NSW GOVERNMENT

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–40)

- Attempt Questions 11–34
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following is equivalent to $\sin^2 5x$?

A. $1 + \cos^2 5x$

B. $1 - \cos^2 5x$

C. $-1 + \cos^2 5x$

D. $-1 - \cos^2 5x$

2 The probability distribution table for a discrete random variable X is shown.

x	$P(X = x)$
1	0.6
2	0.3
3	0.1

What is the expected value of X?

A. 0.6

B. 1.0

C. 1.5

D. 2.0

3 Which of the following represents the domain of the function $f(x) = \ln(1 - x)$?

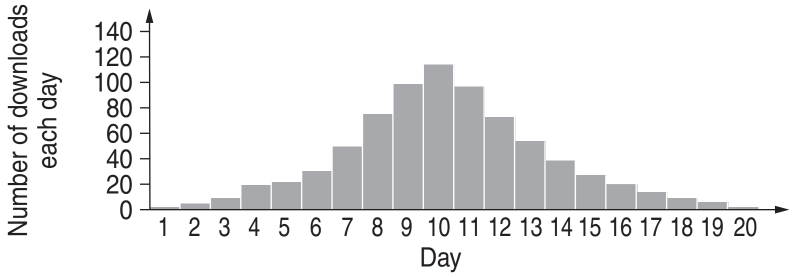
A. $[1, \infty)$

B. $(1, \infty)$

C. $(-\infty, 1]$

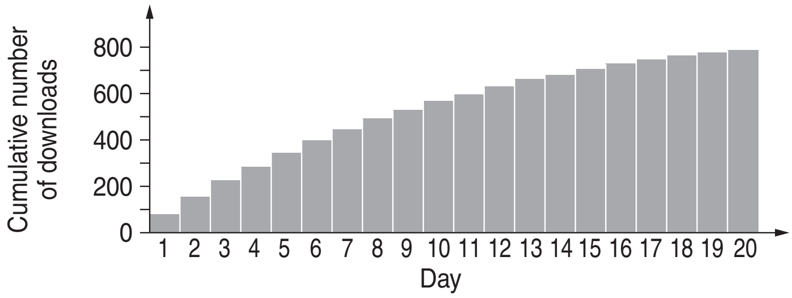
D. $(-\infty, 1)$

4 The number of downloads of a song on each of twenty consecutive days is shown in the following graph.



Which of the following graphs best shows the cumulative number of downloads up to and including each day?

A.



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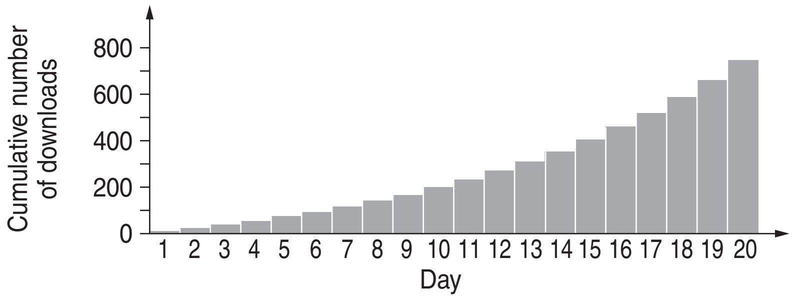

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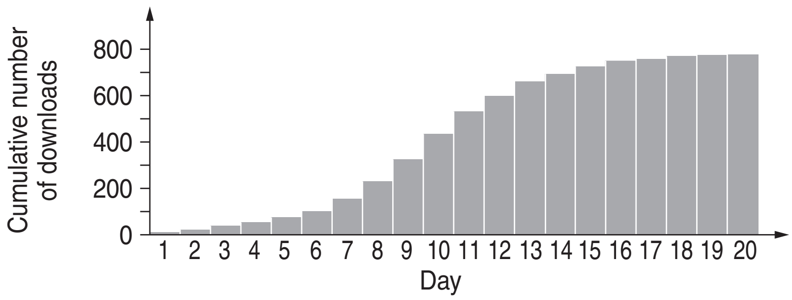
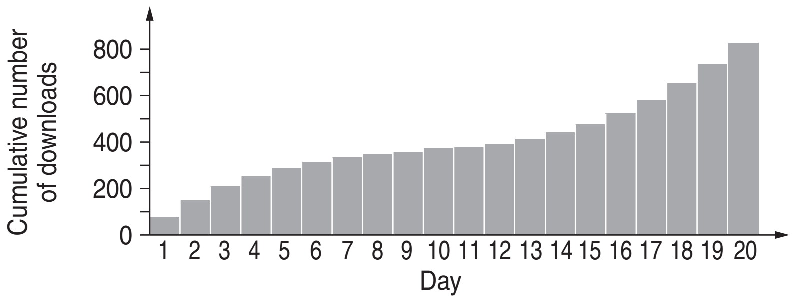
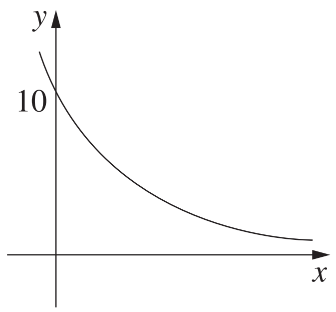


D.

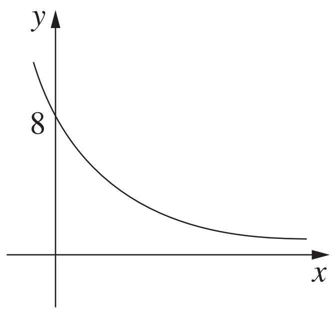


5 Which of the following best represents the graph of $y = 10(0.8)^x$?

A.



B.



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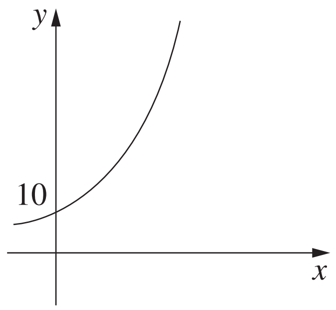

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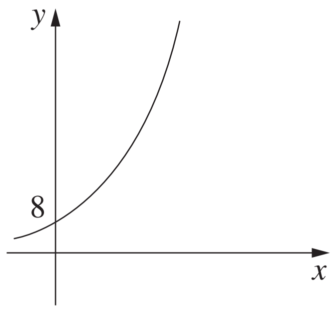
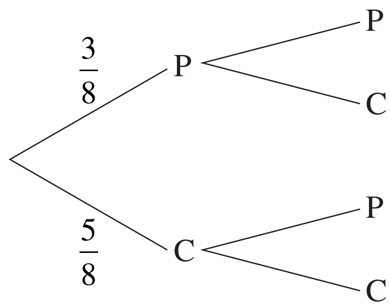


6 There are 8 chocolates in a box. Three have peppermint centres (P) and five have caramel centres (C).

Kim randomly chooses a chocolate from the box and eats it. Sam then randomly chooses and eats one of the remaining chocolates.

A partially completed probability tree is shown.



What is the probability that Kim and Sam choose chocolates with different centres?

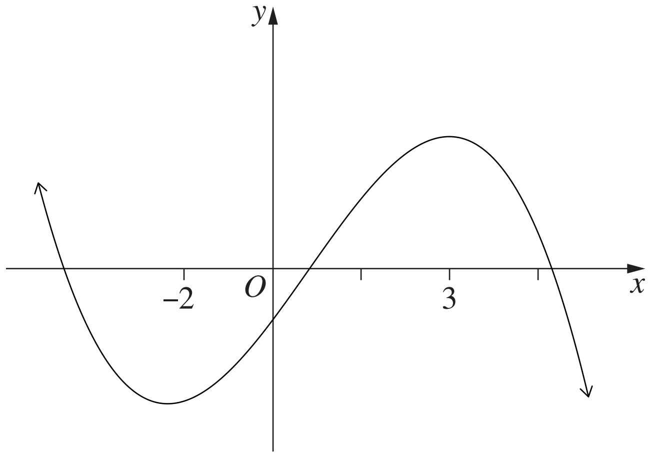
A. $\frac{15}{64}$

B. $\frac{15}{56}$

C. $\frac{15}{32}$

D. $\frac{15}{28}$

7 The diagram shows part of $y = f(x)$ which has a local minimum at $x = -2$ and a local maximum at $x = 3$.



Which of the following shows the correct relationship between $f''(-2)$, $f(0)$ and $f'(3)$?

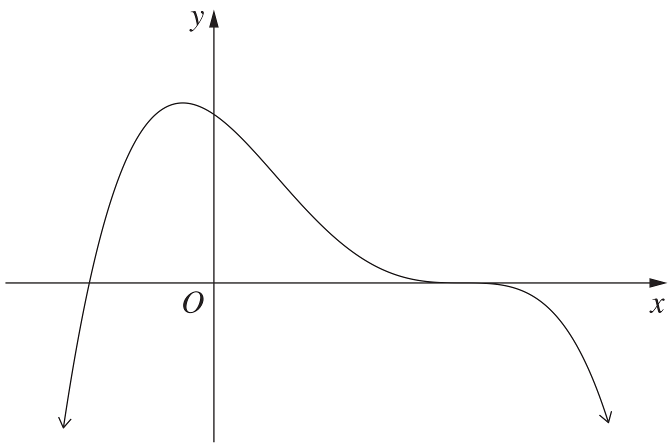
A. $f(0) < f'(3) < f''(-2)$

B. $f(0) < f''(-2) < f'(3)$

C. $f''(-2) < f'(3) < f(0)$

D. $f''(-2) < f(0) < f'(3)$

8 The graph of $y = f(x)$ is shown.



Which of the following could be the equation of this graph?

A. $y = (1 - x)(2 + x)^3$

B. $y = (x + 1)(x - 2)^3$

C. $y = (x + 1)(2 - x)^3$

D. $y = (x - 1)(2 + x)^3$

9 Let $h(x) = f(g(x))$ where the function $f(x)$ is an odd function and the function $g(x)$ is an even function.

The tangent to $y = h(x)$ at $x = k$, where $k > 0$, has the equation $y = mx + c$.

What is the equation of the tangent to $y = h(x)$ at $x = -k$?

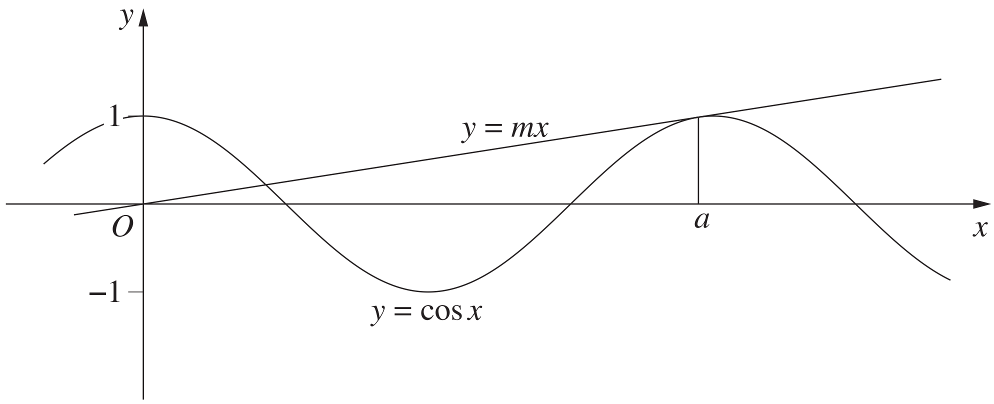
A. $y = mx + c$

B. $y = -mx + c$

C. $y = mx - c$

D. $y = -mx - c$

10 The line $y = mx$ is a tangent to the curve $y = \cos x$ at the point where $x = a$, as shown in the diagram.



Which of the following statements is true?

A. $m < \dfrac{1}{a} < \dfrac{1}{2\pi}$

B. $\dfrac{1}{2\pi} < m < \dfrac{1}{a}$

C. $\dfrac{1}{2\pi} < \dfrac{1}{a} < m$

D. $m < \dfrac{1}{2\pi} < \dfrac{1}{a}$

Do NOT write in this area.

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced
Section II Answer Booklet 1

Section II

90 marks
Attempt Questions 11–34
Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–28 (64 marks)
Booklet 2 — Attempt Questions 29–34 (26 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 28. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Solve $x + \frac{x-1}{2} = 9$. **2**

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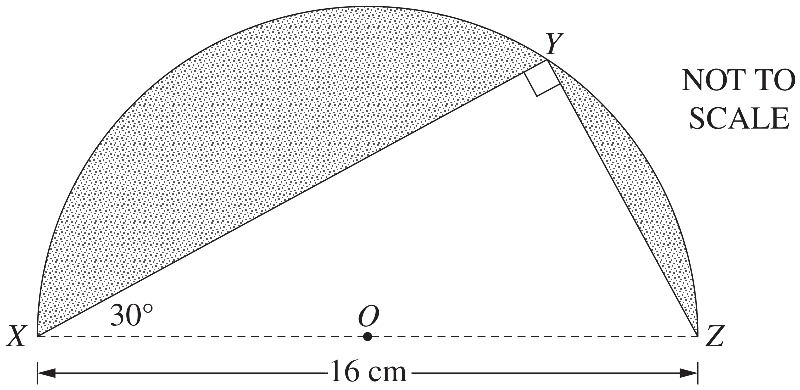
Question 12 (5 marks)

A right-angled triangle XYZ is cut out from a semicircle with centre O. The length of the diameter XZ is 16 cm and $\angle YXZ = 30°$, as shown on the diagram.



(a) Find the length of XY in centimetres, correct to two decimal places. **2**

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(b) Hence, find the area of the shaded region in square centimetres, correct to one decimal place. **3**

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Question 13 (3 marks)

Find the exact gradient of the tangent to the curve $y = x\tan x$ at the point where $x = \frac{\pi}{3}$. **3**

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Question 14 (2 marks)

The first term of an arithmetic sequence is 5. The sum of the first 43 terms is 2021. **2**

What is the common difference of the sequence?

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Question 15 (2 marks)

Evaluate $\int_{-2}^{0} \sqrt{2x+4}\,dx$. **2**

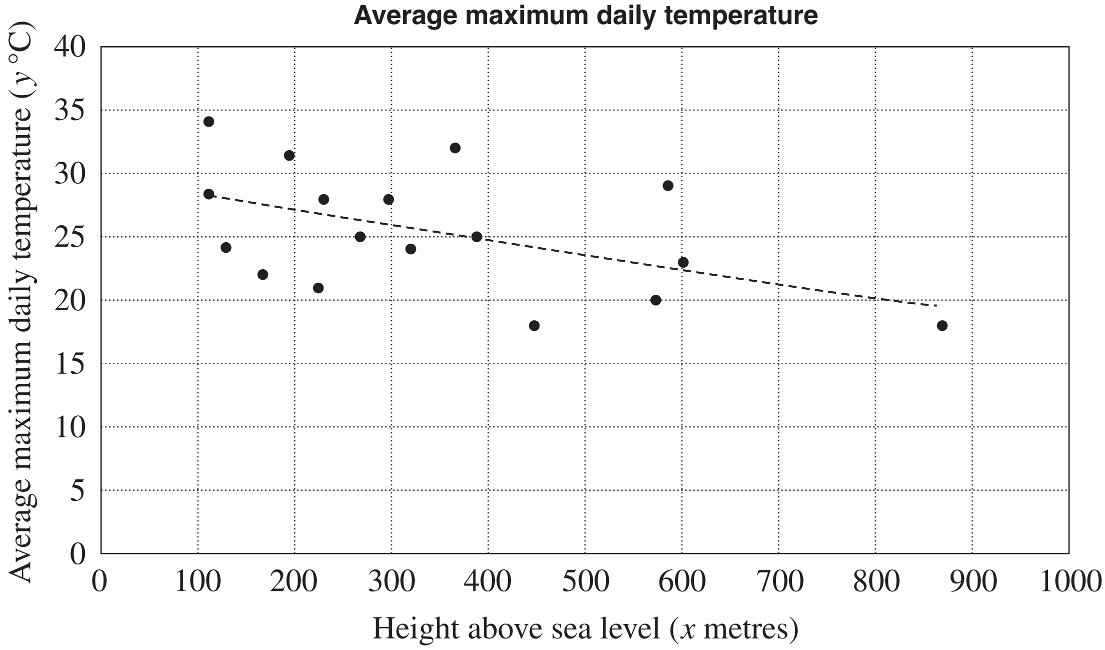
Question 16 (3 marks)

For what values of x is $f(x) = x^2 - 2x^3$ increasing? **3**

Question 17 (4 marks)

For a sample of 17 inland towns in Australia, the height above sea level, x (metres), and the average maximum daily temperature, y (°C), were recorded.

The graph shows the data as well as a regression line.



The equation of the regression line is $y = 29.2 - 0.011x$.

The correlation coefficient is $r = -0.494$.

(a) (i) By using the equation of the regression line, predict the average maximum daily temperature, in degrees Celsius, for a town that is 540 m above sea level. Give your answer correct to one decimal place. **1**

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(ii) The gradient of the regression line is −0.011. Interpret the value of this gradient in the given context. **2**

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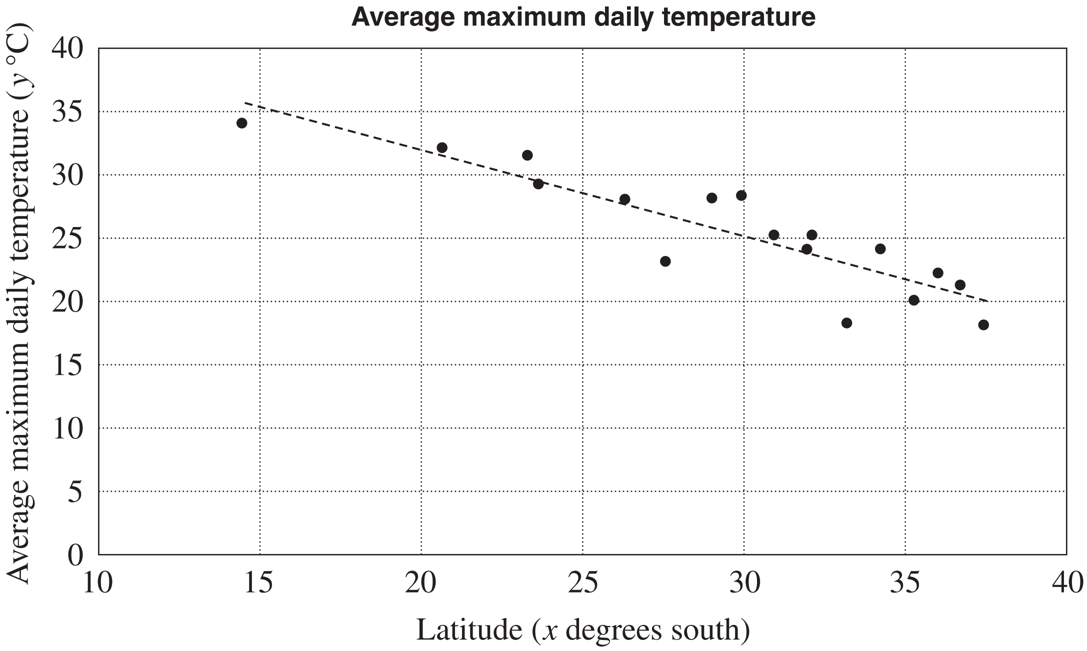
Question 17 continues on page 15

Question 17 (continued)

(b) The graph below shows the relationship between the latitude, x (degrees south), and the average maximum daily temperature, y (°C), for the same 17 towns, as well as a regression line. **1**



The equation of the regression line is $y = 45.6 - 0.683x$.

The correlation coefficient is $r = -0.897$.

Another inland town in Australia is 540 m above sea level. Its latitude is 28 degrees south.

Which measurement, height above sea level or latitude, would be better to use to predict this town's average maximum daily temperature? Give a reason for your answer.

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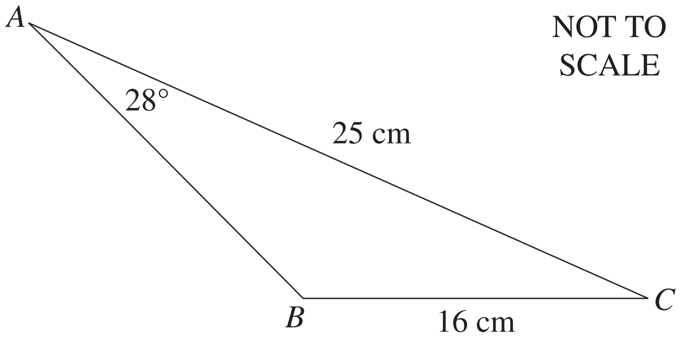
End of Question 17

Question 18 (3 marks)

The diagram shows a triangle ABC where $AC = 25$ cm, $BC = 16$ cm, $\angle BAC = 28°$ and angle ABC is obtuse. **3**



Find the size of the obtuse angle ABC correct to the nearest degree.

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Question 19 (3 marks)

Without using calculus, sketch the graph of $y = 2 + \dfrac{1}{x+4}$, showing the asymptotes and the x and y intercepts. **3**

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Question 20 (2 marks)

For what values of x, in the interval $0 \le x \le \dfrac{\pi}{4}$, does the line $y = 1$ intersect the graph of $y = 2\sin 4x$? **2**

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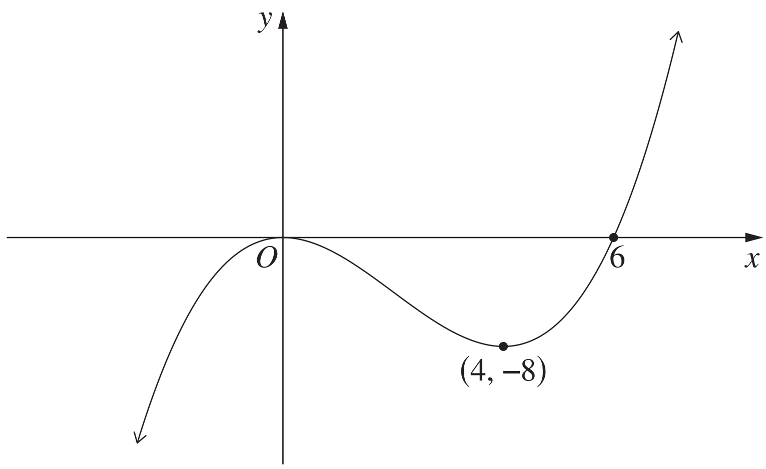
Question 21 (2 marks)

Consider the graph of $y = f(x)$ as shown. **2**



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Sketch the graph of $y = 4f(2x)$ showing the x-intercepts and the coordinates of the turning points.

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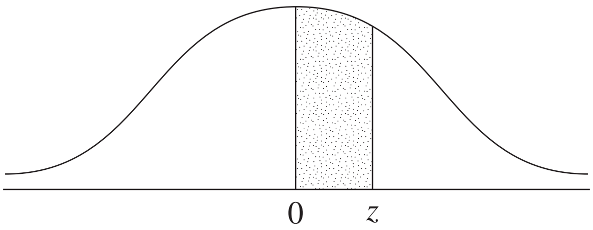
Question 22 (4 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable lies between 0 and z for different values of z.

z	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.0398	0.0793	0.1179	0.1554	0.1915	0.2257

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



(a) Using the table, find the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1 lies between 0.1 and 0.5. **1**

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(b) Birth weights are normally distributed with a mean of 3300 grams and a standard deviation of 570 grams. By first calculating a z-score, find how many babies, out of 1000 born, are expected to have a birth weight greater than 3528 grams. **3**

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Question 23 (4 marks)

A population, P, which is initially 5000, varies according to the formula **4**

$$P = 5000b^{\frac{-t}{10}},$$

where b is a positive constant and t is time in years, $t \geq 0$.

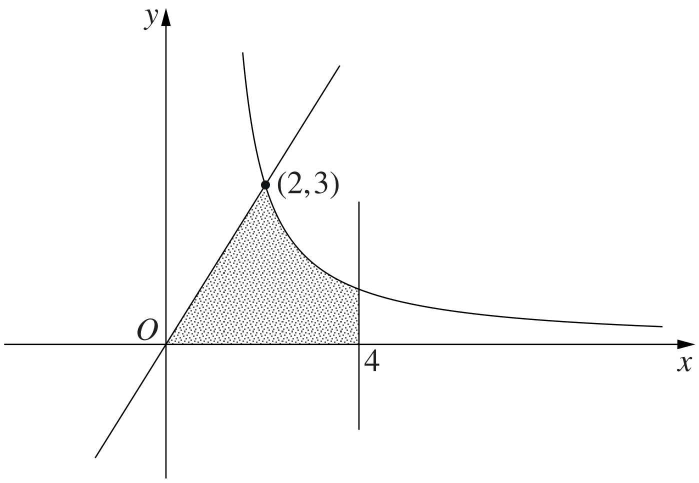
The population is 1250 after 20 years.

Find the value of t, correct to one decimal place, for which the instantaneous rate of decrease is 30 people per year.

Question 24 (3 marks)

The curve $y = \dfrac{3}{x-1}$ intersects the line $y = \dfrac{3}{2}x$ at the point $(2, 3)$. **3**

The region bounded by the curve $y = \dfrac{3}{x-1}$, the line $y = \dfrac{3}{2}x$, the x-axis and the line $x = 4$ is shaded in the diagram.



Find the exact area of the shaded region.

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Question 25 (3 marks)

A table of future value interest factors for an annuity of $1 is shown. **3**

Table of future value interest factors

Number of periods	*Interest rate per period*				
	0.25%	0.5%	0.75%	1%	1.25%
2	2.0025	2.0050	2.0075	2.0100	2.0125
4	4.0150	4.0301	4.0452	4.0604	4.0756
6	6.0376	6.0755	6.1136	6.1520	6.1907
8	8.0704	8.1414	8.2132	8.2857	8.3589
10	10.1133	10.2280	10.3443	10.4622	10.5817

Simone deposits $1000 into a savings account at the end of each year for 8 years. The interest rate for these 8 years is 0.75% per annum, compounded annually.

After the 8th deposit, Simone stops making deposits but leaves the money in the savings account. The money in her savings account then earns interest at 1.25% per annum, compounded annually, for a further two years.

Find the amount of money in Simone's savings account at the end of ten years.

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Question 26 (5 marks)

A particle is shot vertically upwards from a point 100 metres above ground level. The position of the particle, y metres above the ground after t seconds, is given by $y(t) = -5t^2 + 70t + 100$.

(a) Find the maximum height above ground level reached by the particle. **2**

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(b) Find the velocity of the particle, in metres per second, immediately before it hits the ground, leaving your answer in the form $a\sqrt{b}$, where a and b are integers. **3**

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Questions 11–26 are worth 50 marks in total

Question 27 (8 marks)

Kenzo has a solar powered phone charger. Its power, P, can be modelled by the function

$$P(t) = 400\sin\left(\frac{\pi}{12}t\right),\ 0 \le t \le 12,$$

where t is the number of hours after sunrise.

(a) Sketch the graph of P for $0 \le t \le 12$. **2**

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Power is the rate of change of energy. Hence the amount of energy, E units, generated by the solar powered phone charger from $t = a$ to $t = b$, where $0 \le a \le b \le 12$ is given by

$$E = \int_a^b P(t)\,dt.$$

(b) Show that $E = \frac{4800}{\pi}\left(\cos\frac{a\pi}{12} - \cos\frac{b\pi}{12}\right)$. **2**

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Question 27 continues on page 25

Question 27 (continued)

(c) To make a phone call, a phone battery needs at least 300 units of energy. Kenzo woke up 3 hours after sunrise and found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery. **3**

Find the least amount of time he needed to wait before he could make a phone call. Give your answer correct to the nearest minute.

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(d) The next day, Kenzo woke up 6 hours after sunrise and again found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery. **1**

Would it take more time or less time or the same amount of time, compared to the answer in part (c), to charge his phone battery in order to make a phone call? Explain your answer by referring to the graph drawn in part (a).

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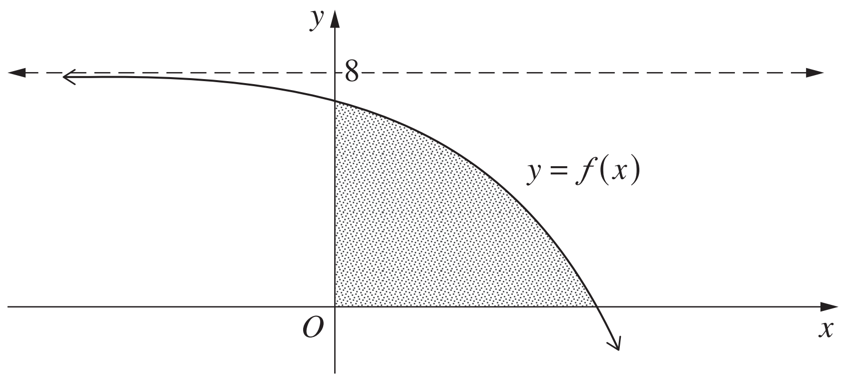
End of Question 27

Question 28 (6 marks)

The region bounded by the graph of the function $f(x) = 8 - 2^x$ and the coordinate axes is shown.



(a) Show that the exact area of the shaded region is given by $24 - \dfrac{7}{\ln 2}$. **3**

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Question 28 continues on page 27

Question 28 (continued)

(b) A new function $g(x)$ is found by taking the graph of $y = -f(-x)$ and translating it by 5 units to the right. **2**

Sketch the graph of $y = g(x)$ showing the x-intercept and the asymptote.

(c) Hence, find the exact value of $\int_2^5 g(x)\,dx$. **1**

End of Question 28

Proceed to Booklet 2 for Questions 29–34

Section II extra writing space

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2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 — Attempt Questions 29–34 (26 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 39–40. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 29 (5 marks)

(a) On the day that Megan was born, her grandfather deposited \$5000 into an account earning 3% per annum compounded annually. On each birthday after this, her grandfather deposited \$1000 into the same account, making his final deposit on Megan's 17th birthday. That is, a total of 18 deposits were made. **2**

Let A_n be the amount in the account on Megan's nth birthday, after the deposit is made.

Show that $A_3 = \$8554.54$.

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Question 29 continues on page 31

(b) On her 17th birthday, just after the final deposit is made, Megan has \$30 025.83 in her account. You are NOT required to show this. **3**

Megan then decides to leave all the money in the same account continuing to earn interest at 3% per annum compounded annually. On her 18th birthday, and on each birthday after this, Megan withdraws \$2000 from the account.

How many withdrawals of \$2000 will Megan be able to make?

End of Question 29

Question 30 (2 marks)

The number of hours for which light bulbs will work before failing can be modelled by the random variable X with cumulative distribution function **2**

$$F(x) = \begin{cases} 1 - e^{-0.01x}, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

Jane sells light bulbs and promises that they will work for longer than exactly 99% of all light bulbs.

Find how long, according to Jane's promise, a light bulb bought from her should work. Give your answer in hours, rounded to two decimal places.

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Question 31 (4 marks)

By considering the equation of the tangent to $y = x^2 - 1$ at the point $(a, a^2 - 1)$, find the equations of the two tangents to $y = x^2 - 1$ which pass through $(3, -8)$. **4**

Question 32 (4 marks)

In a particular city, the heights of adult females and the heights of adult males are each normally distributed. **4**

Information relating to two females from that city is given in Table 1.

Table 1

Height	*Gender*	*Percentage of females in this city shorter than this person*
175 cm	Female	97.5%
160.6 cm	Female	16%

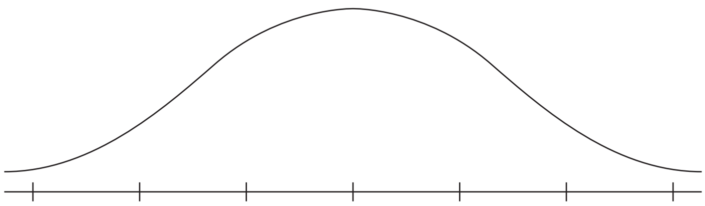
The means and standard deviations of adult females and males, in centimetres, are given in Table 2.

Table 2

	Mean	*Standard deviation*
Females	μ	σ
Males	1.05μ	1.1σ

A selected male is taller than 84% of the population of adult males in this city.

By first labelling the normal distribution curve below with the heights of the two females given in Table 1, calculate the height of the selected male, in centimetres, correct to two decimal places.



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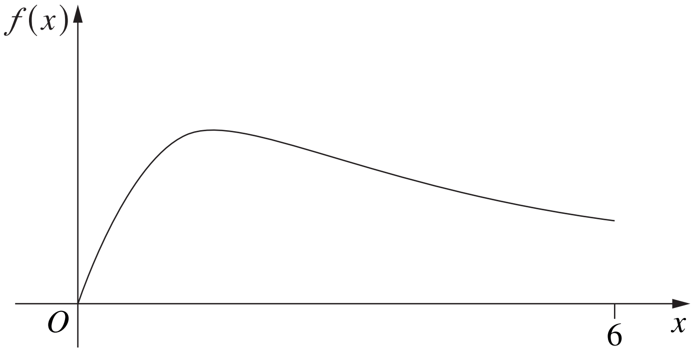
Question 33 (8 marks)

People are given a maximum of six hours to complete a puzzle. The time spent on the puzzle, in hours, can be modelled using the continuous random variable X which has probability density function

$$f(x) = \begin{cases} \dfrac{Ax}{x^2+4}, & \text{for } 0 \le x \le 6, \ (\text{where } A > 0) \\ 0, & \text{for all other values of } x \end{cases}.$$

The graph of the probability density function is shown below. The graph has a local maximum.



(a) Show that $A = \dfrac{2}{\ln 10}$. **2**

Question 33 continues on page 36

(b) Show that the mode of X is two hours. **2**

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(c) Show that $P(X < 2) = \log_{10} 2$. **2**

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Question 33 continues on page 37

Question 33 (continued)

(d) The Intelligence Quotient (IQ) scores of people are normally distributed with a mean of 100 and standard deviation of 15. **2**

It has been observed that the puzzle is generally completed more quickly by people with a high IQ.

It is known that 80% of people with an IQ greater than 130 can complete the puzzle in less than two hours.

A person chosen at random can complete the puzzle in less than two hours.

What is the probability that this person has an IQ greater than 130? Give your answer correct to three decimal places.

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End of Question 33

Please turn over

Question 34 (3 marks)

A discrete random variable has probability distribution as shown in the table where n is a finite positive integer. **3**

x	r	r^2	r^3	$\dots$	r^k	$\dots$	r^n
$P(X = x)$	r^n	r^{n-1}	r^{n-2}	$\dots$	r^{n-k+1}	$\dots$	r

Show that $E(X) = n(2r - 1)$.

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End of paper

Do NOT write in this area.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

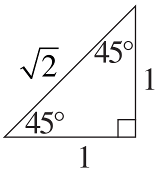
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

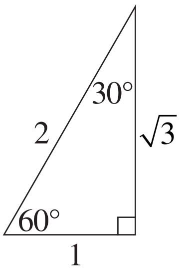
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

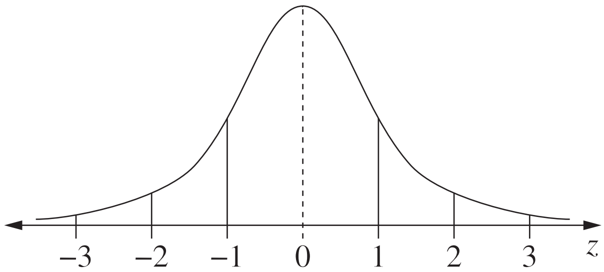
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$