# Mathematics Advanced

## HSC Marking Feedback 2023

### General feedback

Students should:

- show relevant mathematical reasoning and/or calculations
- read the question carefully to ensure that they do not miss important components of the question
- have a clear understanding of key words in the question and recognise the intent of the question and its requirements, such as show, solve, evaluate, hence, calculate, derive
- use the Reference Sheet where appropriate
- ensure the solution is legible and follows a clear sequence
- engage with any stimulus material provided and refer to it in their response when required by the question
- check their solution answers the question
- round off numerical solutions only at the final step of the solution
- construct graphs neatly, with precision and display all relevant information as required by the question
- interpret information presented in graphs across a range of contexts
- understand when to use relevant calculator functions
- carefully note any information in the questions which supplies units of measurement.

### Section II

### Question 11

In better responses, students were able to:

- accurately write down the formula for the term of an arithmetic sequence, or if listing the fifteen terms, ensure that each is correct and that all terms are listed
- substitute the three variables accurately into the correct formula, and apply the correct operations of addition, subtraction and multiplication.

Areas for students to improve include:

- making use of the Reference Sheet to apply the correct formula: $Tn = a + (n - 1)d$
- knowing which formula on the Reference Sheet is for the term of an arithmetic sequence.

## Question 12 (a)

In better responses, students were able to:

- clearly show the sum of the product of $x$ and $P(X = x)$.

Areas for students to improve include:

- knowing how to multiply the values in the two rows of the table to generate $xP(x)$, and then finding the sum to state the expected value
- understanding the difference between mode and mean or expected value
- understanding a 'show that' in the question requires working out to be shown.

## Question 12 (b)

In better responses, students were able to:

- use the values in the table to calculate variance, and then find the standard deviation
- use a calculator to correctly to find the standard deviation.

Areas for students to improve include:

- using the correct formulae for variance
- finding the standard deviation by taking the square root of the variance
- using a calculator to correctly to find the standard deviation.

## Question 13

In better responses, students were able to:

- demonstrate their understanding by finding the primitive function from the derivative of an exponential function
- correctly substitute $t$ = 0 and $P$ = 4000 to evaluate the constant.

Areas for students to improve include:

- making use of the Reference Sheet to correctly find the integration of an exponential function
- evaluating the constant of integration
- using subtraction when solving $4000 = 1500e^{2(0)} + c$, and not division
- understanding that $e^0 = 1$.

## Question 14

In better responses, students were able to:

- were able to find the derivative using the Chain rule
- find the gradient of tangent at $x$ = 0
- substitute the coordinates of the point and the value of the gradient into a formula for the equation of a straight line.

Areas for students to improve include:

- using the Chain rule
- not expanding the original function nor the first derivative
- knowing that, when finding the equation of the tangent, a value is required for the gradient, and not an expression in terms of $x$
- knowing that the tangent is a straight line.

## Question 15 (a)

In better responses, students were able to:

- use the table provided to determine the factor 13.181 and then divide $450 000 by 13.181
- identify the interest factor from the table provided
- divide the FV by the interest factor.

Areas for students to improve include:

- recognising that identification of this factor is the use of the rate and periods so that these do not need to be used again in the calculation of the solution
- setting up an equation to help identify when there should be division or multiplication
- highlighting the interest factor in the table
- using compound interest formula or geometric series correctly.

## Question 15 (b)

In better responses, students were able to:

- demonstrate the conversion of the rate and the periods to four times a year
- select the correct factor from the table and multiply it by $8535
- convert the period to 40 and interest rate to 1.5% or equivalent
- identify the interest factor from the table.

Areas for students to improve include:

- identifying that every three months is four times a year
- using the converted rate and period to identify the factor from the table
- making use of the table of factors provided, not making errors in creating/using a geometric series.

## Question 16

In better responses, students were able to:

- interpret the question correctly to identify the main parts and evaluate the length of arc *PQ* and the length of line segment *PQ*

- set their working out clearly and identify the parts they were calculating, and show their rounded answers to at least three decimal places to assist the calculation for the perimeter
- identify the arc length was 110/360 of a circle and not a semi-circle
- correctly substitute values into the correct formulas (the sine or cosine rule for line segment *PQ* and arc length formula for arc *PQ*) and show full substitution
- round the final solution to 1 decimal place.

Areas for students to improve include:

- selecting and demonstrating appropriate substitutions into the required formula from the Reference Sheet
- understanding that the question requires the calculation of the perimeter of the shape, therefore no area formulas were needed
- using the square root when using the cosine rule
- realising the arc *PQ* has a radius of 2.1 m and is not a semi-circle with interval *PQ* as the diameter
- writing the formula they are using before substituting
- adding all sections when calculating the perimeter.

## Question 17

In better responses, students were able to:

- identify the correct method of integration required
- correctly divide fractions to achieve the solution.

Areas for students to improve include:

- recognising the relationship between the derivative and integral
- using the Reference Sheet to apply the correct integral formula.

## Question 18 (a)

In better responses, students were able to:

- calculate both means showing all calculations
- accurately plot the coordinates of the means and *y*-intercept
- use correct mathematical notation.

Areas for students to improve include:

- reading the question carefully to determine the means for each quantity
- accurately plotting coordinates on the intersection of the grid lines
- recognising that the initial quantity is the *y*-intercept
- reading the grid scale correctly.

### Question 18 (b)

In better responses, students were able to:

- calculate the gradient using $\frac{rise}{run}$
- recognise the *y*-intercept
- correctly substitute into $y = mx + c$.

Areas for students to improve include:

- writing an equation not an expression
- not interpreting the data as bivariate and attempting to use the calculator to find the equation of the regression line
- using the two coordinates from their graph to find the gradient of the line and not reading two inaccurate coordinates off their regression line.

### Question 18 (c)

In better responses, students were able to:

- comment on the prediction of gas usage being negative, which is not possible
- recognise and explain that using extrapolation is inaccurate.

Areas for students to improve include:

- relating their answer to the context of the question, that is, gas usage and temperatures
- providing an accurate explanation relevant to the regression line.

### Question 19 (a)

In better responses, students were able to:

- choose an appropriate scale for the straight line and parabola
- sketch a parabolic shape and points of intersection with the line
- label the vertex of the parabola
- label the *y*-intercept of the parabola.

Areas for students to improve include:

- using a ruler to draw straight lines
- demonstrating an understanding of the appropriate scale
- identifying key points to improve the care and accuracy of each sketch
- knowing the concavity of a parabola from its equation.

## Question 19 (b)

In better responses, students were able to:

- understand the term 'Hence' meant to solve the equation by using the work completed in Q19(a)
- expand a binomial product and then solve the quadratic inequality $\boldsymbol{x^2 + 3x - 4 < 0}$
- express their solutions as a combined inequality or use bracket notation.

Areas for students to improve include:

- correctly solving quadratic inequalities
- demonstrating the connection between algebra and graphs
- interpreting the inequality to be solved, that is, finding the solution for $x$ when the straight line is below the parabola
- understanding inequality symbols and express their solution using the correct notation.

## Question 20

In better responses, students were able to:

- adjust the domain to include all possible values of $\theta$.
- recognise the solutions for $\theta - 60°$ lie in the third and fourth quadrants
- provide all three solutions in degrees.

Areas for students to improve include:

- demonstrating knowledge about changing the domain according to the function in the question
- understanding that if the question is posed in degrees, the solution must be presented in degrees
- recognising that $0°$and $360°$ are the same angle on the unit circle.

## Question 21

In better responses, students were able to:

- use the Reference Sheet to identify and use the correct Geometric Progression formulas
- solve simultaneous equations involving index notation
- identify the positive and negative values for the common ratio and the respective first terms.

Areas for students to improve include:

- understanding the differences between an Arithmetic and Geometric sequence
- understanding that when solving an equation with an 'even power', that there will be two possible answers
- making '$r$' the subject complicates the simultaneous equation solution.

## Question 22

In better responses, students were able to:

- efficiently find the angle *AEM* by only finding *AM* and then using tangent ratio
- view the diagram in a 3-dimensional (3D) context
- label the sides and angles they were finding in the diagram or in their solution.

Areas for students to improve include:

- understanding the properties of quadrilaterals and 3D shapes
- using correct angle and triangle notation in their solution
- using exact values throughout the calculations
- not assuming scale in the diagram and making 45-degree angles appear in the working.

## Question 23

In better responses, students were able to:

- calculate the correct $z$-score using the formula provided on the Reference Sheet
- use the table to obtain the correct probability
- correctly find the proportion of koalas greater than 11.93kg, recognising that the value from the table represents the probability of the weight being less than 11.93kg
- find the number of koalas with weight greater than 11.93kg.

Areas for students to improve include:

- accessing the provided table to find the probability after calculating the $z$-score
- knowing when to use the $z$-score formula and table given opposed to using the normal distribution (empirical rule) to calculate an area under a curve
- completely answering the question by reading what was required, that is, the number of koalas greater than 11.93kg.

## Question 24 (a)

In better responses, students were able to:

- identify the side lengths of the garden in terms of $x$
- find an expression for the area of the garden in terms of $x$ and $y$
- equate the area in terms of $x$ and $y$ with the value given for the area of the garden
- rearrange their area equation to make $y$ the subject
- identify the dimensions of the garden as $(x - 2)$ and $(y - 1)$.

Areas for students to improve include:

- using correct algebraic processes to change the subject of their equation
- recognising the importance of brackets to indicate the correct order of operations

- understanding they were not solving an equation as many students tried to find a numerical value for $y$ or $x$ from the equation provided.

### Question 24 (b)

In better responses, students were able to:

- establish an equation for the area of the path in terms of $x$
- differentiate the area of the path and solve their derivative to find the possible dimensions
- eliminate solutions to the derivative of the area of the path that did not fit the context
- verify that their solution gives the minimum area of the path
- use numerical values in the first derivative test to determine a minimum.

Areas for students to improve include:

- interpreting what is being optimised to develop an equation to represent it
- simplifying the algebraic expression for the area of the path
- using correct algebraic processes to simplify the derivative and solve for $A' = 0$
- identifying, from the context, which solution is relevant
- demonstrating that the solution gives a minimum area
- creating a formula from information provided
- using brackets, expanding brackets, operating with negative integers
- using appropriate notation such as *A*, *A*', *A*'' to help identify the steps.

### Question 25 (a)

In better responses, students were able to:

- show the connection between $A_1$ and $A_2$ by writing either $A_2 = A_1 \times 1.004 - M$ or $A_2 = (10000 \times 1.004 - M) \times 1.004 - M$.

Areas for students to improve include:

- demonstrating understanding of how financial series are built term by term
- showing adequate steps in the working to reflect their reasoning.

### Question 25 (b)

In better responses, students were able to:

- write an explicit and complete expression for $A_n$ showing a geometric series
- use the sum of a Geometric Progression series correctly to show $A_n = 10000(1.004)^n - M\left(\frac{1.004^n - 1}{0.004}\right)$
- use algebra to expand and simplify to show all relevant steps.

Areas for students to improve include:

- using the pattern from Q25(a) to show the expression of $A_n$
- carefully identifying the number of terms in the geometric series
- demonstrating skills in expanding, rearranging, and factorising to arrive at the final expression
- knowledge of fractions to show $\frac{1}{0.004} = 250$.

## Question 25 (c)

In better responses, students were able to:

- identify the substitution of $n = 100$ and $A_n = 0$ into the expression from Q25(b)
- manipulate the expression to obtain the value for $M$.

Areas for students to improve include:

- using the appropriate values of $n$ and $A_n$ to substitute into the expression
- avoiding arithmetic errors when rearranging equations and inequations.

## Question 26 (a)

In better responses, students were able to:

- give the correct trigonometric primitive function
- find the constant using initial conditions
- write the required trigonometric equation.

Areas for students to improve include:

- manipulating algebraic fractions correctly
- showing the substitution of the initial conditions
- solving simple equations
- using the information provided on the Reference Sheet more effectively when finding the expression for $x(t)$.

## Question 26 (b)

In better responses, students were able to:

- calculate the number of periods in the given time
- find the period of the trigonometric function
- sketch either the rate of change trigonometric function or the displacement expression in Q26a)
- solve trigonometric functions involving fractions over the required domain
- understand when the maximum and minimum distances from the camera occur.

Areas for students to improve include:

- correctly calculating the period of a trigonometric function
- understanding the relevance of the period of a trigonometric function to a real-world situation
- providing the correct solution of a trigonometric equation, over an adjusted domain, with multiple solutions.

## Question 27 (a)

In better responses, students were able to:

- recognise the horizontal and vertical translations of the function and the relationship to *b* and *c*
- substitute *b* and *c* with a given coordinate into the function to find *a*
- use the gradient to calculate the dilation and recognise the sign of *a* as negative.

Areas for students to improve include:

- understanding the vertical and horizontal transformation of the function
- recognising that the gradient of this function was the dilation represented by *a*
- avoiding the use of simultaneous equations to solve algebraically.

## Question 27 (b)

In better responses, students were able to:

- recognise the need to calculate two distinct gradients from the origin
- calculate the gradients so that the line intersects the absolute value function twice
- write the correct inequality.

Areas for students to improve include:

- understanding that the line $y = mx$ will pass through (0,0)
- using the understanding of gradients to find the required values rather than simultaneous equations to find points of intersection
- using the correct inequality symbols.

## Question 28

In better responses, students were able to:

- recognise that the tangents at $T$ and $R$ have the same gradient
- equate $\frac{dy}{dx} = 1$ and solve the quadratic to find the $x$ coordinate of $R$
- find the anti-derivation of $\frac{dy}{dx}$ and calculate the correct value of $c$
- correctly substitute the $x$-coordinate of $R$ into $y = f(x)$ to find the $y$ coordinate of $R$.

Areas for students to improve include:

- writing $+c$ in their integration
- recognising that if the tangent is not horizontal, then $R$ is not a stationary point.

### Question 29 (a)

In better responses, students were able to:

- expand the probability density function so that the derivative can be easily determined
- calculate the mode as the $x$-coordinate of the maximum turning point of the probability density function
- identify the mode by substituting the possible turning point of $x$ values into the probability density function and comparing the $y$ values, rather than first or second derivative testing.

Areas for students to improve include:

- distinguishing the difference between a probability density function and a cumulative distribution function
- applying correct differentiation techniques and simplifying functions before differentiating
- taking care with negative values when using the product rule for differentiation
- using an appropriate concavity test if there is more than one solution.

### Question 29 (b)

In better responses, students were able to:

- integrate the probability density function using limits of 0 to $x$ to obtain the cumulative distribution function
- expand the probability density function before integrating the expression.

Areas for students to improve include:

- recognising the relationship between a probability density function and a cumulative distribution function
- knowing what the cumulative distribution function represents.

### Question 29 (c)

In better responses, students were able to:

- correctly substitute the mode into the cumulative distribution function
- understand that the median occurs at the point where the area under the probability density function is 0.5
- conclude the calculation of the cumulative distribution function for the mode is greater than the median value of the cumulative distribution function.

Areas for students to improve include:

- understanding the significance of the cumulative distribution function as a measure of the area under the probability density function
- remembering the total area under the probability density function is 1 square unit.

## Question 30 (a)

In better responses, students were able to:

- differentiate exponential and trigonometric functions
- use the product rule
- solve an equation containing exponentials and trigonometric functions
- calculate the $y$-values of the stationary points
- use exact values in their answer.

Areas for students to improve include:

- solving a trigonometric equation for all angles within a given domain
- recognising that $e^{-x} = 0$ has no solutions
- finding the corresponding $y$-values to the stationary point $x$-values
- converting between degrees and radians
- recognising that if the domain is in radians, then so should the $x$-values.

## Question 30 (b)

In better responses, students were able to:

- sketch a single, smooth graph for the function in part (a), showing stationary points and $x$-intercepts
- correctly label their stationary points and $x$-intercepts.

Areas for students to improve include:

- calculating the correct $x$-intercepts
- sketching a smooth curve and carefully labelling the necessary features
- considering the scale when sketching a graph
- knowing what a stationary point should look like in a sketch
- reading the question carefully to make sure that all the required information is shown in the diagram.

### Question 31 (a)

In better responses, students were able to:

- demonstrate the difference between an independent and dependent event
- know how one event impacts the other
- use $P(F) \neq P\left(F \mid S\right)$ to show that Kim's availability next Friday changes, given Kim is available on Saturday.

Areas for students to improve include:

- recognising the difference between an independent and dependent event
- understanding the meaning of conditional probability notation.

### Question 31 (b)

In better responses, students were able to:

- set out a well-structured response with the aid of conditional probability rules
- calculate the probability intersection of Saturday and Friday.

Areas for students to improve include:

- understanding how to use conditional probability in the context of a show question
- identifying this dependent multi-event probability
- recognising that $P(S \cap F)$ and $P(F \cap S)$ are the same as they represent where the sets intersect.

### Question 31 (c)

In better responses, students were able to:

- find the complement of all four students available.

Areas for students to improve include:

- understanding the technique of complementary probability theory when the expression **'**at least one**…'** is required it is equivalent to **'**1 – Probability (none)**'**
- finding the complement of four students and not just one.

### Question 32 (a)

In better responses, students were able to:

- write an integrand for the area between two curves
- integrate the resulting expression with the correct sign for each term in the expression
- substitute the correct limits to find the area that was given.

Areas for students to improve include:

- reviewing the working when using positive and negative signs

- carefully subtracting functions in the correct order to find the area between to curves
- working with exponentials with logarithmic powers
- showing substitution of limits, and fully evaluating the resulting anti-derivative to show the area given.

## Question 32 (b)

In better responses, students were able to:

- equate the two equations and obtain a quadratic expression equation in $e^{-x}$
- apply the quadratic formula to determine the roots of the exponential function in terms of $k$
- for two solutions the discriminant, 1+ 4$k$ > 0 and stated. $k > -\frac{1}{4}$
- for $e^{-x} > 0, u$sed $1 - \sqrt{1 + 4k}$ > 0 and found $k$ < 0.

Areas for students to improve include:

- solving simultaneous equation using quadratic formula in a general form of a quadratic equation
- understanding that for two points of intersection, the discriminant is greater than zero
- recognising that $e^{-x}$ is positive is also a part of the solution.