 7$

$$= 0$$

$$\frac{d^2y}{dx^2} = 6(2(1) + 1) = 18 > 0$$

$\therefore$ Concave up

$\therefore$ (1, 0) is a local minimum.

When $x = -2$, $y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 7$

$$= 27$$

$$\frac{d^2y}{dx^2} = 6(2(-2) + 1) = -18 < 0$$

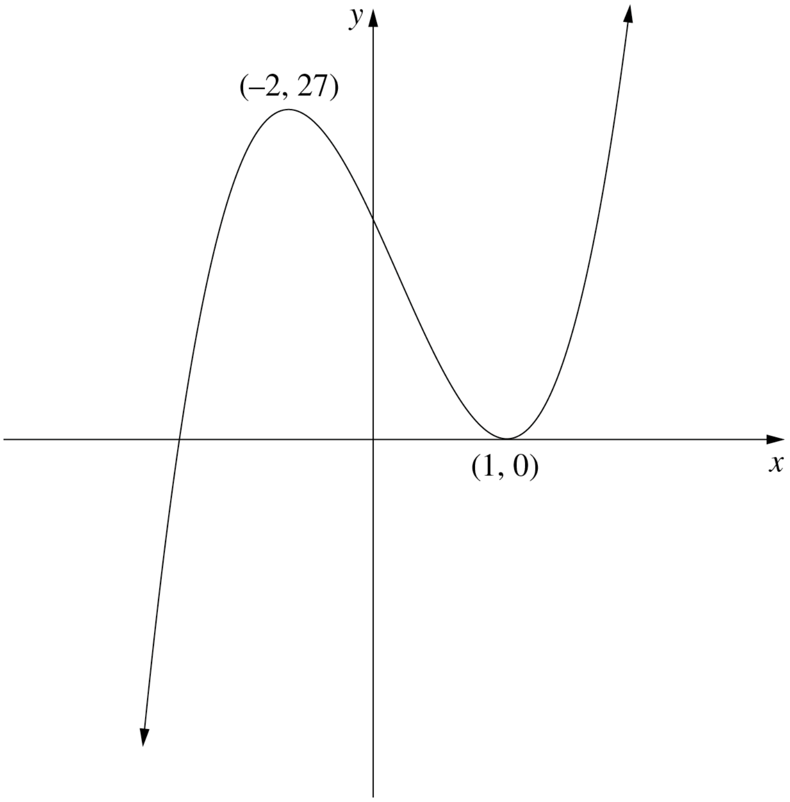
$\therefore$ Concave down

$\therefore$ (–2, 27) is a local maximum.

Question 13 (b) (ii)

Criteria	Marks
• Provides correct sketch	2
• Sketches a cubic curve, or equivalent merit	1

Sample answer:



Question 13 (b) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

From the graph, $\frac{dy}{dx}$ is positive when $x > 1$ or $x < -2$

Question 13 (c)

Criteria	Marks
• Provides correct solution	2
• Obtains the quadratic in *m*, or equivalent merit	1

Sample answer:

$m = t^{\frac{1}{3}}, \quad m^2 = \left(t^{\frac{1}{3}}\right)^2 = t^{\frac{2}{3}}$

$$t^{\frac{2}{3}} + t^{\frac{1}{3}} - 6 = 0$$

$$\therefore \quad m^2 + m - 6 = 0$$

$$(m-2)(m+3) = 0$$

$$\therefore \ m = 2 \quad \text{or} \quad m = -3$$

$$\therefore \ t^{\frac{1}{3}} = 2 \quad \text{or} \quad t^{\frac{1}{3}} = -3$$

$$t = 8 \qquad\qquad t = -27$$

Question 13 (d)

Criteria	Marks
• Provides correct solution	3
• Provides correct primitive, or equivalent merit	2
• Attempts to integrate the expression, or equivalent merit	1

Sample answer:

$\frac{dV}{dt} = \frac{2t}{1+t^2}$ when $t = 0, \ V = 0$

$$V = \int_0^{10} \frac{2t}{1+t^2}\,dt$$

$$= \left[\ln\left(1+t^2\right)\right]_0^{10}$$

$$= \ln(101) - \ln 1$$

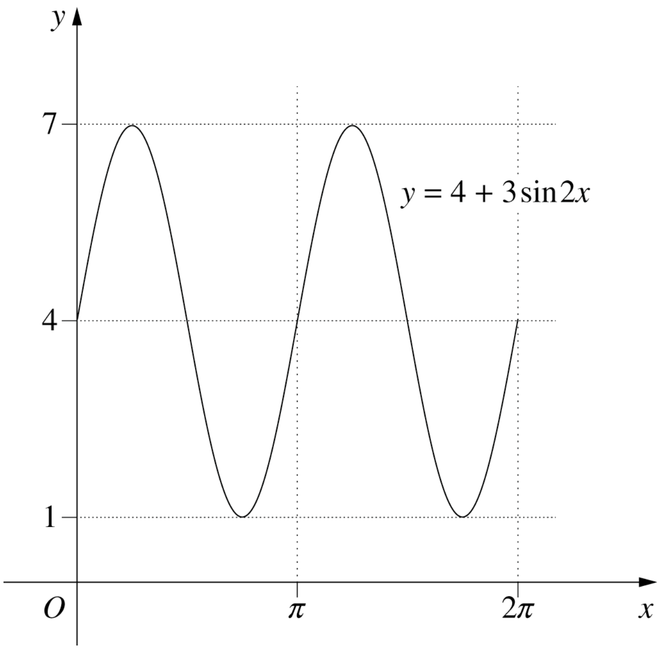
$$= \ln(101) \qquad (\doteqdot 4.615\ldots)$$

After 10 seconds the volume of water in the tank is $\ln 101$ litres.

Question 14 (a)

Criteria	Marks
• Provides correct sketch	3
• Indicates correct amplitude and period, or equivalent merit	2
• Indicates correct amplitude, or equivalent merit	1

Sample answer:



Question 14 (b) (i)

Criteria	Marks
• Provides correct answer	1

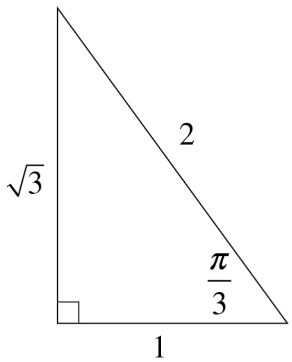
Sample answer:

$$\int_0^{\frac{\pi}{3}} \cos x \, dx$$

$$= \Big[\sin x \Big]_0^{\frac{\pi}{3}}$$

$$= \sin\left(\frac{\pi}{3}\right) - \sin(0)$$

$$= \frac{\sqrt{3}}{2}$$



Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to apply Simpson's rule, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{3}} \cos x \, dx \doteq \frac{\frac{\pi}{3}-0}{6}\left[\cos(0)+4\cos\left(\frac{\pi}{6}\right)+\cos\frac{\pi}{3}\right]$$

$$= \frac{\pi}{18}\left[1+4\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\right]$$

$$= \frac{\pi}{18}\left[\frac{3}{2}+\frac{4\sqrt{3}}{2}\right]$$

$$= \frac{\pi}{18}\left[\frac{3+4\sqrt{3}}{2}\right]$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\frac{\sqrt{3}}{\not{2}} \doteqdot \frac{\pi}{18}\left[\frac{3+4\sqrt{3}}{\not{2}}\right]$$

$$18\sqrt{3} \doteqdot \pi\left[3+4\sqrt{3}\right]$$

$$\pi \doteqdot \frac{18\sqrt{3}}{3+4\sqrt{3}}$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$C(t) = Ae^{kt}$$

$$\frac{dC}{dt} = k \times Ae^{kt}$$

$$= kC\text{, as required}$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Recognises the significance of the half-life, or equivalent merit	1

Sample answer:

$$t = 5730 \qquad C = \frac{1}{2}C_0$$

$$C(0) = A$$

$$\frac{1}{2}C(0) = \frac{1}{2}A$$

$$\therefore \quad \frac{1}{2}\not{A} = \not{A}e^{5730k}$$

$$\ln\frac{1}{2} = 5730k$$

$$k = \ln\left(\frac{1}{2}\right) \div 5730$$

$$k = -0.00012$$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains a correct exponential equation for *t*, or equivalent merit	1

Sample answer:

$C(t) = 0.9A \qquad t = ?$

$$0.9\not{A} = \not{A}e^{kt}$$

$$\ln(0.9) = kt$$

$$t = \frac{\ln(0.9)}{k}$$

$$\begin{aligned} t &= \frac{\ln(0.9)}{k} \\ &= \frac{\ln(0.9)}{-0.00012\ldots} \\ &\approx 870.9777\ldots \\ &= 870 \text{ years} \end{aligned}$$

or using $k = -0.00012$

$t = 878.0042\ldots$

$t = 880$ years

Question 14 (d)

Criteria	Marks
• Provides correct solution	3
• Obtains an expression for the area involving k with integration complete, or equivalent merit	2
• Attempts to use integration to find the area of the shaded region, or equivalent merit	1

Sample answer:

$$\text{Area} = 2\int_0^1 \left(k\left(1-x^2\right)-2k\left(x^2-1\right)\right)dx$$

$$= 2\int_0^1 \left(3k-3kx^2\right)dx$$

$$= 6k\int_0^1 \left(1-x^2\right)dx$$

$$= 6k\left[x-\frac{1}{3}x^3\right]_0^1$$

$$= 4k$$

$$\therefore\ 4k = 8$$

$$k = 2$$

Question 15 (a)

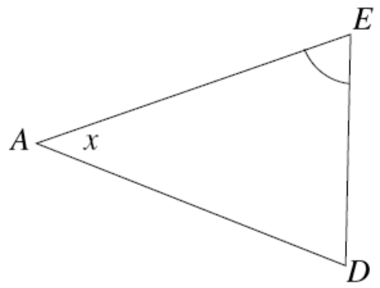
Criteria	Marks
• Provides correct solution	3
• Finds a correct equation for *x*, or equivalent merit	2
• Makes some progress	1

Sample answer:

$\triangle ABC$, $\triangle ACD$, $\triangle ADE$ are congruent (given)

So angles $\angle BAC$, $\angle CAD$, and $\angle DAE$, are equal (corresponding angles in congruent triangles)

$\angle AED$ is the base angle of an isosceles triangle.



$2 \times \angle \mathrm{AED} + \mathrm{x} = 180$ (angle sum of a triangle)

So $\angle AED = \dfrac{180 - x}{2}$

$$\begin{aligned} \angle AED + \angle EAB &= 180 \quad (\text{cointerior angle} - AB \,||\, ED) \\ \frac{180 - x}{2} + 3x &= 180 \\ 180 - x + 6x &= 360 \\ 5x &= 180 \\ x &= 36 \end{aligned}$$

Question 15 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains $M_1 = X(1.0035) - 2500$, or equivalent merit	1

Sample answer:

$$M_1 = X \times \left(1 + \frac{0.042}{12}\right) - 2500$$

$$= X(1.0035) - 2500$$

$$M_2 = \big((X(1.0035) - 2500) + X\big)1.0035 - 2500$$

$$= X(1.0035)^2 + X(1.0035) - 2500(1.0035) - 2500$$

$$= X\left(1.0035^2 + 1.0035\right) - 2500(1.0035 + 1)$$

Question 15 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Obtains an expression for M_{48} with at least one series summed, or equivalent merit	2
• Obtains an expression for M_{48}, or equivalent merit	1

Sample answer:

At the end of 4 years = 48 months, $M_{48} = 80\,000$

$$X\left(1.0035^{48} + \cdots + 1.0035\right) - 2500\left(1.0035^{47} + \cdots + 1\right) = 80\,000$$

$$X\left[\frac{1.0035\left(1.0035^{48} - 1\right)}{0.0035}\right] - \frac{2500\left(1.0035^{48} - 1\right)}{0.0035} = 80\,000$$

$$X\left[\frac{1.0035\left(1.0035^{48} - 1\right)}{0.0035}\right] = 210\,421.2054$$

$$52.351X = 240\,421.2054$$

$$X = 4019.42$$

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Finds correct primitive, or equivalent merit	1

Sample answer:

Particle 1 when $t = 0$ $x_1 = 0$ $v_1 = 3$ and $a_1 = 6t + e^{-t}$

v_1 is a primitive of a_1

$$v_1 = \frac{6t^2}{2} + -e^{-t} + k$$

$$= 3t^2 - e^{-t} + k$$

When $t = 0$ $v_1 = 3$

$$3 = 3 \times 0 - e^0 + k$$

$$= -1 + k$$

$$4 = k$$

So $v_1 = 3t^2 + 4 - e^{-t}$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Equates velocities, or equivalent merit	1

Sample answer:

$$v_1 = v_2$$

$$3t^2 + 4 - e^{-t} = 6t + 1 - e^{-t}$$

$$3t^2 - 6t + 3 = 0$$

$$3\left(t^2 - 2t + 1\right) = 0$$

$$3(t-1)^2 = 0$$

$$t = 1$$

Question 15 (c) (iii)

Criteria	Marks
• Provides correct solution	3
• Obtains cubic equation in t, or equivalent merit	2
• Finds x_1 or x_2, or equivalent merit	1

Sample answer:

Show that the particles do not meet for $t > 0$

$$v_1 = 3t^2 + 4 - e^{-t} \quad \text{so}$$

$$x_1 = \frac{3t^3}{3} + 4t + e^{-t} + C$$

When $t = 0 \quad x_1 = 0$

$$0 = 0 + 0 + 1 + C$$

So $C = -1$

$$x_1 = t^3 + 4t + e^{-t} - 1$$

$$v_2 = 6t + 1 - e^{-t}$$

$$x_2 = \frac{6t^2}{2} + t + e^{-t} + k$$

$$= 3t^2 + t + e^{-t} + k$$

When $t = 0 \quad x_2 = 0$

$$3 \times 0^2 + 0 + 1 + k = 0$$

$$k = -1$$

So $x_2 = 3t^2 + t + e^{-t} - 1$

If particles meet $x_1 = x_2$

So

$$t^3 + 4t + e^{-t} - 1 = 3t^2 + t + e^{-t} - 1$$

$$t^3 - 3t^2 + 4t - t = 0$$

$$t^3 - 3t^2 + 3t = 0$$

$$t\left(t^2 - 3t + 3\right) = 0$$

$t = 0$ (at origin)

or $t = \dfrac{3 \pm \sqrt{9 - 4 \times 3}}{2}$

$9 - 12 < 0$ so quadratic has no solutions.

$\therefore$ particles do not meet for $t > 0$

(alternative method)

Since both particles start at the origin,

$$v_1 = 3t^2 + 3 + \left(1 - e^{-t}\right)$$

$$= 3t^2 + 3 + \left(v_2 - 6t\right)$$

$$= v_2 + 3\left(t^2 - 2t + 1\right)$$

$$= v_2 + 3(t - 1)^2$$

So P_1 is never slower than P_2.

They start together. P_1 starts faster than P_2 and never gets slower.

$\therefore$ P_1 will always be ahead of P_2

$\therefore$ The particles never meet ($t > 0$).

Question 16 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

By Pythagoras' theorem

$AE = \sqrt{x^2 + 25}$

$DE = 9 - x$, so

$BE = \sqrt{49 + (9 - x)^2}$

$\therefore \quad L = \sqrt{x^2 + 25} + \sqrt{49 + (9 - x)^2}$

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	3
• Obtains $\dfrac{x}{\sqrt{x^2 + 25}} = \dfrac{9 - x}{\sqrt{49 + (9 - x)^2}}$, or equivalent merit	2
• Correctly differentiates $\sqrt{x^2 + 25}$, or equivalent merit	1

Sample answer:

$$\frac{dL}{dx} = \frac{x}{\sqrt{x^2 + 25}} - \frac{(9 - x)}{\sqrt{49 + (9 - x)^2}}$$

$$\frac{dL}{dx} = 0 \quad \Rightarrow \quad \frac{x}{\sqrt{x^2 + 25}} = \frac{9 - x}{\sqrt{49 + (9 - x)^2}}$$

$\Rightarrow \quad \sin\alpha = \sin\beta$

since in $\triangle ACE$, $\sin\alpha = \dfrac{CE}{AE}$

$$= \frac{x}{\sqrt{25 + x^2}}$$

in $\triangle BDE$, $\sin\beta = \dfrac{ED}{EB}$

$$= \frac{9 - x}{\sqrt{(9 - x)^2 + 49}}$$

Question 16 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Makes some progress	1

Sample answer:

$\sin\alpha = \sin\beta \quad \Rightarrow \quad \alpha = \beta \qquad$ (since α, β acute)

$\therefore \quad \triangle ACE \,|||\, \triangle BDE$ (equiangular)

$$\therefore \quad \frac{x}{5} = \frac{9-x}{7}$$

$$\Rightarrow \quad 7x = 45 - 5x$$

$$x = \frac{45}{12}$$

$$= 3\frac{3}{4}$$

Question 16 (a) (iv)

Criteria	Marks
• Provides correct solution	1

Sample answer:

From parts (ii) and (iii) there is only one value of x for which $\frac{dL}{dx} = 0$ for $0 < x < 9$

For $x = 0 \qquad \frac{dL}{dx} = 0 - \frac{9}{\sqrt{130}} < 0$

For $x = 9 \qquad \frac{dL}{dx} = \frac{9}{\sqrt{106}} > 0$

Hence, since $x = 3\frac{3}{4}$ lies between $x = 0$ and $x = 9$ then this must be a minimum.

Question 16 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains $-1 < 1 - \frac{a}{2} < 1$, or equivalent merit	2
• Recognises $\frac{a}{1-r} = 2$, or equivalent merit	1

Sample answer:

$$S_\infty = \frac{a}{1-r} = 2$$

$$\therefore \quad \frac{a}{2} = 1 - r$$

$$\therefore \quad r = 1 - \frac{a}{2}$$

Now $|r| < 1$ so $\left|1 - \frac{a}{2}\right| < 1$

$$-1 < 1 - \frac{a}{2} < 1$$

$$-2 < 2 - a < 2$$

$$2 > a - 2 > -2$$

$$4 > a > 0$$

$$\therefore \quad 0 < a < 4$$

Question 16 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Shows $BD = DE$, or equivalent merit	2
• Shows $\triangle BDM$ is similar to $\triangle BEC$, or equivalent merit	1

Sample answer:

Since $BM = MC$ (given)
then $BD = DE$ (equal intercept)

$\therefore \quad BD = DA + AE$
and $\quad BD = DA + AC$ (given)

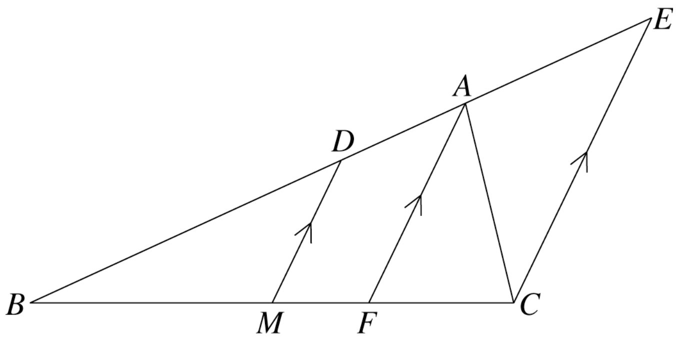
ie $\quad DA + AE = DA + AC$
$\therefore \quad AE = AC$

Hence, $\triangle ACE$ is isosceles (two equal sides)

Question 16 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows $\angle FAC = \angle ACE$, or equivalent merit	1

Sample answer:



Since $AF \parallel DM$ and
$EC \parallel DM$ then
$AF \parallel EC$

$\angle BAF = \angle AEC$ (corresponding angles, parallel lines $AF \parallel EC$)
$\angle AEC = \angle ACE$ (base angles of isosceles triangle ACE)
$\angle ACE = \angle CAF$ (alternate angles, parallel lines AF and EC)

So $\angle BAF = \angle CAF$, ie AF bisects $\angle BAC$

2017 HSC Mathematics Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	6.2	P5
2	1	1.3	P3
3	1	12.5	H3
4	1	10.4	H6
5	1	12.2	H3
6	1	4.3	P4
7	1	5.2	P3
8	1	4.4, 6.4	P4
9	1	10.8	H7
10	1	14.3	H6

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	1.1	P3
11 (b)	1	11.2	H5
11 (c)	2	8.8, 13.5	H5
11 (d)	2	8.8, 12.5	H3
11 (e) (i)	1	2.3	P4
11 (e) (ii)	1	13.1	H5
11 (f)	2	9.5	P5
11 (g)	2	1.2	P4
11 (h)	2	4.1	P5
12 (a)	2	8.4	P6, P7
12 (b)	3	11.4	H8
12 (c)	3	7.1	H5
12 (d) (i)	1	6.5	P4
12 (d) (ii)	2	6.8	H5
12 (e) (i)	1	3.1	H5
12 (e) (ii)	1	3.3	H5
12 (e) (iii)	1	3.2	H5
12 (e) (iv)	1	3.3	H5
13 (a)	3	1.4, 5.5	P3, P4
13 (b) (i)	4	10.2	H6
13 (b) (ii)	2	10.5	H6

Question	Marks	Content	Syllabus outcomes
13 (b) (iii)	1	10.1	H6
13 (c)	2	9.4	P4
13 (d)	3	14.1	H5
14 (a)	3	13.3	H5
14 (b) (i)	1	13.6	H5
14 (b) (ii)	2	11.3	H5
14 (b) (iii)	1	11.3	P3, H5
14 (c) (i)	1	14.2	H5
14 (c) (ii)	2	14.2	H5
14 (c) (iii)	2	14.2	H5
14 (d)	3	11.4	H8
15 (a)	3	2.4–6	H2, H5
15 (b) (i)	2	7.5	H5
15 (b) (ii)	3	7.5	H5
15 (c) (i)	2	14.3	H5
15 (c) (ii)	2	14.3	H5
15 (c) (iii)	3	9.2, 14.3	H5
16 (a) (i)	1	2.3	P4
16 (a) (ii)	3	10.6	H5
16 (a) (iii)	2	2.3, 2.6	P4, H5
16 (a) (iv)	1	10.6	H5
16 (b)	3	7.3	H5
16 (c) (i)	3	2.4–6	H2, H5
16 (c) (ii)	2	2.4–6	H2, H5