

Mathematics Advanced

HSC Marking Feedback 2020

Question 11 (a)

Students should:

- interpret given information to draw the linear function
- use initial conditions to construct the graph representing volume
- calculate the intercepts of a linear function at each axis.

In better responses, students were able to:

- draw an accurate graph using the given information
- label the graph as instructed.

Areas for students to improve include:

- using a ruler to draw linear graphs
- starting at the stated volume.

Question 11 (b)

Students should:

- interpret a direct variation relationship to model a second linear function on the same Cartesian plane
- use graphical techniques to find the point of intersection
- use algebraic techniques to solve linear equations.

In better responses, students were able to:

- interpret the given description to produce the correct graph
- represent a practical situation graphically
- construct an accurate graph to find the correct point of intersection
- read the intersection point correctly.

Areas for students to improve include:

- graphically representing practical situations
- finding the point of intersection using the graph
- developing an equation using given information
- substituting coordinates to find unknown values
- appreciating that time is positive.

Question 11 (c)

Students should:

- solve a practical problem graphically
- construct an equation to solve algebraically.

In better responses, students were able to:

- interpret the question as the cumulative volume
- solve a practical problem graphically.

Areas for students to improve include:

- understanding the graphical interpretation of linear graphs
- interpreting the question as the cumulative volume
- understanding that finding a solution graphically is dependent on the accuracy of the graph.

Question 12

Students should:

- use the Reference Sheet to write down the formulae for arithmetic terms and sums
- show substitution into formulae
- identify a and d
- find how many terms there were in the series using the last given term
- use the formula to find the sum of an arithmetic series.

In better responses, students were able to:

- use the Reference Sheet to obtain the correct formulae for arithmetic series
- identify the value of a and d
- use the n th term formula to find the number of terms in the series
- use one of the sum formulae of an arithmetic series to find the sum
- use the Reference Sheet to obtain the correct formulae for arithmetic series.

Areas for students to improve include:

- showing how to find the value of n using the first term, common difference and last term
- understanding that n needs to be an integer, greater than 4
- using general algebra skills in factorising and solving equations
- using the calculator correctly for brackets and fractions.

Question 13

Students should:

- use the Reference Sheet to find standard integrals
- understand the need for radians when calculus is involved
- show $\left[\tan x\right]_0^{\frac{\pi}{4}}$ before their substitution step
- show the substitution into their primitive.

In better responses, students were able to:

- use calculus notation correctly

- give the correct primitive function
- substitute limits into primitives correctly
- evaluate exact trigonometric ratios
- use radians in the context of trigonometric integration.

Areas for students to improve include:

- using the Reference Sheet to find standard integrals
- understanding the need to use radians for calculus involving trigonometric functions
- using exact trigonometric ratios correctly
- fully evaluating the resulting anti-derivative to give a numerical solution.

Question 14 (a)

Students should:

- label Venn diagrams clearly
- read the question carefully
- check that all numbers in the Venn diagram add to the required total
- understand probability may be written as a fraction or percentage.

In better responses, students were able to:

- construct correct, complete, labelled Venn diagrams or 2-way tables
- find that the number of students doing both History and Geography was 5
- ensure that the total number of students added to 40
- answer the question, realising that the answer to a probability question must be a number from 0 to 1.

Areas for students to improve include:

- creating Venn diagrams confidently and correctly
- ensuring that their Venn diagram (or equivalent) represents 40 students
- reading questions carefully
- answering the question asked.

Question 14 (b)

Students should:

- find the students that studied Geography only
- understand how to use the formula for conditional probability
- recognize that 'given study Geography, does not study history' means $P(\bar{H}|G)$.

In better responses, students were able to:

- understand the language of conditional probability
- extract the correct information from their Venn diagram, 2-way table or by other means
- use the conditional probability formula correctly.

Areas for students to improve include:

- applying conditional probability concepts
- interpreting that the probability required only concerned the 18 students doing Geography and not the whole cohort of 40, ie $\frac{13}{18}$ not $\frac{13}{40}$

- using Venn diagrams in preference to algebraic formulae for conditional probability whenever possible.

Question 14 (c)

Students should:

- find the probability of multiple events in questions that involve situations without replacement
- understand the multiplication required in two-step probability.

In better responses, students were able to:

- interpret the meaning of the question
- use their Venn diagram or a tree diagram and the original information given in the question to find the probabilities needed
- multiply the probabilities, displaying understanding of the language of probability.

Areas for students to improve include:

- using tree diagrams to assist with understanding if needed
- learning and applying the language of 'or' and 'and' used in probability
- interpreting questions involved probabilities without replacement
- ensuring their fractional probabilities match the totals in their Venn diagrams.

Question 15 (a)

Students should:

- interpret information given in written form
- solve a two-dimensional problem using graphical techniques
- use true bearings to find an angle measured in degrees.

In better responses, students were able to:

- show given information on the diagram provided
- transfer true bearings onto the diagram
- use supplementary and alternate angles in the diagram.

Areas for students to improve include:

- showing all working
- indicating the given information on the diagram provided.

Question 15 (b)

Students should:

- establish the cosine rule to solve practical problems involving true bearings and compass bearings
- correctly substitute side lengths and the included angle into the cosine rule.

In better responses, students were able to:

- substitute the angle found into the cosine rule
- apply the cosine rule correctly
- show all working.

Areas for students to improve include:

- evaluating expressions correctly after substitution into the cosine rule
- using the correct mode on the calculator
- substituting correctly into the cosine rule
- showing working out in the space provided
- using the Reference Sheet to obtain correct the trigonometric formula.

Question 15 (c)

Students should:

- use the sine rule correctly to solve practical problems
- use trigonometric ratios to solve problems involving true bearings
- evaluate trigonometric expressions using angles and side lengths.

In better responses, students were able to:

- use the sine or cosine rule correctly to find an angle
- calculate the bearing using the internal angle of a triangle
- use the diagram to identify the correct quadrant of the required bearing.

Areas for students to improve include:

- correctly substituting corresponding angles and sides into the sine or cosine rule
- using an internal angle to determine the corresponding true bearing.

Question 16

Students should:

- avoid only using a table of values to sketch a curve
- ensure calculus is used to find stationary points and inflection points when sketching polynomial functions
- determine the nature of all stationary point they have found
- clearly label information derived for their curve on their graph
- set working out clearly and logically.

In better responses, students were able to:

- find the first and second derivatives
- find the stationary points by setting the first derivative equal to 0 and solving
- solve a quadratic equation
- find the point of inflection by setting the second derivative equal to 0 and solving
- find y -coordinates
- determine the nature of stationary points
- prove the concavity change for the point of inflection
- draw smooth curves and label important information on the drawing
- show all working.

Areas for students to improve include:

- practising simple differentiation, factorisation and substitution
- practising solving quadratic equations with a negative leading term
- drawing large diagrams that are fully labelled, with some thought about scale

- understanding how to determine the nature of stationary points
- understanding the difference between a stationary point and a point of inflection
- understanding the difference between a point of inflection and a horizontal point of inflection.

Question 17

Students should:

- identify the question as a logarithmic integral
- rearrange the algebraic expression using $\frac{f'(x)}{f(x)}$.

In better responses, students were able to:

- manipulate the given expression into an appropriate form to integrate
- use brackets or absolute value signs in the correct position
- demonstrate their understanding that the antiderivative required a logarithmic function.

Areas for students to improve include:

- appropriately adjusting the numerator and denominator of the integral
- correctly using brackets following a logarithmic expression
- consulting the Reference Sheet to find an appropriate integral
- adding the constant term.

Question 18 (a)

Students should:

- understand and use the product rule to differentiate functions of the form $f(x)g(x)$
- use brackets around u or v when they have more than term.

In better responses, students were able to:

- recognise the function as a product
- apply the product rule by identifying u and v and their derivatives respectively
- apply the product rule with correct use of brackets
- fully factorise the derivative.

Areas for students to improve include:

- using the product rule on the Reference Sheet to differentiate
- writing each component of the product rule explicitly
- using brackets correctly to show a product involving more than one term
- fully factorising the derivative to support their attempt in the next part
- consulting the Reference Sheet for the derivative of exponential functions.

Question 18 (b)

Students should:

- appreciate that the wording of the question involves 'hence'
- recognise the relationship between integration and differentiation.

In better responses, students were able to:

- engage with part (a) to demonstrate the reciprocal combination

- find the integral using a fully factorised derivative
- manipulate the previous answer to enable working backwards to find the requested integral.

Areas for students to improve include:

- factorising completely
- identifying 'hence' as a key word requiring use of information obtained in the previous part
- understanding that questions worth one mark require a simple step.

Question 19

Students should:

- use the Reference Sheet to find the trigonometric identities
- work from the left-hand side to prove the right-hand side or vice versa
- set out their work clearly and logically
- identify the trigonometric relationship used, for example, $1 - \cos^2 \theta = \sin^2 \theta$
- manipulate fractions carefully.

In better responses, students were able to:

- identify the trigonometric relationships involved in the question, including the inverse trigonometric relationships
- establish a common denominator
- set out their work and simplify correctly
- use clear techniques to provided correct solution
- demonstrate a strong understanding of manipulating fractions and common denominators.

Areas for students to improve include:

- practising the manipulation of fractions involving trigonometric identities
- showing each step when rearranging trigonometric functions
- practising working with an expression on the left-hand side of an equation to prove that it equals the right-hand side
- understanding the difference between a trigonometric identity proof and a trigonometric equation.

Question 20

Students should:

- demonstrate understanding of equal sub-intervals in the Trapezoidal rule
- identify the correct number of function values
- substitute the strip heights correctly into the Trapezoidal rule.

In better responses, students were able to:

- identify the correct number of values to use in the Trapezoidal rule
- substitute directly into the Trapezoidal rule
- find a correct numerical expression.

Areas for students to improve include:

- determining the correct number of function values
- simplifying fractions

- understanding the substitutions necessary for the Trapezoidal rule which is found on the Reference Sheet
- using strip heights rather than t -values in the substitution.

Question 21 (a)

Students should:

- use a calculator to find a numerical solution.

In better responses, students were able to:

- show substitution of $t = 4$ into the correct equation for temperature
- state the correct rounded answer for temperature.

Areas for students to improve include:

- refraining from writing a bald answer only
- correctly writing the answer from the calculator display.

Question 21 (b)

Students should:

- differentiate exponential functions involving a base other than e
- understand the two-step solution requiring a substitution into the derivative
- differentiate a power function to find the instantaneous rate of change in a real-life situation
- state an appropriate conclusion regarding the instantaneous rate of change.

In better responses, students were able to:

- use the Reference Sheet to differentiate $70(1.5)^{-0.4t}$ correctly, showing all factors
- show substitution of $t = 4$ into $\frac{dT}{dt}$
- conclude with the correct answer for the rate.

Areas for students to improve include:

- understanding the difference between average rate and instantaneous rate
- identifying instantaneous rate as a derivative
- using the Reference Sheet to correctly differentiate exponentials with base other than e
- checking the calculator display when rounding.

Question 21 (c)

Students should:

- develop, manipulate and solve an equation involving exponential functions
- recognise and use the inverse relationship between logarithms and exponential functions in a practical context
- use the formula $\frac{d}{dx}(a^x) = (\ln a)a^x$.

In better responses, students were able to:

- make $(1.5)^{-0.4t}$ the subject correctly
- take logarithms of both sides correctly
- apply logarithmic properties efficiently

- divide by a negative value correctly.

Areas for students to improve include:

- applying order of operations in the solution of an equation involving an exponential function
- demonstrating correct use of logarithmic laws
- refraining from using guess and check
- associating the concept of a rate to a derivative
- carefully checking values from one line of working to the next.

Question 22

Students should:

- determine the length of one side of a regular decagon given the perimeter
- use the sine rule to calculate the side length of a triangle
- find the area of a triangle using the sine rule
- find the area of the decagon using multiple steps.

In better responses, students were able to:

- calculate the internal angles of the triangle
- find the perpendicular height of one triangle correctly
- use the sine rule to find the length of the triangle.

Areas for students to improve include:

- recognising that the triangles are isosceles
- rounding to one decimal place
- distinguishing between the perpendicular height and slant height of a triangle
- using correct substitutions into the sine rule.

Question 23 (a)

Students should:

- understand the relationship between probability density functions and integration
- use the Reference Sheet to find the correct anti-derivative of $\sin x$
- recognise that the sum of all probabilities of a probability density function is equal to 1, ie the definite integral is equal to 1
- know how to find k using the probability density function.

In better responses, students were able to:

- display an understanding of the definition of a probability density function
- integrate correctly
- substitute bounds correctly
- solve the trigonometric equation, and recognise the need for radians in the context of calculus.

Areas for students to improve include:

- using the Reference Sheet for probability density functions
- solving trigonometric equations
- using the Reference Sheet to correctly express the anti-derivative of $\sin x$ as $-\cos x$ as opposed to $\cos x$

- knowing that the area under a probability density function is equal to 1
- understanding the need to use radians in calculus.

Question 23 (b)

Students should:

- understand the link between continuous random variables and probability
- use calculus to find probabilities
- show all steps involved in evaluating a definite integral
- understand the need for radians in calculus.

In better responses, students were able to:

- write down the correct definite integral
- integrate and substitute bounds correctly
- evaluate correctly, using radians
- round answer to 4 decimal places correctly as stated in the question.

Areas for students to improve include:

- finding the correct definite integral
- showing the ability to integrate correctly and apply correct limits
- giving a solution correctly in radians
- understanding the meaning of $P(X \leq 1)$ as equating to evaluating the integral of the probability density function between 0 and 1.

Question 24

Students should:

- transform the equation by completing the square on both x and y terms
- reflect in the x -axis by algebraically replacing y with $(-y)$, or graphically after sketching the original circle
- sketch a reflected circle showing the centre and radius.

In better responses, students were able to:

- show all necessary working to complete the square
- state the coordinates of the centre and radius of the original circle
- substitute y in the original equation with $(-y)$, ensuring that $y^2 > 0$
- state the coordinates of the centre and radius of the reflected circle
- sketch the original circle and then the reflected circle clearly showing the centre and radius
- use appropriate scale to show suitable x - and y -intercepts
- find the horizontal width and vertical height of the circle (N-E-W-S).

Areas for students to improve include:

- reflecting in the x -axis, either graphically or by replacing y in the original equation with $(-y)$
- answering the question by including a sketch
- completing the square and drawing circles when the centre is not at the origin
- recognising that the radius is the square root of the right-hand side of the equation
- checking working if the equation involves a negative radius
- sketching circles by showing the centre and radius.

Question 25 (a)

Students should:

- use algebraic techniques to construct expressions representing area and perimeter of familiar shapes
- rearrange equations involving fractions to change the subject
- substitute the expression for y in terms of x into P
- simplify algebraically to achieve the desired result.

In better responses, students were able to:

- define variables and develop functions representing the area and perimeter of the composite shape in terms of x and y
- make y the subject and simplify this expression before substituting into P
- simplify the expression for y before substituting into P
- substitute and simplify the algebraic expressions effectively
- show all working.

Areas for students to improve include:

- finding the arc length in terms of x
- simplifying, collecting like terms, and taking a common denominator when manipulating complex algebraic equations
- deriving the area and perimeter equations in terms of π for a quadrant
- solving simultaneously to find P in terms of x
- acknowledging the 'show that' as a checking mechanism rather than forcing equations to achieve the desired result.

Question 25 (b)

Students should:

- solve optimisation problems involving perimeter
- use calculus to find the derivative
- solve the equation after equating the derivative to zero
- verify the minimum value of x by using either the first or second derivative
- substitute the value of x obtained from calculus to find the minimum perimeter
- provide reasoning to support conclusions in the given context.

In better responses, students were able to:

- rearrange an expression using index laws before differentiating
- differentiate correctly to solve $P' = 0$
- use the first derivative table using numeric values for P' to verify minimum perimeter
- calculate a positive second derivative after substituting the value of x to determine concavity verifying minimum perimeter
- answer the question by calculating the minimum P
- minimise using calculus despite being unsuccessful in part (a).

Areas for students to improve include:

- differentiating fractions involving x in the denominator
- checking calculations when the side length is negative

- explicitly including correct notation of P' and calculating numeric values in the first derivative table
- formulating relevant conclusions by stating that P is minimised when $P''(x) > 0$
- answering the question to find the minimum perimeter.

Question 26 (a)

Students should:

- understand the mathematics of a recurrence relationship
- use a calculator efficiently, including correct rounding
- substitute the values correctly to evaluate the final answer
- understand subscript notation.

In better responses, students were able to:

- calculate the value of the first and the second amount
- use the recurrence relationship correctly to calculate A_3
- use an algebraic approach to the recurrence relationship and evaluate with a calculator in the final step only.

Areas for students to improve include:

- practising using a recurrence relationship
- showing clearly set out working.

Question 26 (b)

Students should:

- remember that parts of questions are often related
- use the solution of part (a) to calculate the solution to this part
- calculate each successive year's interest and adding them.

In better responses, students were able to:

- recognise that the interest was the difference between the repayments and the reduction of the principal value
- calculate interest for each successive month
- understand that this section was not a simple interest or compound interest calculation
- obtain the balance reduced by subtracting the initial amount from their amount of money in the account immediately after the third withdrawal.

Areas for students to improve include:

- avoiding overcomplicating their thinking
- understanding the different ways of calculating interest.

Question 26 (c)

Students should:

- set up the geometric series using the first 3 terms of the series: A_1, A_2 and A_3
- write the 94th term of the series
- use the sum of a geometric series with the correct number of terms
- avoid using a calculator 94 times to find the answer.

In better responses, students were able to:

- show their working clearly to demonstrate the formation of the geometric series
- understand that the series has 94 terms to sum
- remove the common factor of 800 correctly
- use the sum of a geometric series to calculate the solution.

Areas for students to improve include:

- practising using a calculator for the sum of a geometric series
- showing clear and concise working out.

Question 27

Students should:

- read longer questions more than once, highlighting or selecting important information from the question
- not be distracted by words and phrases irrelevant to the solution of the question
- show all working in a clear and organised manner
- break down the key parts to the question.

In better responses, students were able to:

- find the value of \bar{x} and \bar{y}
- substitute the values of \bar{x} and \bar{y} into the formula provided on the exam booklet
- solve the equation after substitution to calculate the value of b
- use $x = 19$ to calculate the number of chirps and obtain the solution: chirps = 29
- round once only, at the very end of the question
- recognise that this question did not require gradient of rise over run to be calculated, nor did it require any integration.

Areas for students to improve include:

- understanding the difference between the median and mean
- practising longer, worded, un-scaffolded problems
- showing their working clearly
- finding parts of solutions to complete if finding a problem difficult
- practising solving equations and substituting correctly
- understanding the need to only round off once only at the end of a question.

Question 28 (a)

Students should:

- use the empirical results of a normal distribution
- link probability with a normal distribution
- find the probability of a given event using z -scores.

In better responses, students were able to:

- find the required z -scores
- calculate the probability of one adult within the given range of values from the normal distribution
- recognise the problem as a two-stage event

- use a tree diagram to illustrate the sample space for a two-stage event
- recognise that 'at least one' means one or more
- recognise 'at least' as a complementary event
- understand that the area above the mean in a normal distribution is $\frac{1}{2}$.

Areas for students to improve include:

- using a diagram to represent the normal distribution
- referring to the Reference Sheet for percentages between z-scores
- understanding that the normal distribution is a bell-shaped curve
- using a tree diagram for multi-stage events
- showing all steps in calculation of the required probability.

Question 28 (b)

Students should:

- use complementary events to calculate probability
- recognise that the sum of the probabilities is equal to 1
- find the probability of two simultaneous events.

In better responses, students were able to:

- interpret a ratio as a probability
- identify the mean as having a z-score of 0 and a probability of $\frac{1}{2}$
- clearly state the probabilities as given in the question
- recognise that $P(A \text{ and } B) = P(A) \times P(B)$.

Areas for students to improve include:

- recognising the difference between two-stage probability and conditional probability
- defining the independent probabilities
- writing the % sign if the value represents a percentage
- understanding the role of the mean in a probability distribution
- identifying the conjunction 'and' in a probability context to imply multiplication.

Question 29 (a)

Students should:

- differentiate a logarithmic function
- determine the gradient at a given point
- calculate the y-coordinate at a given point
- derive the equation of a tangent using the point-gradient formula.

In better responses, students were able to:

- find the correct derivative of the given log function
- calculate the gradient of the tangent at $x = p$
- find the y-coordinate at $x = p$
- use point-gradient formula to find the equation of the tangent.

Areas for students to improve include:

- identifying c as a constant when taking the derivative of the logarithmic function

- differentiating logarithmic functions
- stating the derivative in terms of x
- noting that a point coincides with a line if it satisfies the equation of that line
- substituting the x -coordinate into the derivative to find the gradient before substituting into a formula
- showing the substitution of values into the formulae.

Question 29 (b)

Students should:

- substitute a coordinate into the equation of a tangent to find an unknown value
- solve a logarithmic equation.

In better responses, students were able to:

- set the gradient from part (a) equal to 1
- write the relationship between c and p
- substitute $(0, 0)$ into the equation of the tangent
- solve an equation involving logarithms
- use simultaneous equations to solve the equation for c .

Areas for students to improve include:

- identifying the gradient in the equation of a straight line
- showing correct substitution of a point into a linear equation
- solving equations containing logarithms
- clearly writing the solution for the required variable.

Question 30 (a)

Students should:

- show their working clearly and coherently moving from one step to the next
- include many steps in their proofs
- read the question carefully to understand what result they are required to reach.

In better responses, students were able to:

- solve the quadratic equations simultaneously
- factorise carefully
- use brackets correctly
- clearly write the steps involved to 'show' the given result.

Areas for students to improve include:

- practising 'show' questions
- knowing the different methods of solving quadratic equations.

Question 30 (b)

Students should:

- know and apply each step involved in finding a definite integral
- substitute both the limits into a primitive in the correct order
- solve carefully for a , being mindful of careless algebraic errors

- check their two solutions for the value of a to ensure their final answer is valid
- factorise like terms before finding a primitive.

In better responses, students were able to:

- understand the link to part (a) in writing the correct definite integral
- understand how to find the area between two curves
- obtain a correct integral containing correct quadratic expression and integrate correctly
- substitute the correct limits, including 0
- recognise the link made through factorising out x^2 before the integration step, or factorising out x^3 before solving for a
- recognise the fact that the value of a must be positive in their final solution.

Areas for students to improve include:

- using general algebra skills in factorising and solving equations
- knowing the processes involved in evaluating a definite integral.

Question 31 (a)

Students should:

- understand that the trigonometric graph is written with radians as the unit of measurement, not degrees
- understand what the terms a and b represent in a trigonometric function
- know the correct position of amplitude in a trigonometric equation.

In better responses, students were able to:

- understand the terms amplitude and translation in the process, while linking to the sine curve
- understood the geometrical significance of a and b and read their values from the graphs
- solve simultaneous equations effectively.

Areas for students to improve include:

- taking care to avoid careless errors when finding the median point on the graph at the y -intercept
- understanding amplitude and the translation of trigonometric functions
- reading information from a graph.

Question 31 (b)

Students should:

- know how to work with trigonometric functions in term of π
- know how to find the derivative of trigonometric functions.

In better responses, students were able to:

- use the graphs to find where both populations were increasing, making a clear link to what they understood from Question 30 (a)
- find both $c'(t)$ and $m'(t)$ and interpret their application to the term 'increasing'
- apply an understanding of where the stationary points on a trigonometric graph exist (for sine and cosine curves), and link these to the correct notation for displaying an inequality

- use inequalities correctly to express the solution, understanding the difference between 'less than' and 'less than or equal to'.

Areas for students to improve include:

- understanding the trigonometric graphs and how to draw and interpret them
- understanding the rules for finding the derivative of trigonometric functions
- understanding the difference between differentiation and integration and their uses in solving problems.

Question 31 (c)

Students should:

- show numerical results when completing the test for the maximum or minimum
- appreciate rate of change as the first derivative of $m(t)$
- accurately substitute and evaluate expressions
- show their substitution step.

In better responses, students were able to:

- find the correct derivative of $m'(t)$ and substitute accordingly
- link their understanding of what they were being asked to do in parts (a) and (b)
- find $t = 36$ as the maximum value and the substitute into $m'(t)$
- reach a correct maximum value through calculation relating to a derivative, or through sketching the correct trigonometric graph
- draw a graph to show the population of cats and then found correct inequalities.

Areas for students to improve include:

- understanding the rules for differentiation of complex trigonometric functions
- drawing clear graphs.