Mathematics Advanced

HSC Marking Feedback 2021

Question 11

Students should:

- know how to solve equations involving fractions
- find a common denominator or multiply correctly to eliminate the denominator
- apply inverse operations to solve the equation.

In better responses, students were able to:

- establish a common denominator or multiply each term by 2
- carefully demonstrate each step of their solution
- use substitution to check a solution.

Areas for students to improve include:

- correctly finding the common denominator of algebraic fractions
- working with equations which have two terms in the numerator of a fraction
- applying inverse operations to solve equations.

Question 12 (a)

Students should:

- use SOHCAHTOA in right-angled triangles
- establish the correct trigonometric ratio to use
- evaluate trigonometric expressions using angles and side lengths
- use a calculation to find a numerical solution in degrees.

In better responses, students were able to:

- identify the correct trigonometric ratio and substitute the values and pronumerals correctly into the ratio
- manipulate the ratio to calculate the required unknown side length
- use the sine rule to calculate the length of XY
- have their calculator in degree mode.

Areas for students to improve include:

- using the Reference Sheet to ensure that the trigonometric ratio is correct
- identifying which side is the hypotenuse and which is the adjacent
- showing adequate steps in their working to reflect their reasoning.

Question 12 (b)

Students should:

- find the radius of the semi-circle
- find the area of the semi-circle
- find the area of the triangle
- find the shaded area by calculating the area of the semicircle minus the area of the triangle.

In better responses, students were able to:

- demonstrate all calculations necessary to find the solution
- show working for lengths not provided in the diagram but used in their solution
- identify the perpendicular sides of ΔXYZ.

Areas for students to improve include:

- identifying the correct area formulae supplied on the Reference Sheet
- recognising the difference between a semicircle, a segment, a sector, to correctly calculate the area
- recognising the perpendicular sides of a triangle when using $A = \frac{1}{2}bh$, or the correct angle when using $A = \frac{1}{2}ab\sin C$
- carrying all digits through to the final result before rounding.

Question 13

Students should:

- use the product rule to differentiate a product involving a trigonometric function
- show the substitution of $\frac{\pi}{3}$ into the derivative
- evaluate the derivative in exact form to give the gradient of the tangent.

In better responses, students were able to:

- understand that $y = x\tan x$ is a product of two functions of x
- show u, u', v, v'
- show the substitution of $\frac{\pi}{3}$ into their derivative
- use $\sec^2 x = \frac{1}{\cos^2 x}$
- understand that exact answers may contain surds and π
- evaluate $\frac{1}{\left(\frac{1}{2}\right)^2}$.

Areas for students to improve include:

- recognising when to use the product rule
- being familiar with radians and the exact trig values for them
- finding the exact value of the square of a reciprocal trigonometric ratio
- working with fractions with a fractional denominator
- using radians confidently in the context of trigonometric calculus
- using the Reference Sheet to find correct derivatives and trigonometric ratios
- showing all working, particularly the substitution step.

Question 14

Students should:

- select the equation from the Reference Sheet for the sum of an arithmetic series
- substitute values from the question into the formula to form an equation
- solve the equation to find the common difference.

In better responses, students were able to:

- identify the correct variables and substitute into the correct formulae
- solve the equation efficiently, showing all necessary steps.

Areas for students to improve include:

- knowing the terminology for the components of an arithmetic series
- using general algebra skills in expanding brackets and solving equations.

Question 15

Students should:

- express the integrand using index notation
- use the Reference Sheet to find standard integrals
- show the substitution into their primitive
- use a calculator to find a simplified numerical solution
- differentiate the anti-derivative to check that it matches the given integrand.

In better responses, students were able to:

- correctly use calculus notation
- express the integrand in terms of a fractional power
- correctly substitute limits into primitives
- evaluate numerical expressions involving a fractional index
- evaluate using a calculator.

Areas for students to improve include:

- appropriately manipulating the numerator and denominator of the integral
- distinguishing between the chain rule and the reverse chain rule
- integrating functions involving roots and fractional indices
- simplifying fractions involving a fractional denominator

- explicitly showing substitutions into the anti-derivative with the use of brackets
- evaluating the resulting anti-derivative to give the fully simplified numerical solution
- developing a habit of checking the anti-derivative of integrals by differentiating.

Question 16

Students should:

- use calculus to determine the first derivative
- state that $f(x)$ is increasing when $f'(x) > 0$
- solve the resulting quadratic inequation by graphing or using a sign diagram
- set working out clearly and logically
- state the domain.

In better responses, students were able to:

- state the condition for an increasing function
- find the first derivative
- factorise the derivative to find the x-intercepts
- solve the quadratic inequation by graphing a concave down parabola
- solve the quadratic inequation by testing critical points on the number line
- find the domain using set notation or interval notation.

Areas for students to improve include:

- differentiating cubic functions
- recognising that the first derivative corresponds to the gradient of a function
- associating an increasing function with $f'(x) > 0$
- solving quadratic equations and inequations with a negative leading term
- distinguishing between the graph of a function and the signs of $f(x)$, $f'(x)$ and $f''(x)$
- expressing the domain using interval notation or inequality signs.

Question 17 (a) (i)

Students should:

- establish the meaning of the question by reading carefully
- substitute the given value into the correct equation
- use a calculator to find the solution and round correctly.

In better responses, students were able to:

- show their substitution into an equation
- show an approximate answer
- show a correctly rounded answer.

Areas for students to improve include:

- taking care with substitution
- taking care with rounding

- refraining from writing a bald answer only.

Question 17 (a) (ii)

Students should:

- use the context to describe both the rise and the run of the gradient
- use the given graph for guidance
- write concisely and legibly.

In better responses, students were able to:

- describe the relationship between the variables using the names of the variables in the given context and the values of the rise and the run
- give clear indication of both data sets increasing and decreasing according to the graph
- give specific value changes.

Areas for students to improve include:

- using the correct terms to describe the relationship between the variables
- recognising the difference between gradient and correlation
- comprehending the information given in both written and mathematical forms to address the question effectively.

Question 17 (b)

Students should:

- draw a conclusion and justify it using the information provided about correlation.

In better responses, students were able to:

- compare the strength of the correlation of the graphs in (a) and (b)
- understand that $r = -0.897$ represents a stronger relationship than $r = -0.494$
- keep explanations simple.

Areas for students to improve include:

- using the appropriate terms in the right context
- providing a clear and succinct mathematical justification for their conclusion
- drawing on the information provided in the question
- understanding that questions worth one mark require a simple reason.

Question 18

Students should:

- use the sine ratio to find an angle in a triangle
- show substitution into the sine rule
- calculate the acute angle
- apply the ambiguous case to find the obtuse angle.

In better responses, students were able to:

- demonstrate substitution into the sine rule
- use $\frac{\sin A}{a} = \frac{\sin B}{b}$ to find an angle
- use a calculator to find the acute angle
- calculate the obtuse angle by using $\sin \theta = \sin(180 - \theta)$.

Areas for students to improve include:

- selecting the appropriate trigonometric ratio
- applying trigonometric ratios of angles of any magnitude
- solving an equation involving fractions.

Question 19

Students should:

- calculate the intercepts of the hyperbolic function on the x- and y-axis
- identify the horizontal and vertical asymptotes
- carefully sketch the hyperbolic graph showing the asymptotes and intercepts.

In better responses, students were able to:

- draw a hyperbolic graph labelling intercepts and asymptotes
- sketch a hyperbolic shape with the branches approaching the asymptotes
- demonstrate an understanding of vertical and horizontal shift of a basic hyperbola.

Areas for students to improve include:

- labelling essential features on a graph
- carefully drawing the hyperbolic branches without crossing the asymptotes
- taking care to avoid arithmetic errors with positive and negative values
- using a ruler to draw the axes and asymptotes on a sufficiently large diagram.

Question 20

Students should:

- find the intersecting points between two functions
- consider the given domain has two solutions
- answer using radian measure.

In better responses, students were able to:

- form the equation $2 \sin 4x = 1$
- solve for $4x$ in the domain $0 \leq 4x \leq \pi$
- solve for x in the domain $0 \leq x \leq \frac{\pi}{4}$
- solve the trigonometric equation within the specified domain
- show all working to find the related angles.

Areas for students to improve include:

- finding solutions satisfying the given domain
- appreciating that the question is asking for values of x rather than the number of solutions
- developing confidence in working entirely in radian measure when solving trigonometric equations
- appreciating that when a question refers to a graph, it does not necessarily imply that a graphical solution is required
- understanding that sketching a graph will find the number of solutions.

Question 21

Students should:

- identify that a horizontal and a vertical dilation are required
- deduce that $y = f(2x)$ dilates the function horizontally with scale factor $\frac{1}{2}$
- deduce that $y = 4f(x)$ dilates the function vertically with scale factor 4
- sketch a cubic graph showing the transformed x-intercepts and turning points.

In better responses, students were able to:

- understand the geometrical significance of the 4 and the 2 in the new function
- calculate the horizontal and vertical dilation
- appreciate that the origin is maintained as a maximum turning point after dilations
- compress the cubic curve by factor $\frac{1}{2}$ to transform $(6, y)$ to $(3, y)$
- stretch the cubic curve by factor 4 to transform $(x, -8)$ to $(x, -32)$
- draw successive graphs representing progressive transformations
- use appropriate scale to show x- and y-values
- draw a continuous cubic curve and label turning points on the graph.

Areas for students to improve include:

- separating the transformation into horizontal and vertical components, rather than attempting both simultaneously
- distinguishing between dilations and translations in the horizontal and vertical directions
- understanding horizontal dilations affect the abscissa
- understanding vertical dilations affect the ordinate
- systematically focusing on the critical points provided in a graph
- labelling critical points on a graph drawn not to scale
- developing a systematic method for applying transformations to sketch functions
- developing an understanding of the manipulation of simple functions by using graphing software.

Question 22 (a)

Students should:

- read the whole question carefully to understand what is represented by the data in the given table
- use the given information to calculate the desired probability
- understand the relationship between the probability of two z-scores and the position of the z-scores on the bell curve
- subtract the probability value for a z-score of 0.1 from the probability value of a z-score of 0.5 using the table provided.

In better responses, students were able to:

- understand that the shaded area represents the probability that a random variable lies between the mean and the z-score
- understand how probabilities interact with z-scores
- identify the correct probability values to use from the table
- use the given graph depicting the normal distribution to find the difference between the two probabilities.

Areas for students to improve include:

- connecting the probability between two z-scores and the position of the z-scores on the bell curve
- understanding that the probability between two z-scores requires a subtraction
- practising questions with z-scores and the normal distribution to be familiar with the language used in probability questions
- appreciating that worded questions provide necessary information
- reading and understanding worded questions that examine course content using simple scenarios
- understanding that questions worth one mark require a simple step.

Question 22 (b)

Students should:

- use the Reference Sheet to find the z-score formula
- calculate the z-score
- use the z-score to find the corresponding probability from the given table
- understand the connection between the results provided in the table and the shaded area under the normal distribution bell curve
- draw the bell curve to understand the value of their probability in terms of the position of the mean
- find the expected number by multiplying the probability by 1000.

In better responses, students were able to:

- identify each variable in the question: x, σ, μ
- calculate the z-score
- generate a probability from a z-score given a table of probabilities

- recognise that the right side of the normal distribution represents the required probability
- exclude half the scores below the mean
- multiply the probability by 1000.

Areas for students to improve include:

- calculating a z-score
- understanding what each variable (x, σ, μ) represents
- equating a normal distribution to a continuous probability distribution that is symmetrical on both sides of the mean
- connecting z-scores with probability of scores being above or below that z-score
- understanding what is represented by the empirical rule and the area under a normal distribution curve
- linking probability to the area under a normal distribution bell curve
- avoiding the empirical rule when presented with a table of probabilities
- reading the question again to check if it has been answered.

Question 23

Students should:

- substitute the given values into the equation and solve to find the value of b
- differentiate the resulting equation
- set $\frac{dP}{dt}$ equal to -30 and solve by applying the logarithmic laws appropriately to solve for t.

In better responses, students were able to:

- demonstrate excellent command of index rules to calculate b
- accurately find the derivative of $P = 5000(2)^{-\frac{t}{10}}$
- appreciate the need to set $\frac{dP}{dt}$ equal to -30 as the rate was decreasing
- apply the logarithmic laws appropriately to solve for t.

Areas for students to improve include:

- familiarising themselves with the derivative of $f(x) = ab^x$ where $b \neq e$
- identifying that a decrease means that the value of the differential is negative
- solving equations with negative indices
- improving skills in manipulating logarithms and solving logarithmic equations.

Question 24

Students should:

- determine the area of a shaded region by solving indefinite integrals
- divide the shaded region into two shapes
- calculate the area of each shape using integration or geometric ideas
- label the diagram to assist with calculations.

In better responses, students were able to:

- interpret the shaded area as a combination of a triangle and area under a curve
- integrate the hyperbolic function between $x = 2$ and $x = 4$ to obtain the logarithmic expression
- substitute upper and lower boundaries into the logarithmic expression
- add the two areas to find the area of the shaded region.

Areas for students to improve include:

- understanding the primitive function of $y = \frac{3}{x-1}$ is logarithmic
- showing substitution of upper and lower boundaries into the anti-derivative.

Question 25

Students should:

- identify the interest rate per period from the table of future values
- apply the compound interest formula
- convert a percentage into a decimal.

In better responses, students were able to:

- identify the correct interest rate of 8.2132 from the table
- multiply this rate by $\$1000$
- deduce that the final two years did not require a second interest rate from the table
- apply two years of compound interest at 1.25% per annum.

Areas for students to improve include:

- practising using a future value table
- avoiding generating a financial series when a future value table is provided
- distinguishing between an annuity and compound interest
- converting percentage rates to a decimal.

Question 26 (a)

Students should:

- find $y'(t)$ and solve $y'(t) = 0$ to find the time when maximum height occurs
- or use $t = \frac{-b}{2a}$ to find time of max height
- substitute the found value of t into $y(t)$ to find maximum height.

In better responses, students were able to:

- interpret the question and perform the algebra needed to produce the correct solution
- find time from a derivative and substitute it into the original equation to find the maximum height
- show clear working and substitution.

Areas for students to improve include:

- using a calculator to evaluate substitutions
- practising calculus differentiation

- understanding that maximum height occurs when $y'(t) = 0$
- reading the question carefully and understanding that finding t is a step towards the solution, not the required answer.

Question 26 (b)

Students should:

- solve $y(t) = 0$ to find the times the particle will hit the ground
- understand that time must be positive
- substitute the exact value of t into $y'(t)$ to find velocity
- simplify, leaving the answer in simplified surd form.

In better responses, students were able to:

- simplify and use surds carefully
- understand that solving $y(t) = 0$ would provide the time immediately before hitting the ground
- substitute into the quadratic formula to determine value of t, recognising $t > 0$
- correctly substitute into derivative and expanded brackets to get correct answer
- recognise they needed to keep the square root answer for t, as the question requested, rather than changing to a decimal/approximation.

Areas for students to improve include:

- taking greater care when applying the quadratic formula, simplifying surds and expanding expressions
- practising differentiation, factorisation and substitution
- practising various methods of solving quadratic equations.

Question 27 (a)

Students should:

- state the amplitude of the function is 400
- calculate the period of the function is 24 hours
- draw a sine curve starting at the origin
- label the amplitude and related time.

In better responses, students were able to:

- sketch a half-period of the curve showing the amplitude
- label horizontal and vertical axes
- sketch a smooth curve depicting a sine wave
- use the space provided to sketch a sufficiently large graph.

Areas for students to improve include:

- sketching a sine curve with correct concavity
- finding values for time in radian measure
- developing an understanding of period and amplitude when sketching trigonometric functions.

Question 27 (b)

Students should:

- set up the integral as $\int_a^b 400\sin\left(\frac{\pi t}{12}\right)dt$
- use the Reference Sheet as a guide to set up the integration step
- state the additional coefficient of $-\frac{12}{\pi}$
- show the order of substitutions of the upper and lower boundaries.

In better responses, students were able to:

- rearrange the integral $\int_a^b 400\sin\left(\frac{\pi t}{12}\right)dt = 400\int_a^b \sin\left(\frac{\pi t}{12}\right)dt$
- find the correct primitive function
- show the substitution of a and b.

Areas for students to improve include:

- showing every step of working
- understanding integration result is the primitive function value after substitution of limits
- using the formula $\int_a^b f(x)dx = F(b) - F(a)$
- taking care when multiplying and dividing with fractions.

Question 27 (c)

Students should:

- substitute $a = 3$ and $E = 300$ into the equation in part (b)
- solve the trigonometric equation using radians
- answer to the nearest minute.

In better responses, students were able to:

- substitute the correct values for a and E into the equation given in part (b)
- perform arithmetic and trigonometric calculations to find $b = 3$ hours 57 minutes
- subtract 3 hours to find the minimum waiting time.

Areas for students to improve include:

- carefully reading the question to identify the appropriate substitutions
- equating the rate to $P(t)$ and the energy to $E(t)$
- developing skills to solve resulting equations involving multiple trigonometric and arithmetic calculations
- practising algebraic manipulations of trigonometric equations
- using exact trigonometric ratios and simplified fractions when solving complex trigonometric equations.

Question 27 (d)

Students should:

- interpret their graph of $P(t)$ from part (a) to formulate an answer
- state that $P(t)$ has a maximum charging capacity when $t = 6$ hours, which is more than the charging power at $t = 3$ hours.

In better responses, students were able to:

- refer to the graph of $P(t)$ to indicate a maximum power of 400 when $t = 6$ hours
- conclude that it would take less time to charge at $t = 3$
- use the maximum turning point at $t = 6$ to compare it to the answer found in part (c).

Areas for students to improve include:

- referring to the original graph to compare the times
- stating a reason to support their answer in relation to the graph
- referencing the power in the graph rather than the sun.

Question 28 (a)

Students should:

- find the x-intercept for the function
- write the correct definite integral representing the area under the curve
- use the Reference Sheet when integrating a^x
- calculate the area by careful substitution of the limits into the anti-derivative
- check that their final answer matched the given value.

In better responses, students were able to:

- find the x-intercept efficiently without the need of logarithms
- able to use the Reference Sheet correctly to find the integral of base 2
- show all the steps in their working including the substitution of limits using brackets.

Areas for students to improve include:

- becoming more familiar with the integral other than base e offered on Reference Sheet
- using brackets as part of their worked solution of definite integrals
- knowing the processes involved in evaluating definite integrals.

Question 28 (b)

Students should:

- demonstrate an understanding of the reflections associated with replacing x with $-x$ and y with $-y$
- demonstrate an understanding of translation to the right
- rule and label the axes
- clearly show the x-intercept and asymptote
- draw their graph carefully, taking up one third of a page.

In better responses, students were able to:

- correctly apply all the required transformations
- draw a small sketch for each of the transformations, then draw a finally sketch
- ensure their sketch shows all requested detail.

Areas for students to improve include:

- familiarising themselves with the transformations associated with changes to a function
- showing the x-intercept and asymptote and labelling their axes
- drawing clear graphs, making sure the graph extends past the axes
- including the asymptote in the transformations.

Question 28 (c)

Students should:

- recognise the question required the integral not area
- see the area in parts a and c are identical but just under the x-axis
- find the negative of the area from part (a)
- understand that 'hence' implies the use of previous parts of a question.

In better responses, students were able to:

- recognise that the answer was the negative of part (a)
- recognise that the amount of writing space and the 1-mark value means that there is not a lot of work involved.

Areas for students to improve include:

- understanding the difference between area and integrals
- understanding the difference between exact value and absolute value
- understanding that 'hence' means use a previous answer.

Question 29 (a)

Students should:

- show the development of calculations for A_1, A_2 and A_3
- ascertain if deposits are made at the beginning or end of the year
- calculate the values for A_1, A_2 and A_3.

In better responses, students were able to:

- identify that the first payment of $1000 did not accrue interest in the first year
- find the value of $A_1 = \$6150$
- apply the recurrence relation to generate $A_2 = A_1(1.03) + 1000$ before substituting A_1
- apply the recurrence relation to generate $A_3 = A_2(1.03) + 1000$ before substituting the expression for A_2.

Areas for students to improve include:

- differentiating between payments made at the beginning or end of a period
- developing a sequential pattern starting with A_1
- understanding a 'show' question requires systematic working out for A_1, A_2 and A_3.

Question 29 (b)

Students should:

- manually construct a reverse annuity with regular, equal contributions and interest compounding at the end of each period
- generate a series involving the first three terms of an annuity
- sum the resulting series using the geometric series formula
- resolve the equation to a zero balance
- solve the resulting logarithmic equation.

In better responses, students were able to:

- generate the first three amounts to find the series A_n
- use their series to create a general series for n terms
- use the substitution $A_n = 0$
- solve an exponential equation using algebraic properties of logarithms.

Areas for students to improve include:

- relating a financial question involving regular deposits or withdrawals will generate a series with n terms
- following an established pattern to create a geometric series
- carefully identifying the number of terms in a geometric series
- solving complex exponential and logarithmic equations.

Question 30

Students should:

- understand the meaning of a cumulative distribution function
- extract the necessary information from the question for substitution into the cumulative distribution function
- solve the resulting exponential equation using logarithms.

In better responses, students were able to:

- write down a correct exponential equation using the information given
- solve the exponential equation correctly and efficiently using logarithms.

Areas for students to improve include:

- understanding of cumulative distribution functions in a practical context
- solving exponential equations
- recognising the difference between $F(x)$ and $f(x)$
- reading of questions.

Question 31

Students should:

- find the derivative function
- find the gradient at $x = a$
- use point gradient formula to find equation of tangent
- substitute $(3, -8)$ into equation and solve for a
- substitute both values of a into equation of tangent to find the two equations required.

In better responses, students were able to:

- draw a diagram of the information
- clearly show each step of working
- show correct calculations for the gradient and coordinate substitution
- recognise the tangent equation in its different forms.

Areas for students to improve include:

- understanding what a tangent to a curve is
- drawing a sketch or model to visualise the problem
- demonstrating substitution steps clearly
- working with tangents using variables
- practising general algebra skills including expansions, brackets and simplifications
- practising simple differentiation, factorisation and substitution
- practising solving quadratic equations using factorising, completing the squares, and using the quadratic formula
- understanding how to use calculus to find the gradient of the tangent at any given point
- understanding how to use the point-gradient formula to find an equation.

Question 32

Students should:

- convert the female heights to z-scores
- understand how to use the empirical rule
- position the female heights on a normal distribution curve
- calculate the mean of female heights using the required number of standard deviations
- calculate the mean and standard deviation of male heights using the relationship given in the table
- use the mean and standard deviation to calculate the height of the selected male.

In better responses, students were able to:

- use Table 1 to calculate z-scores
- label the normal distribution curve with the female heights
- find the mean of female heights
- use Table 2 to calculate the mean and standard deviation of male heights
- identify that the selected male is one standard deviation above the mean height for males

- add the correct amount of standard deviations to the male mean.

Areas for students to improve include:

- using the empirical rule on the Reference Sheet to aid calculation
- comparing z-scores of different data sets to understand the connection between z-scores, μ and σ
- applying the empirical rule to a variety of problems
- practising worded problems to focus on key words
- checking the reasonableness of their answers.

Question 33 (a)

Students should:

- state the property of a probability density function $\int_0^6 f(x)dx = 1$
- correctly integrate $\int_0^6 \frac{Ax}{x^2+4} dx = \left[\left(\frac{A}{2}\right) \ln(x^2+4)\right]_0^6$ and set it equal to 1
- show the substitution of 6 and 0 into the integral
- solve the equation to find A, clearly showing the use of the logarithmic laws.

In better responses, students were able to:

- understand the relationship between probability density functions and integration
- clearly state that the integral is equal to 1
- use brackets in the expression for $\ln(x^2+4)$ so that further substitution was accurate
- write an integral using fractions, and to maintain it in later steps
- show that $\ln 40 - \ln 4 = \ln \frac{40}{4} = \ln 10$.

Areas for students to improve include:

- knowing that the area under a probability density function is equal to 1
- using the Reference Sheet to identify the correct integral
- integrating logarithms
- showing all working in a clear and organised manner.

Question 33 (b)

Students should:

- recognise that the mode is the global maximum of a probability density function
- find the derivative of the function using the quotient rule
- solve the derivative equal to 0 to find the x value that gives the maximum value of the function.

In better responses, students were able to:

- use the quotient rule correctly and efficiently, without ignoring the constant
- recognise A was a constant and could multiply the differential and could be left as A until simplified
- find the maximum point by solving the equation with the numerator of the derivative equal

to zero.

Areas for students to improve include:

- ensuring that the 'A' value is included in the differential
- keeping both the constant and the denominator when using the quotient rule
- showing u, u', v, v' and showing substitution into the quotient rule
- solving equations with fractions equal to zero
- showing all steps of working to 'show' answer.

Question 33 (c)

Students should:

- recognise the link between this question and part (a)
- use the probability density function and the value of A to evaluate the integral from 0 to 2
- use integration resulting in a natural logarithmic function
- evaluate the integral from 0 to 2
- use logarithmic laws to simplify
- use change of base rule for logarithms.

In better responses, students were able to:

- write the definite integral correctly
- evaluate the definite integral correctly
- use logarithmic laws to simplify the definite integral correctly
- demonstrate the change of base law for $\frac{\ln 2}{\ln 10} = \log_{10} 2$.

Areas for students to improve include:

- understanding of probability density functions and integration
- evaluating definite integrals
- using logarithmic laws.

Question 33 (d)

Students should:

- find a z-score by interpreting the question using the information given
- use knowledge of the empirical approximations for normal functions
- identify conditional probability $P(IQ > 130|\text{completes in} < 2 \text{ hours}) = \frac{P(IQ>130 \cap X<2)}{P(X<2)}$
- find the probability needed for the numerator (80% of 2.5%)
- recognise the need to use the value given in part (c) for the denominator of the conditional probability fraction.

In better responses, students were able to:

- recognise the question involves conditional probability
- use a diagram as an aid to find the percentage of the population above a z-score of 2
- find the probability of two events occurring

- take and apply the value found in part (c) to conditional probability.

Areas for students to improve include:

- using the empirical values of normal distributions correctly
- recognising conditional probability statements
- applying probability in a complex situation involving conditional probability.

Question 34

Students should:

- use the Reference Sheet to write down the formula for $E(X)$
- fully simplify the series $E(X)$
- equate the expression for the sum of probabilities $P(X = x) = 1$
- express the sum of the probabilities as a geometric sequence
- manipulate algebraic fractions carefully
- make the connection between the expression for the sum of the probabilities and $E(X)$ to show the desired result.

In better responses, students were able to:

- use $E(X) = np$ to generate a series
- recognise the pattern to simplify $E(X) = nr^{n+1}$
- equate the algebraic sum of the probabilities to 1
- use the sum of a geometric sequence to show that $r^{n+1} = 2r - 1$
- rearrange an algebraic expression
- link both equations and connect to show required result.

Areas for students to improve include:

- familiarising with the Reference Sheet to write down the geometric sequence formula
- understanding the difference between $E(x) = E(x)p(x)$ and the arithmetic mean
- understanding that the sum of probabilities is equal to 1
- distinguishing between an arithmetic and geometric series
- distinguishing between a geometric series with and without a limiting sum
- reversing a geometric series to simplify the substitutions in the formula
- fully simplifying algebraic expressions
- working with index laws to simplify patterns
- appreciating that the question is asking for 'show that'
- showing all necessary steps in a 'show' question.