2018 HSC Mathematics
Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	C
3	A
4	D
5	D
6	C
7	C
8	D
9	B
10	D

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Attempts to use $3-\sqrt{2}$, or equivalent merit	1

Sample answer:

$$\frac{3}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{9-3\sqrt{2}}{9-2}$$

$$= \frac{9-3\sqrt{2}}{7}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Obtains $-3x > 9$, or equivalent merit	1

Sample answer:

$$1-3x > 10$$

$$\frac{-3x}{-3} > \frac{9}{-3}$$

$$x < -3$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Attempts to factorise numerator, or equivalent merit	1

Sample answer:

$$\frac{8x^3-27y^3}{2x-3y} = \frac{(2x)^3-(3y)^3}{2x-3y}$$

$$= \frac{(2x-3y)\left(4x^2+6xy+9y^2\right)}{2x-3y}$$

$$= 4x^2+6xy+9y^2$$

Question 11 (d) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$a + 2d = 8 \qquad ①$$
$$a + 19d = 59 \qquad ②$$
$$②-①: \frac{17d}{17} = \frac{51}{17}$$
$$d = 3$$

Question 11 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds first term, or equivalent merit	1

Sample answer:

$$d = 3, \quad \text{so} \quad a + 2(3) = 8$$
$$a + 6 = 8$$
$$a = 2$$

$$a + 49d = 2 + 49 \times 3$$
$$= 2 + 147$$
$$= 149$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Provides correct primitive, or equivalent merit	1

Sample answer:

$$\int_0^3 e^{5x}\,dx = \left[\frac{1}{5}e^{5x}\right]_0^3$$
$$= \frac{1}{5}\left(e^{15} - e^{0}\right)$$
$$= \frac{1}{5}\left(e^{15} - 1\right)$$

Question 11 (f)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use product rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(x^2 \tan x\right) = x^2 \times \sec^2 x + \tan x \times (2x)$$

$$= x\left(x \sec^2 x + 2\tan x\right)$$

Question 11 (g)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use quotient rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(\frac{e^x}{x+1}\right) = \frac{(x+1)\times e^x - e^x \times 1}{(x+1)^2}$$

$$= \frac{xe^x + e^x - e^x}{(x+1)^2}$$

$$= \frac{xe^x}{(x+1)^2}$$

Question 12 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\angle ABC = 50^\circ + 60^\circ$$

$$= 110^\circ$$

Question 12 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Correctly substitutes into cosine rule, or equivalent merit	1

Sample answer:

$$d^2 = 320^2 + 190^2 - 2(320)(190)\cos 110°$$

$$d = \sqrt{320^2 + 190^2 - 2(320)(190)\cos 110°}$$

$$d = 424.3697084$$

$$d = 420 \text{ km}$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains slope at $x = \frac{\pi}{6}$, or equivalent merit	2
• Obtains correct derivative, or equivalent merit	1

Sample answer:

$$y = \cos 2x$$

$$y' = -2\sin 2x$$

$$M_T = -2\sin\left(\frac{2\pi}{6}\right)$$

$$M_T = -2\sin\left(\frac{\pi}{3}\right)$$

$$= -\sqrt{3} \quad \text{at } x = \frac{\pi}{6}$$

$$\text{when } x = \frac{\pi}{6}, \quad y = \cos\left(\frac{2\pi}{6}\right)$$

$$= \frac{1}{2}$$

Equation of tangent

$$y - \frac{1}{2} = -\sqrt{3}\left(x - \frac{\pi}{6}\right)$$

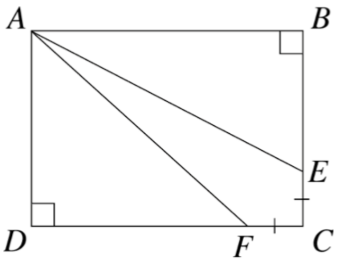
$$y - \frac{1}{2} = -\sqrt{3}x + \frac{\sqrt{3}\pi}{6}$$

$$y = -\sqrt{3}x + \frac{\sqrt{3}\pi}{6} + \frac{1}{2}$$

Question 12 (c) (i)

Criteria	Marks
• Provides correct proof	2
• Observes that $AD = AB$, with a reason, or equivalent merit	1

Sample answer:



In $\triangle ADF$ and $\triangle ABE$

$AD = AB$ (equal sides of square $ABCD$)

$\angle ADF = \angle ABE = 90°$ (equal angles of square $ABCD$)

$DF = DC - FC$ (equal sides of square $ABCD,\ FC = EC$ given)

$= BC - EC$

$= BE$

$\therefore\ \triangle ADF \equiv \triangle ABE$ (SAS)

Question 12 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds area of $\triangle ADF$, or equivalent merit	1

Sample answer:

Area square $= 14^2$
$= 196 \text{ cm}^2$

Area triangle $ABE = \frac{1}{2} \times 14 \times 10$
$= 70 \text{ cm}^2$

Area $AECF$ $= 196 - (2 \times 70)$
$= 56 \text{ cm}^2$

Question 12 (d) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$x = \frac{t^3}{3} - 2t^2 + 3t$$

$$\dot{x} = t^2 - 4t + 3$$

when $t = 0$

$$\dot{x} = 3 \text{ m/s}$$

Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to solve $v = 0$, or equivalent merit	1

Sample answer:

Need $\dot{x} = 0$

$$t^2 - 4t + 3 = 0$$

$$(t-3)(t-1) = 0$$

$$t = 3, \quad t = 1$$

Question 12 (d) (iii)

Criteria	Marks
• Provides correct solution	2
• Finds the time when $a = 0$, or equivalent merit	1

Sample answer:

$$\ddot{x} = 2t - 4$$

$$2t - 4 = 0$$

$$2t = 4$$

$$t = 2$$

when $t = 2$

$$x = \frac{2^3}{3} - 2(2)^2 + 3(2)$$

$$= \frac{2}{3} \text{ m}$$

Question 13 (a) (i)

Criteria	Marks
• Provides correct solution	3
• Finds the stationary points, or equivalent merit	2
• Attempts to solve $\frac{dy}{dx} = 0$, or equivalent merit	1

Sample answer:

$$y = 6x^2 - x^3$$
$$y' = 12x - 3x^2$$
$$y'' = 12 - 6x$$

stationary points occur when $y' = 0$

$$12x - 3x^2 = 0$$
$$3x(4 - x) = 0$$

$$3x = 0 \qquad 4 - x = 0$$
$$x = 0 \qquad x = 4$$

stationary points at (0, 0) and (4, 32)

$$\text{when} \quad x = 0, \quad y'' = 12 > 0 \quad \therefore \text{minimum at } (0,0)$$
$$x = 4, \quad y'' = -12 < 0 \quad \therefore \text{maximum at } (4,32)$$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows second derivative vanishes at $x = 2$, or equivalent merit	1

Sample answer:

Point of inflexion when $y'' = 0$

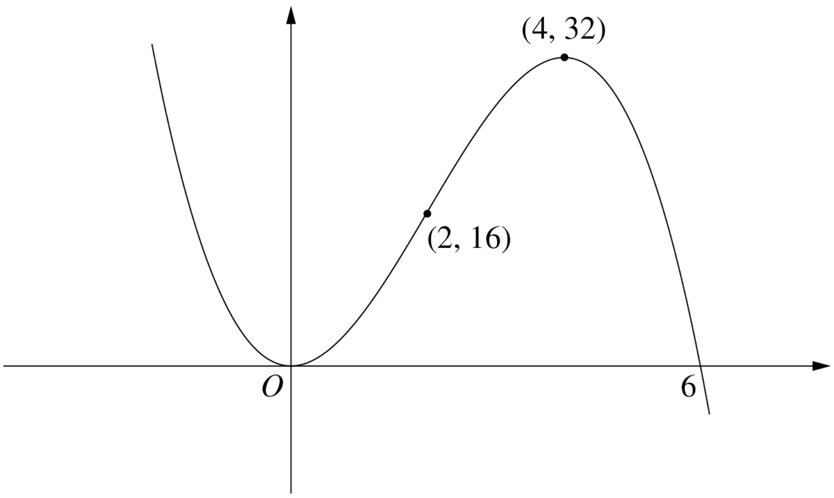
$$12 - 6x = 0$$
$$6x = 12$$
$$x = 2$$

$$\text{when} \quad x = 2 - \varepsilon, \quad y'' > 0$$
$$\text{when} \quad x = 2 + \varepsilon, \quad y'' < 0 \quad \therefore (2,\ 16) \text{ is a point of inflextion}$$

Question 13 (a) (iii)

Criteria	Marks
• Provides correct sketch	2
• Provides curve with correct shape, or equivalent merit	1

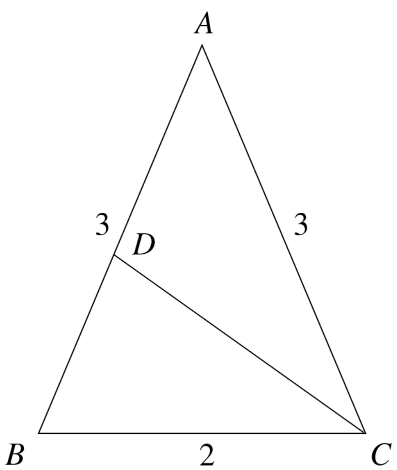
Sample answer:



Question 13 (b) (i)

Criteria	Marks
• Provides correct proof	2
• Finds one pair of congruent angles, or equivalent merit	1

Sample answer:



In $\triangle ABC$ and $\triangle CBD$ angle B is common.

Both triangles are isosceles so $\angle CDB = \angle CBD = \angle ABC = \angle ACB$ hence $\triangle ABC$ is similar to $\triangle CBD$ (AA).

Question 13 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use ratios to find a correct equation involving BD, or equivalent merit	1

Sample answer:

$AD = AB - BD$

From part (i)

$$\frac{BD}{BC} = \frac{BC}{BA}$$

$$\frac{BD}{2} = \frac{2}{3}$$

$$BD = \frac{4}{3}$$

$$AD = 3 - \frac{4}{3}$$

$$= \frac{5}{3}$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains $184 = 92e^{50k}$, or equivalent merit	1

Sample answer:

$p(t) = 92e^{kt}$

when $t = 50, \quad p = 184$

$92e^{50k} = 184$

$e^{50k} = 2$

$50k = \ln(2)$

$k = \frac{1}{50}\ln(2)$

$= 0.01386294361$

$\doteqdot 0.0139$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses $t = 110$, or equivalent merit	1

Sample answer:

When $t = 110$, $p = ?$

$$p = 92e^{110 \times 0.0139}$$
$$= 424.4476077$$
$$\doteqdot 424$$

Question 14 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$A = \frac{1}{2} \times 3 \times 6 \times \sin 60°$$
$$= \frac{9\sqrt{3}}{2} \text{ square units}$$

Question 14 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds the area of another triangle, or equivalent merit	1

Sample answer:

$$\frac{1}{2} \times 3 \times x \times \sin 30° + \frac{1}{2} \times 6 \times x \times \sin 30° = \frac{9\sqrt{3}}{2}$$
$$\frac{3x}{2} + 3x = 9\sqrt{3}$$
$$\frac{9x}{2} = 9\sqrt{3}$$
$$x = 2\sqrt{3}$$

Question 14 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains correct integral for volume in terms of *y* only, or equivalent merit	2
• Attempts to use an integral of the form $\pi \int x^2 \, dy$, or equivalent merit	1

Sample answer:

$$y = x^4 + 1$$
$$x^4 = y - 1$$
$$x^2 = \sqrt{y-1}$$

$$V = \pi \int_1^{10} x^2 \, dy$$
$$= \pi \int_1^{10} (y-1)^{\frac{1}{2}} \, dy$$
$$= \pi \left[\frac{2(y-1)^{\frac{3}{2}}}{3 \times 1} \right]_1^{10}$$
$$= \frac{2\pi}{3}\left(9^{\frac{3}{2}} - 0\right)$$
$$= \frac{2\pi}{3} \times 27$$

$$\text{Volume } = 18\pi \text{ cubic units}$$

Question 14 (c)

Criteria	Marks
• Provides correct solution	3
• Obtains correct discriminant, or equivalent merit	2
• Obtains correct derivative, or equivalent merit	1

Sample answer:

$f(x) = x^3 + kx^2 + 3x - 5$

$f'(x) = 3x^2 + 2kx + 3$

For stationary points need $f'(x) = 0$

$\triangle = (2k)^2 - 4 \times 3 \times 3$

$= 4k^2 - 36$

For no stationary points

$$\begin{aligned} \triangle &< 0 \\ 4k^2 - 36 &< 0 \\ k^2 - 9 &< 0 \\ (k+3)(k-3) &< 0 \\ \therefore -3 < k &< 3 \end{aligned}$$

Question 14 (d) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned} T_1 &= 2^1 + 1 \\ &= 3 \\ T_2 &= 2^2 + 2 \\ &= 6 \\ T_3 &= 2^3 + 3 \\ &= 11 \end{aligned}$$

Question 14 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Correctly sums one series, or equivalent merit	1

Sample answer:

$$\left(2^1+1\right)+\left(2^2+2\right)+\left(2^3+3\right)+\cdots+\left(2^n+n\right)$$

$$=\left(2^1+2^2+2^3+\cdots+2^n\right)+(1+2+3+\cdots+n)$$

For $2+2^2+2^3+\cdots+2^n$

$a=2,\ r=2,\ n=20$

$$S_{20}=\frac{2\left(2^{20}-1\right)}{2-1}$$

$$=2097150$$

For $1+2+3+\cdots+n$

$a=1,\ d=1,\ n=20$

$$S_{20}=\frac{20}{2}(1+20)$$

$$=210$$

$$\therefore \text{Total}=2097150+210$$

$$=2097360$$

Question 14 (e) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\text{Probability }=1-0.9\times0.95$$

$$=1-0.855$$

$$=0.145$$

Question 14 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Demonstrates knowledge of complementary events, or equivalent merit	1

Sample answer:

$$\begin{aligned}\text{Probability} &= 0.5 \times 0.9 \times 0.9 + 0.5 \times 0.95 \times 0.95 \\ &= 0.405 + 0.45125 \\ &= 0.85625\end{aligned}$$

Question 15 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Let $t = 0$

$$\begin{aligned}L(0) &= 12 + 2\cos\left(\frac{2\pi \times 0}{366}\right) \\ &= 12 + 2\cos 0 \\ &= 12 + 2 \\ &= 14\end{aligned}$$

Day is 14 hours long

Question 15 (a) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$-1 \le \cos\left(\frac{2\pi t}{366}\right) \le 1$$

$$-2 \le 2\cos\left(\frac{2\pi t}{366}\right) \le 2$$

Shortest length of daylight when $2\cos\left(\frac{2\pi t}{366}\right) = -2$

$L(t) = 12 - 2 = 10$ hours

Shortest length of daylight is 10 hours.

Question 15 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains $\cos\left(\frac{2\pi t}{366}\right) = -\frac{1}{2}$, or equivalent merit	1

Sample answer:

$$11 = 12 + 2\cos\left(\frac{2\pi t}{366}\right)$$

$$-1 = 2\cos\left(\frac{2\pi t}{366}\right)$$

$$-\frac{1}{2} = \cos\left(\frac{2\pi t}{366}\right)$$

$$\frac{2\pi t}{366} = \frac{2\pi}{3} \quad \text{or} \quad \frac{4\pi}{3}$$

$$\frac{\cancel{2}\cancel{\pi} t}{366} = \frac{\cancel{2}\cancel{\pi}}{3}$$

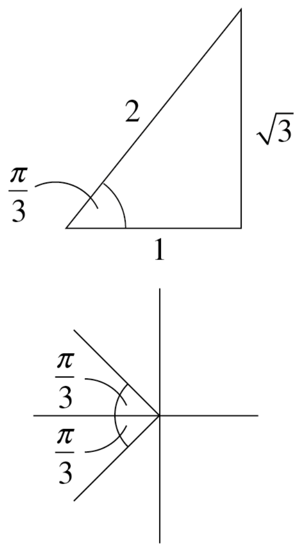
$$t = \frac{366}{3}$$

$$t = 122 \text{ days after 21 December}$$

$$\frac{2\cancel{\pi} t}{366} = \frac{4\cancel{\pi}}{3}$$

$$t = \frac{2 \times 366}{3}$$

$$= 244 \text{ days after 21 December}$$



Question 15 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains correct equation involving logarithms, or equivalent merit	2
• Obtains correct equation involving integrals, or equivalent merit	1

Sample answer:

$$\int_0^k \frac{dx}{x+3} + \int_k^{45} \frac{dx}{x+3} = \int_0^{45} \frac{dx}{x+3}$$

$$\int_0^k \frac{dx}{x+3} = \int_k^{45} \frac{dx}{x+3}$$

$$2\int_0^k \frac{dx}{x+3} = \int_0^{45} \frac{dx}{x+3}$$

$$2\Big[\ln(x+3)\Big]_0^k = \Big[\ln(x+3)\Big]_0^{45}$$

$$2\Big[\ln(k+3) - \ln 3\Big] = \ln(48) - \ln 3$$

$$\frac{(k+3)^2}{9} = 16$$

$$(k+3)^2 = 144$$

$$k+3 = \pm 12 \qquad (k>0)$$

$$k+3 = 12$$

$$k = 9$$

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains correct integral for the area, or equivalent merit	1

Sample answer:

$$A = \int_0^3 2x - \left(x^3 - 7x\right) dx$$

$$A = \int_0^3 2x - x^3 + 7x \, dx$$

$$= \int_0^3 9x - x^3 \, dx$$

$$= \left[\frac{9x^2}{2} - \frac{x^4}{4} \right]_0^3$$

$$= \left(\frac{9 \times 9}{2} - \frac{81}{4} \right) - (0 - 0)$$

$$= \frac{81}{2} - \frac{81}{4}$$

$$= \frac{81}{4}$$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses correct function values in Simpson's Rule, or equivalent merit	1

Sample answer:

$$\int_0^3 9x - x^3\,dx = \frac{3-0}{6}\left[f(0) + 4f\left(\frac{3}{2}\right) + f(3)\right]$$

$$= \frac{1}{2}\left[0 + 4\left[\frac{9\times 3}{2} - \left(\frac{3}{2}\right)^3\right] + 0\right]$$

$$= \frac{1}{2}\times 4 \times\left(\frac{27}{2} - \frac{27}{8}\right)$$

$$= 2\times\left(\frac{4\times 27}{8} - \frac{27}{8}\right)$$

$$= \not{2}\times\frac{81}{\not{8}4}$$

$$= \frac{81}{4}$$

Question 15 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains the equation $3x^2 - 7 = 2$, or equivalent merit	1

Sample answer:

P is on curve $y = x^3 - 7x$ where tangent to curve is parallel to $y = 2x$.

$\frac{dy}{dx} = 3x^2 - 7$ slope of line $= 2$

want slope of tangent to be 2

$$3x^2 - 7 = 2$$
$$3x^2 = 9$$
$$x^2 = 3$$
$$x = \pm\sqrt{3} \quad \text{want } x > 0 \text{ so}$$
$$x = +\sqrt{3}$$

when $x = +\sqrt{3}$ $\quad y = x^3 - 7x = \left(\sqrt{3}\right)^3 - 7 \times \sqrt{3}$

$$= 3\sqrt{3} - 7\sqrt{3}$$
$$= -4\sqrt{3}$$

Coordinates of P are $\left(\sqrt{3}, -4\sqrt{3}\right)$

Question 15 (c) (iv)

Criteria	Marks
• Provides correct solution	2
• Finds the distance from p to the line, or equivalent merit	1

Sample answer:

Perpendicular distance from P to $y = 2x$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|2\sqrt{3} - 1 \times \left(-4\sqrt{3}\right)|}{\sqrt{2^2 + (-1)^2}}$$

$$= \frac{|2\sqrt{3} + 4\sqrt{3}|}{\sqrt{5}}$$

$$= \frac{6\sqrt{3}}{\sqrt{5}}$$

distance from (0, 0) to (3, 6)

$$d = \sqrt{(3-0)^2 + (6-0)^2}$$

$$= \sqrt{9 + 36}$$

$$= \sqrt{45}$$

$$= 3\sqrt{5}$$

$$\text{Area of triangle} = \frac{1}{2} \times \frac{6\sqrt{3}}{\sqrt{\not{5}}} \times 3\sqrt{\not{5}}$$

$$= 9\sqrt{3}$$

Alternative solution

Take vertical distance between P and the line as base of two triangles.

When $x = \sqrt{3}, \quad y = 2x \Rightarrow y = 2\sqrt{3}$

Vertical distance is $2\sqrt{3} - -4\sqrt{3} = 6\sqrt{3}$

$$A_1 = \frac{1}{2}b \times h_1 = \frac{1}{2} \times 6\sqrt{3} \times \sqrt{3}$$

$$A_2 = \frac{1}{2}b \times h_2 = \frac{1}{2} \times 6\sqrt{3} \times \left(3 - \sqrt{3}\right)$$

$$A = \frac{1}{2} \times 6\sqrt{3} \times \left[\sqrt{3} + \left(3 - \sqrt{3}\right)\right]$$

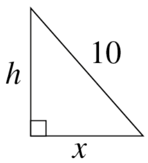
$$= 3\sqrt{3} \times 3 = 9\sqrt{3}$$

Question 16 (a) (i)

Criteria	Marks
• Provides correct proof	1

Sample answer:

$$V = \frac{1}{3}\pi r^2 h$$



$$10^2 = h^2 + x^2 \qquad ; \ \text{radius} = x$$

$$h = \sqrt{100 - x^2}$$

$$V = \frac{1}{3}\pi x^2 \sqrt{100 - x^2}$$

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Correctly applies product rule, or equivalent merit	1

Sample answer:

$$V = \frac{1}{3}\pi x^2 \sqrt{100 - x^2}$$

$$u = \frac{1}{3}\pi x^2 \qquad V = \left(100 - x^2\right)^{\frac{1}{2}}$$

$$\frac{du}{dx} = \frac{2}{3}\pi x \qquad \frac{dV}{dx} = \frac{1}{2} \times -2x \times \left(100 - x^2\right)^{-\frac{1}{2}}$$

$$= \frac{-x}{\sqrt{100 - x^2}}$$

$$\frac{dV}{dx} = \frac{-\pi x^3}{3\sqrt{100 - x^2}} + \frac{2\pi x\sqrt{100 - x^2}}{3}$$

$$= \frac{-\pi x^3 + 2\pi x\left(100 - x^2\right)}{3\sqrt{100 - x^2}}$$

$$= \frac{-\pi x^3 + 200\pi x - 2\pi x^3}{3\sqrt{100 - x^2}}$$

$$= \frac{200\pi x - 3\pi x^3}{3\sqrt{100 - x^2}}$$

$$= \frac{\pi x\left(200 - 3x^2\right)}{3\sqrt{100 - x^2}}$$

Question 16 (a) (iii)

Criteria	Marks
• Provides correct solution	3
• Verifies that $x = \frac{10\sqrt{6}}{3}$ gives the maximum, or equivalent merit	2
• Finds the values of x at the stationary points, or equivalent merit	1

Sample answer:

$$\frac{dV}{dx} = 0$$

$$\frac{\pi x\left(200 - 3x^2\right)}{3\sqrt{100 - x^2}} = 0$$

$$\pi x\left(200 - 3x^2\right) = 0$$

$$\pi x = 0 \quad , \quad 200 - 3x^2 = 0$$

$$x = 0 \qquad 3x^2 = 200$$

Not a solution, $\quad x = \pm\sqrt{\frac{200}{3}}$

$$x > 0 \qquad x > 0$$

$$x = \frac{10\sqrt{2}}{\sqrt{3}}$$

$$x = \frac{10\sqrt{6}}{3}$$

Nature ?

x	8	$\frac{10\sqrt{6}}{3}$	9
$\frac{dV}{dx}$	11.1701	0	–92.974

$\therefore$ Maximum volume when $x = \frac{10\sqrt{6}}{3}$

$\ell = r\theta$

$\ell = 10\theta$

$\therefore 2\pi r = 10\theta$

$2\pi x = 10\theta$

$\theta = \dfrac{\pi x}{5}$

Therefore

$\theta = \dfrac{\pi}{5} \times \dfrac{10\sqrt{6}}{3}$

$\theta = \dfrac{2\sqrt{6}\pi}{3}$

Question 16 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Enumerates some of the possibilities, or equivalent merit	1

Sample answer:

Cannot win if there are no numbers between the first two.

Numbers between

	1	2	3	4	5	6
1	×	×	1	2	3	4
2	×	×	×	1	2	3
3	1	×	×	×	1	2
4	2	1	×	×	×	1
5	3	2	1	×	×	×
6	4	3	2	1	×	×

So there are 16 rolls that cannot win before third roll.

Probability $= \dfrac{16}{36} = \dfrac{4}{9}$

Question 16 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Correctly deals with one case, or equivalent merit	1

Sample answer:

If 1 number between

probability of win is $\frac{1}{36} \times \frac{1}{6}$

8 cases

2 number between

probability is $\frac{1}{36} \times \frac{2}{6}$

6 cases

3 number between

probability is $\frac{1}{36} \times \frac{3}{6}$

4 cases

4 number between

probability is $\frac{1}{36} \times \frac{4}{6}$

2 cases

$$P(\text{win}) = 8 \times \frac{1}{36} \times \frac{1}{6} + 6 \times \frac{1}{36} \times \frac{2}{6} + 4 \times \frac{1}{36} \times \frac{3}{6} + 2 \times \frac{1}{36} \times \frac{4}{6}$$

$$= \frac{1}{216}(8 + 12 + 12 + 8)$$

$$= \frac{40}{216}$$

$$= \frac{5}{27}$$

Question 16 (c) (i)

Criteria	Marks
• Provides correct proof	1

Sample answer:

$$A_1 = 300\,000(1.04) - P$$

$$A_2 = A_1(1.04) - P(1.05)$$

$$= 300\,000(1.04)^2 - P(1.04) - P(1.05)$$

$$= 300\,000(1.04)^2 - P\big[(1.04) + (1.05)\big]$$

Question 16 (c) (ii)

Criteria	Marks
• Provides correct proof	1

Sample answer:

$$A_3 = A_2(1.04) - P(1.05)(1.05)$$

$$= 300\,000(1.04)^3 - P(1.04)^2 - P(1.04)(1.05) - P(1.05)^2$$

$$= 300\,000(1.04)^3 - P\Big[(1.04)^2 + (1.04)(1.05) + (1.05)^2\Big]$$

Question 16 (c) (iii)

Criteria	Marks
• Provides correct solution	3
• Sums the series for A_n, or equivalent merit	2
• Obtains an expression for A_n, or equivalent merit	1

Sample answer:

Continuing this pattern

$$A_n = 300\,000(1.04)^n - P\left[(1.04)^{n-1} + (1.04)^{n-2}(1.05)^1 + (1.04)^{n-3}(1.05)^2 + \cdots + (1.05)^{n-1}\right]$$

The second term is a geometric series with n terms, $a = (1.04)^{n-1}$ and $r = \frac{1.05}{1.04} = \frac{105}{104}$

$$= 300\,000(1.04)^n - P(1.04)^{n-1}\left[\frac{\left(\frac{105}{104}\right)^n - 1}{\left(\frac{105}{104}\right) - 1}\right]$$

$$= 300\,000(1.04)^n - P(1.04)^{n-1}\frac{\left[\left(\frac{105}{104}\right)^n - 1\right]}{\left(\frac{1}{104}\right)}$$

$$= 300\,000(1.04)^n - 104P(1.04)^{n-1}\left[\left(\frac{105}{104}\right)^n - 1\right]$$

For money to be in the account we require $A_n > 0$

$$\text{ie } 300\,000(1.04)^n > 100P(1.04)^n\left[\left(\frac{105}{104}\right)^n - 1\right]$$

$$\therefore \left(\frac{105}{104}\right)^n - 1 < \frac{300\,000}{100P}$$

$$\therefore \left(\frac{105}{104}\right)^n < 1 + \frac{3000}{P}$$

2018 HSC Mathematics Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	1.1	P3
2	1	6.7	P4
3	1	6.1	P4
4	1	6.5	P4
5	1	12.5, 13.5	H5
6	1	3.3	H5
7	1	11.4	H8
8	1	9.3	P4
9	1	10.8	H6, H7
10	1	13.3, 13.6	H5

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	1.1	P3
11 (b)	2	1.2	P3
11 (c)	2	1.3	P3
11 (d) (i)	1	7.1	H5
11 (d) (ii)	2	7.1	H5
11 (e)	2	12.5	H3, H5
11 (f)	2	8.8, 13.5	H5
11 (g)	2	8.8, 12.4	H3, H5
12 (a) (i)	1	5.4	P4
12 (a) (ii)	2	5.5	P4
12 (b)	3	6.2, 13.5	H5
12 (c) (i)	2	2.3, 2.4	P4
12 (c) (ii)	2	2.3, 2.5	H5
12 (d) (i)	1	14.3	H5
12 (d) (ii)	2	14.3	H5
12 (d) (iii)	2	14.3	H5
13 (a) (i)	3	10.2	H6
13 (a) (ii)	2	10.2	H6
13 (a) (iii)	2	10.5	H6
13 (b) (i)	2	2.3	P4
13 (b) (ii)	2	2.3	P4
13 (c) (i)	2	14.2	H3, H4
13 (c) (ii)	2	14.2	H3, H4

Question	Marks	Content	Syllabus outcomes
14 (a) (i)	1	5.3, 5.5	P3
14 (a) (ii)	2	5.3, 5.5	P3
14 (b)	3	11.4	H8
14 (c)	3	8.9, 9.1, 9.2, 10.2	P4, P5, P7, H6
14 (d) (i)	1	7.5	H5
14 (d) (ii)	2	7.5	H5
14 (e) (i)	1	3.3	H5
14 (e) (ii)	2	3.3	H5
15 (a) (i)	1	13.2	H4
15 (a) (ii)	1	13.2	H4
15 (a) (iii)	2	5.3, 13.2	H4
15 (b)	3	11.4, 12.2, 12.5	H8
15 (c) (i)	2	11.4	H8
15 (c) (ii)	2	11.3	H8
15 (c) (iii)	2	10.7	H6
15 (c) (iv)	2	6.5	P4
16 (a) (i)	1	2.3	P3
16 (a) (ii)	2	8.8	P7
16 (a) (iii)	3	10.6, 13.1	H5
16 (b) (i)	2	3.1, 3.2, 3.3	H5
16 (b) (ii)	2	3.1, 3.2, 3.3	H5
16 (c) (i)	1	7.5	H5
16 (c) (ii)	1	7.5	H5
16 (c) (iii)	3	7.5	H5