Mathematics Advanced

HSC Marking Feedback 2022

Question 11 (a)

Students should:

- know how to add values in a frequency column one at a time to complete a cumulative frequency column
- know how to use the cumulative frequency to find the cumulative percentage
- be able to calculate the required values by observing the given values.

In better responses, students were able to:

- correctly calculate the indicated values from the table
- find the values of A and B through correct calculations.

Areas for students to improve include:

- identifying the necessary values needed to establish correct calculations
- understanding how to calculate a percentage
- ensuring that their answer makes sense within the context of the question.

Question 11 (b)

Students should:

- know how to read and interpret a Pareto chart
- know how to identify which columns are included within the 80% cumulative percentage
- identify both types of complaints.

In better responses, students were able to:

- identify the types of complaints that represented 80% of complaint types
- correctly state the complaints addressed.

Areas for students to improve include:

- understanding the difference between a Pareto chart and a cumulative frequency ogive
- understanding how to read a Pareto chart.

Question 12 (a)

Students should:

- understand what is meant by inverse variation
- calculate the value of the constant of proportionality
- know how to write and solve an inverse variation equation using the information and values given.

In better responses, students were able to:

- relate M and T in a correct inverse equation
- calculate the value of the proportionality constant
- solve the equation for k and rewrite in the form $M = \frac{180}{T}$.

Areas for students to improve include:

- associating inverse variation with reciprocal functions
- recognising the difference between inverse and direct variation
- substituting the correct values for the variables used in the question.

Question 12 (b)

Students should:

- complete the table of values using their function found in (a)
- plot the points correctly from the table of values
- join their points with a smooth curve.

In better responses, students were able to:

- correctly complete a table of values
- plot their points from the table of values accurately on the given number plane and draw a smooth curve rather than 2 straight lines.

Areas for students to improve include:

- plotting points from a table of values accurately
- checking the scale used on the grid provided, so that the points are plotted correctly
- drawing one smooth curve rather than a ruler to connect the points in a straight line.

Question 13

Students should:

- use the trapezoidal rule for 2 applications
- find function values by substituting in $x = 0, x = 1, x = 2$.

In better responses, students were able to:

- recognise that 2 applications requires 3 function values
- determine the exact function values at $x = 0$, $x = 1$ and $x = 2$
- use the trapezoidal rule formula correctly.

Areas for students to improve include:

- understanding the relationship between the number of applications and the number of function values required
- showing substitution into the trapezoidal formula
- understanding that the trapezoidal rule finds the approximate value of the integral and therefore integration is not required.

Question 14

Students should:

- read the amplitude from the graph for the value of k
- read the period from the graph (6π)
- recognise that the period is given by $\frac{2\pi}{a}$.

In better responses, students were able to:

- understand that the vertical dilation equals the amplitude, which is a distance
- understand the scale factor for the horizontal dilation and how it relates to a.

Areas for students to improve include:

- understanding that the period is $\frac{2\pi}{a}$
- understanding the relationship between a and the scale factor for horizontal dilation.

Question 15 (a)

Students should:

- identify the relevant selections
- use a probability tree or list possible outcomes
- recognise the need to add multiple probabilities to get a final answer.

In better responses, students were able to:

- demonstrate their understanding of probability calculations
- correctly add and multiply fractions to reach the correct answer.

Areas for students to improve include:

- reading the question carefully to obtain the correct sample space and favourable events
- using their calculator correctly when working with fractions.

Question 15 (b)

Students should:

- find the probability needed for the numerator $P(\text{special die } \cap \text{ } 5)$
- use the value given in part (a) for the denominator of the conditional probability.

In better responses, students were able to:

- apply the conditional probability formula, including finding the intersection on the numerator
- correctly substitute their answer found in part (a) as the denominator.

Areas for students to improve include:

- recognising conditional probability statements
- applying probability in a complex situation involving conditional probability.

Question 16

Students should:

- determine the area of a shaded region by subtracting functions in the correct order
- show the resulting definite integral
- substitute the correct limits to find the area.

In better responses, students were able to:

- write an integrand for the area between the two curves
- integrate the resulting expression
- substitute the correct limits to find the area.

Areas for students to improve include:

- subtracting functions in the correct order to find the area
- showing substitution of limits, including using brackets for negative values.

Question 17 (a)

Students should:

- identify the number of cards by forming an arithmetic sequence with the first term $a = 3$, common difference $d = 3$ and number of terms $n = 12$
- use the summation formula $S_n = \frac{n}{2}\{2a + (n-1)d\}$ or find the 12th term and use $S_n = \frac{n}{2}(a + l)$
- check their answer matches the given answer in a 'show' question.

In better responses, students were able to:

- identify that the situation involved an arithmetic series to be calculated
- apply the summation formula correctly to show the correct sum of 234 is achieved when $n = 12$, or compile a list of all 12 terms that when added showed that $S = 234$, or solve a quadratic equation (using the quadratic formula, by factorisation or completing the square

- demonstrate appropriate substitutions into the formula used from the Reference Sheet after correctly identifying a and d.

Areas for students to improve include:

- writing at least 3 terms to determine that an arithmetic series and not a geometric series was to be calculated
- making use of the Reference Sheet to apply the correct formula
- using brackets appropriately when substituting into the formula
- understanding that when the word 'total' is used it is referring to the sum and not the n^{th} term.

Question 17 (b)

Students should:

- use the summation formula, using $S = 828$ to find n
- obtain the correct quadratic equation and solve for n by either factorization or by using the quadratic formula and stating the positive result for $n = 23$ as $n > 0$.

In better responses, students were able to:

- select the correct formula from the Reference Sheet and obtain and solve the resulting quadratic equation, accepting the positive value and rejecting the negative value
- compile a list of all 23 terms that were added to show $S = 828$
- use the summation formula using $S = 828$, $a = 3$ and $d = 3$ to guess and check values for n, correctly stating that $n = 23$.

Areas for students to improve include:

- developing the algebraic skills to solve a quadratic equation after starting with $S_n = 828$
- showing a summation to verify that $S_n = 828$, if a list has been used
- checking their solution matches the required answer by substitution back into the original formula
- recognising that the number of rows needs to be a positive integer.

Question 18 (a)

Students should:

- use the chain rule to differentiate a function of the form $f(g(x))$.

In better responses, students were able to:

- use the chain rule to determine the derivative.

Areas for students to improve include:

- recognising when to use the chain rule
- using algebraic skills in simplifying expressions.

Question 18 (b)

Students should:

- recognise the link to the previous question, from the term “Hence”
- understand the link between differentiation and integration
- use the reverse chain rule to find the anti-derivative of the function.

In better responses, students were able to:

- recognise the link from the previous question
- demonstrate their understanding that the anti-derivative required the reverse chain rule
- manipulate the given expression into an appropriate form to integrate.

Areas for students to improve include:

- applying the reverse chain rule with the relevant working
- recognising the term “Hence” implies the need to use the previously obtained result
- adjusting the integral and relating it to the derivative from part (a).

Question 19

Students should:

- take note of the order of transformation
- write an expression for $g(x)$ in terms of m and k
- find m and k for the given quadratic by completing the square or expanding and equating coefficients to solve for m and k.

In better responses, students were able to:

- express $g(x)$ in terms of m and k
- expand and equate coefficients
- complete the square of a non-monic quadratic
- identify m by finding the vertex.

Areas for students to improve include:

- expanding perfect squares
- completing the square of a non-monic quadratic
- equating coefficients of a quadratic expression.

Question 20 (a)

Students should:

- substitute $t = 0$ into the function to find the initial population.

In better responses, students were able to:

- use their calculator to correctly evaluate the exponential expression.

Areas for students to improve include:

- understanding that 'initial' means $t = 0$
- recognising that $e^0 = 1$.

Question 20 (b)

Students should:

- substitute $t = 24$ into the function to find the required population.

In better responses, students were able to:

- use their calculator to correctly evaluate the exponential expression.

Areas for students to improve include:

- showing the substitution $t = 24$ into the expression
- taking care with transcription of all numerals.

Question 20 (c)

Students should:

- find the derivative of an exponential function
- calculate the rate of increase by substituting $t = 24$ into the derivative.

In better responses, students were able to:

- understand that the instantaneous rate of change is given by the derivative
- show the substitution $t = 24$ into the derivative.

Areas for students to improve include:

- finding the derivative of exponential functions with a coefficient
- using their calculator to correctly evaluate the exponential expression
- recognising the difference between instantaneous rate of change and average rate of change.

Question 21 (a)

Students should:

- apply the compound interest formula, showing clear substitution of values
- calculate the number of compounding periods

- calculate the correct interest rate and time periods for compounding monthly
- use a calculator to find the solution.

In better responses, students were able to:

- clearly demonstrate the correct use of the compound interest formula
- show their substitution clearly
- use a calculator to find the solution and round their answer correctly.

Areas for students to improve include:

- finding the interest rate and the number of compound periods
- distinguishing compound interest investments from other investment types
- applying the compound interest formula
- adjusting interest rates and time periods based on the compounding period.

Question 21 (b)

Students should:

- find the future value of an annuity by selecting the correct cell on a table of future values and multiplying by the regular deposit
- convert the annual interest rate to a quarterly interest rate as a decimal, and the number of periods for the investment
- subtract the future values from option 1 and option 2 to determine the difference.

In better responses, students were able to:

- identify the correct cell from the future value table and multiply by the regular deposit
- subtract the two future values to calculate the difference
- identify the correct value from the table and multiply it by 1000
- clearly show a numerical expression for the difference between Option 1 and Option 2.

Areas for students to improve include:

- converting the interest rate and calculating the number of periods
- using the table to find the future value of an annuity
- expressing the difference as an expression before the final calculation
- ensuring they have answered the entire question.

Question 22

Students should:

- know how to find stationary points and the y –values of the endpoints
- compare the y –values of the stationary points and the endpoints to identify the global maximum and minimum values

- find the first derivative and factorise to find the stationery points.

In better responses, students were able to:

- find the first derivative, use this to find the stationary points, find the y −values of the endpoints and identify the global maximum and minimum
- set out their solution clearly, demonstrating each step of working
- show the substitution as a numerical expression when finding the y −values.

Areas for students to improve include:

- understanding the difference between a global and local maximum and minimum
- recognising the need for calculus to be applied in order to find stationary points
- deriving accurately and using algebra skills to factorise and solve $\frac{dy}{dx} = 0$
- substituting into the function to find the y −values of relevant points.

Question 23 (a)

Students should:

- establish the centre of oscillation for a trigonometric function
- use amplitude to find the maximum and minimum values of a trigonometric function.

In better responses, students were able to:

- identify that the maximum and minimum values of a basic cosine function are ±1
- use the amplitude to find the maximum and minimum values of a wave function.

Areas for students to improve include:

- identifying the amplitude of the given wave function
- showing the use of the amplitude to find the maximum and minimum values of the given wave function.

Question 23 (b)

Students should:

- understand how to find the period of a trigonometric function
- be able to perform simple calculations involving π.

In better responses, students were able to:

- use $\frac{2\pi}{a}$ to find the period of a cosine curve
- perform a multiplication of fractions containing π.

Areas for students to improve include:

- using the period to find the time between successive low tides
- calculations involving π.

Question 23 (c)

Students should:

- use the given information to write a trigonometric equation
- solve the trigonometric equation for t
- find the required length of time.

In better responses, students were able to:

- write a correct trigonometric equation
- solve the trigonometric equation to find all relevant solutions for t
- correctly subtract fractions to find the required time.

Areas for students to improve include:

- showing all working to find relevant solutions for a trigonometric equation using radians
- simplifying calculations involving exact trigonometric ratios in radians
- identifying what is required to answer the given question.

Question 24

Students should:

- use correct terminology when describing correlation and refer to the correlation coefficient in formulating their answer
- recognise that the question asks them to interpret the data, meaning that they must comment on the context of the research
- know how to read and interpret a scatter plot, line of best fit and correlation coefficient
- recognise that a four-mark question is likely to require four unique facts.

In better responses, students were able to:

- use the appropriate language for bivariate data and other statistical terms and notations
- respond to the question with accurate, unique and contextual answers, keeping answers brief and to the point
- describe the relationship between the variables in the given context using unique and accurate statements observed from the data
- recognise the limitations of the line of best fit.

Areas for students to improve include:

- comparing bivariate data using the two dimensions given and not finding a statistical measure of one variable only
- using their understanding of the strength, direction and type of correlation to interpret and report on the data in context
- using all information given in formulating their answer (graph, equation, correlation coefficient).

Question 25

Students should:

- apply trigonometric differentiation rules to $f(x) = \sin 2x$ to find $f'(x)$ and $f''(x)$
- determine correct quadrants for each possible solution of the two trigonometric equations ($f'(x)$ and $f''(x)$) and that each equation will have two possible answers, then identify a single answer/ quadrant which produces the correct result for both equations
- recognise the question required a solution in radians
- interpret the meaning of 'AND' in the question as the intersection of $x = \frac{5\pi}{12}, \frac{7\pi}{12}$ and $x = \frac{7\pi}{12}, \frac{11\pi}{12}$.

In better responses, students were able to:

- differentiate $f(x) = \sin 2x$ and solve for the correct values of x in the given domain, then verify the two possible solutions with the second derivative
- solve both first and second derivative equations correctly and find the common x value
- correctly differentiate the trigonometric function twice and solve the resulting equations.

Areas for students to improve include:

- providing a solution in radians rather than degrees
- changing the domain to $0 \leq 2x \leq 2\pi$ to find the required $2x$ angles using an appropriate angles of any magnitude technique
- using the Reference Sheet to correctly differentiate trigonometric equations
- checking their final answer satisfies the initial conditions of the question.

Question 26

Students should:

- use the mean and standard deviation to determine what score is 1 standard deviation from the mean, and the normal distribution to determine the percentage of batteries within the given range
- identify that 34% of the data lie between 840 and 920, and calculate that 10% of the data lie between 820 and 840
- add the probabilities to find the answer.

In better responses, students were able to:

- subtract 50% from 60% to find the percentage of batteries within the given lifespan below the mean and add this to the 34% of batteries within the given lifespan above the mean to find the approximate percentage of batteries within the given lifespan
- represent the relevant information on the normal distribution curve
- show their calculations clearly when writing their answer.

Areas for students to improve include:

- identifying that there are 68% of scores between the first standard deviations either side of the mean, dividing 68% by 2 to get 34% as the distribution is symmetrical

- identifying the values of the mean and integer z-scores when using a scale to show normal distribution
- distinguishing between the percentage and number of batteries to make relevant calculations, checking the reasonableness of their answer.

Question 27 (a)

Students should:

- use the Reference Sheet to use the product rule for the second term
- collect like terms and factorise $f''(x)$ to show the required result.

In better responses, students were able to:

- understand that the product rule was necessary to solve the problem and determine the two functions required in the product rule
- correctly work with the negative terms throughout the solution
- factorise exponentials and negatives appropriately.

Areas for students to improve include:

- expanding grouping symbols involving a negative factor carefully and correctly
- recognising when to use the product rule as the best approach to differentiating
- differentiating exponentials correctly
- setting their work out clearly, showing all steps.

Question 27 (b)

Students should:

- factorise and solve an exponential equation
- find the stationary point
- determine the nature of their stationary point correctly using $f'(x)$ or $f''(x)$.

In better responses, students were able to:

- recognise the first derivative had only one solution
- find the stationary point by setting the first derivative equal to 0 and solving the equation by realising that $e^{-2x} \neq 0$
- determine the nature of stationary points by using the second derivative test, classifying the stationary point as a maximum.

Areas for students to improve include:

- understanding that the terms ‘maximum’ and ‘minimum’ should be used to classify stationary points
- using either the first or second derivative to determine the nature of the stationary points
- setting their work out clearly.

Question 27 (c)

Students should:

- sketch the curve, clearly labelling all relevant information
- use an appropriate scale on their graph
- find and verify points of inflection, $x-$ and $y-$ intercepts and demonstrate that an asymptote exists.

In better responses, students were able to:

- show that the positive $x-$ axis is an asymptote
- sketch the curve and clearly label the stationary points, axes intercept and point of inflection
- find the point of inflection using $f''^{(x)} = 0$, showing a change in concavity either side of the point.

Areas for students to improve include:

- using an appropriate scale on their graph
- understanding the difference between a point of inflection and a horizontal point of inflection
- sketching the graph as a smooth curve with clear labelling of all relevant features.

Question 28 (a)

Students should:

- find the radius of the circle
- calculate the area of the sector using $A = \frac{1}{2}r^2\theta$
- calculate the area of the triangle using $A = \frac{1}{2}bh$
- calculate the shaded area by subtracting the area of the triangle from the area of the sector.

In better responses, students were able to:

- work in radians rather than degrees
- use $A = \frac{1}{2}bh$ rather than $A = \frac{1}{2}ab\sin C$
- use trigonometry to find $\theta = \frac{\pi}{4}$
- leave answers in exact form.

Areas for students to improve include:

- working in radians
- understanding the Cartesian plane
- understanding that the formula $A = \frac{1}{2}r^2\theta$ requires θ in radians.

Question 28 (b)

Students should:

- substitute $x = 0$ and $y = 0$ into the function to show $a = b$
- substitute $x = 1$ and $y = 1$ into the function to obtain $\frac{a}{b-1} = 2$.

In better responses, students were able to:

- solve simultaneous equations
- clearly show all algebraic steps
- work with algebraic fractions.

Areas for students to improve include:

- not using the given answer to solve a 'show' question
- manipulating algebraic fractions.

Question 28 (c)

Students should:

- find the area under the curve by evaluating $\int_0^1 (\frac{2}{2-x} - 1)\, dx$
- find the total area by adding the solution of the integral to their answer from part (a).

In better responses, students were able to:

- integrate $\frac{2}{2-x}$ and -1 to reach the correct primitive
- leave answers in exact form.

Areas for students to improve include:

- finding primitives involving logs and another expression
- understanding the primitive of -1 is $-x$
- reading the question for guidance
- understanding that 'using part (a)' means the result from part (a) can be used without recalculation
- understanding that some areas cannot be found using integration.

Question 29 (a)

Students should:

- recognise the question was a limiting sum of a geometric series
- use the limiting sum formula to show the limiting sum is 2.

In better responses, students were able to:

- identity the common ratio and first term.
- clearly show the substitution into the limiting sum formula.

Areas for students to improve include:

- stating the values for the common ratio and first term
- substitution and simplifying fractions.

Question 29 (b)

Students should:

- identify the question as an integral that can be solved using the Reference Sheet
- substitute in the limits of the integral.

In better responses, students were able to:

- use the Reference Sheet to integrate the exponential function
- manage the negative index
- correctly substitute the limits into the integral to obtain the given result.

Areas for students to improve include:

- using the Reference Sheet to integrate exponential functions
- understanding and applying the laws for negative indices
- manipulating algebraic and arithmetic fractions
- substituting upper and lower limits.

Question 29 (c)

Students should:

- identify the question as a comparison of the rectangles and the exponential curve
- use the correct inequality statement to compare the results of parts (a) and (b)
- rearrange the inequality using logarithmic and index laws correctly for the required solution.

In better responses, students were able to:

- demonstrate their understanding by comparing parts (a) and (b)
- use an efficient starting statement to form their solution
- correctly rearrange the inequality using the required logarithmic and index laws.

Areas for students to improve include:

- using algebraic skills when simplifying inequalities
- using logarithmic and index laws
- showing all steps in calculations.

Question 30 (a)

Students should:

- identify the question as a cumulative distribution function (CDF)
- solve the CDF = 1 to show the required k value
- substitute into the CDF for the correct limits to evaluate a correct response.

In better responses, students were able to:

- manipulate the given CDF expression into an appropriate form to evaluate the k value
- use efficient algebraic strategies involving fractions and equations.

Areas for students to improve include:

- understanding the difference between a probability density function (PDF) and a cumulative distribution function (CDF)
- recognising the CDF evaluated at its limits is equated to 1
- not substituting $k = 3$ into the CDF; rather finding the value of k.

Question 30 (b)

Students should:

- recognize $P(X < c) + P(X > c) = 1$
- solve the given equation using the identity $P(X > c) = 1 - P(X < c)$
- solve the resulting logarithmic equation.

In better responses, students were able to:

- identify $P(X < c) = \frac{2}{3}$
- simplify the expression before substituting the given equation
- solve the resulting logarithmic equation.

Areas for students to improve include:

- applying index and logarithmic laws
- understanding the meaning of the notation $P(X < c)$
- recognising $P(X > c) = 1 - P(X < c)$.

Question 31 (a)

Students should:

- write an equation using equivalent expressions
- carefully manipulate the resulting equation to obtain the required result.

In better responses, students were able to:

- equate two equivalent expressions for the gradient, equate the ratios of corresponding sides in similar triangles or form an equation using the ratio of the areas of similar triangles
- manipulate a simple equation to obtain the required result.

Areas for students to improve include:

- recognising that a relationship between two equivalent expressions is required
- showing all working to obtain the required result.

Question 31 (b)

Students should:

- write an equation for the required area in terms of x
- find the derivative of the resulting equation
- find stationary points and test their nature to find the minimum x value
- use the minimum x value to find the required area.

In better responses, students were able to:

- use the information given in part (a) to write an equation for the area of the triangle
- correctly differentiate the resulting equation using the quotient rule or product rule
- find all stationary points and correctly test their nature
- use the minimum value to find the required area.

Areas for students to improve include:

- writing a simple equation for the area of the triangle
- using related information to write the area equation in x
- showing all working when using the derivative to obtain the stationary points
- showing derivative values (if using a table) and using suitable x values close to the stationary point to test their nature
- showing all working to find the required area of the triangle.

Question 32 (a)

Students should:

- recognise that $1 + (1.0025)^1 + \ldots + (1.0025)^{179} \approx 226.97$ is the sum of a geometric progression (GP)
- use an appropriate formula from the Reference Sheet to find the sum using $n = 180$ and $M\left[\frac{(1.0025)^{180}-1}{1.0025-1}\right]$ or $M\left[\frac{1-(1.0025)^{180}}{1-1.0025}\right]$, then rewrite the equation with M as the subject to show the appropriate value

- show the necessary steps to demonstrate understanding of the process in order to arrive at the given value.

In better responses, students were able to:

- substitute $A_n = 0$, $P = 200000$, $r = 1.0025$ and $n = 180$ correctly into the given formula
- identify the connection to GPs and select a suitable formula from the Reference Sheet
- make M the subject in the equation $2000000(1.0025)^{180} = M\left[\frac{(1.0025)^{180}-1}{1.0025-1}\right]$, then check the calculation provides the given value for M.

Areas for students to improve include:

- understanding a 'show' question requires demonstrated working out
- checking their final answer matches the given answer in a 'show' question
- not proving a result when instructed 'Do NOT prove this'.

Question 32 (b)

Students should:

- recognise the technique used in part (a) is relevant for part (b)
- substitute the values $P = 100032$, $r = 1.0035$, $M = 1381.16$ and $A_n = 0$ into the amount owing formula and solve for n
- note that the interest rate has changed to 0.35%, 100 months into the loan, and that the question asked to find how many more months the loan period was extended for

In better responses, students were able to:

- manage the complex logarithmic and exponential work required to solve the question
- collect the like terms involving $(1.0035)^n$, make this the subject, then take the logarithm of both sides or change from index form to logarithmic form to determine a decimal approximation for n.
- calculate both A_{83} and A_{84} to conclude that 83 full monthly payments are made.

Areas for students to improve include:

- not proving a result that has clearly been stated 'Do NOT prove this'
- calculating both A_{83} and A_{84} to conclude that 83 full monthly payments are made
- not rounding their answer up to $n = 84$ as this would result in overpaying the loan
- more care when setting out solutions
- avoiding arithmetic errors when rearranging equations involving indices
- reading questions carefully, in this case not mistaking the value of P for the value of A_n.

Question 32 (c)

Students should:

- use their rounded answer from part (b) to determine the amount owing after 83 full payments
- apply the interest charged for 1 month on the final amount owing.

In better responses, students were able to:

- use their rounded answer from part (b) to ensure the final repayment is less than $1381.16
- demonstrate an understanding of the necessary additional interest calculation $(A_{83} \times 1.0035)$
- identify the two components of the question, rounding their part (b) answer for n down to 83, then use the formula correctly where $P = 100032$, $r = 1.0035$ and $M = \$1381.16$ to find the amount owing, then multiply the final answer by 1.0035 to give $A_n = \$931.54$
- understand the significance of getting a non-integer value of n, indicating there is less than the full payment amount in the last month.

Areas for students to improve include:

- considering if their final solution is reasonable within the context of the question; in this case the final payment should not be larger than the original payments for the loan
- remembering to multiply the final amount owing by 1.0035 as interest for that month needs to be added to obtain the final payment amount
- checking their solution answers the question.