



NSW Education Standards Authority

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Centre Number

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Student Number

**2023** HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Advanced

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## General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

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## Total marks: 100

### Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

### Section II – 90 marks (pages 9–40)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

## Section I

10 marks

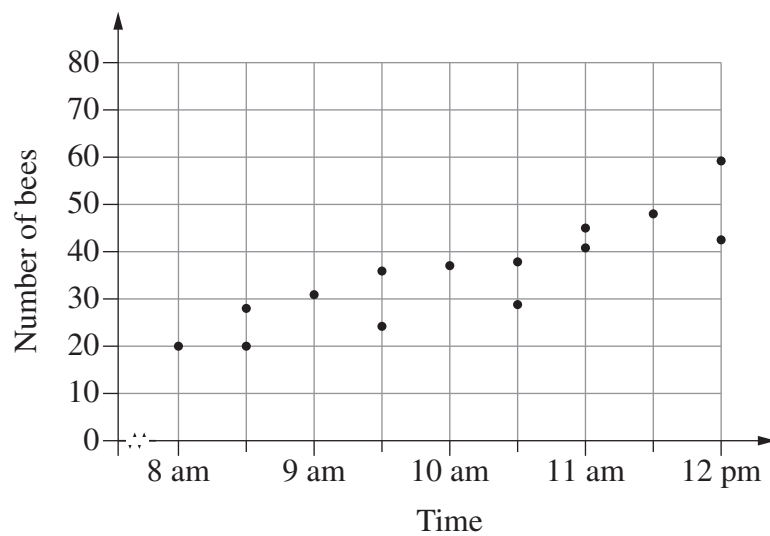
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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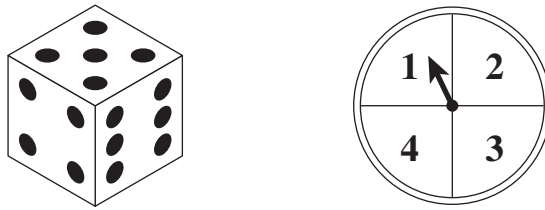
- 1 The number of bees leaving a hive was observed and recorded over 14 days at different times of the day.



Which Pearson's correlation coefficient best describes the observations?

- A.  $-0.8$
- B.  $-0.2$
- C.  $0.2$
- D.  $0.8$

- 2 A game involves throwing a die and spinning a spinner.



The sum of the two numbers obtained is the score.

The table of scores below is partially completed.

		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

What is the probability of getting a score of 7 or more?

- A.  $\frac{1}{6}$
- B.  $\frac{1}{4}$
- C.  $\frac{5}{18}$
- D.  $\frac{5}{12}$

3 What is the domain of  $f(x) = \frac{1}{\sqrt{1-x}}$ ?

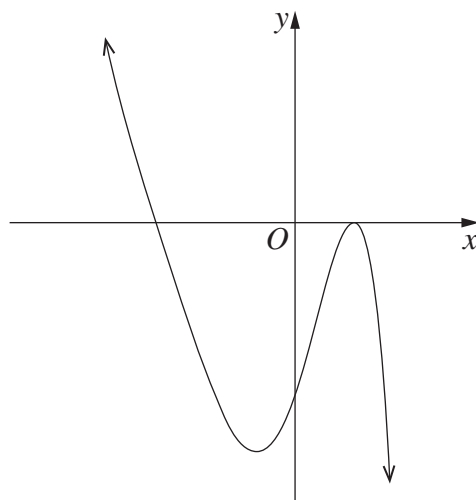
A.  $x < 1$

B.  $x \leq 1$

C.  $x > 1$

D.  $x \geq 1$

4 The graph of a polynomial is shown.



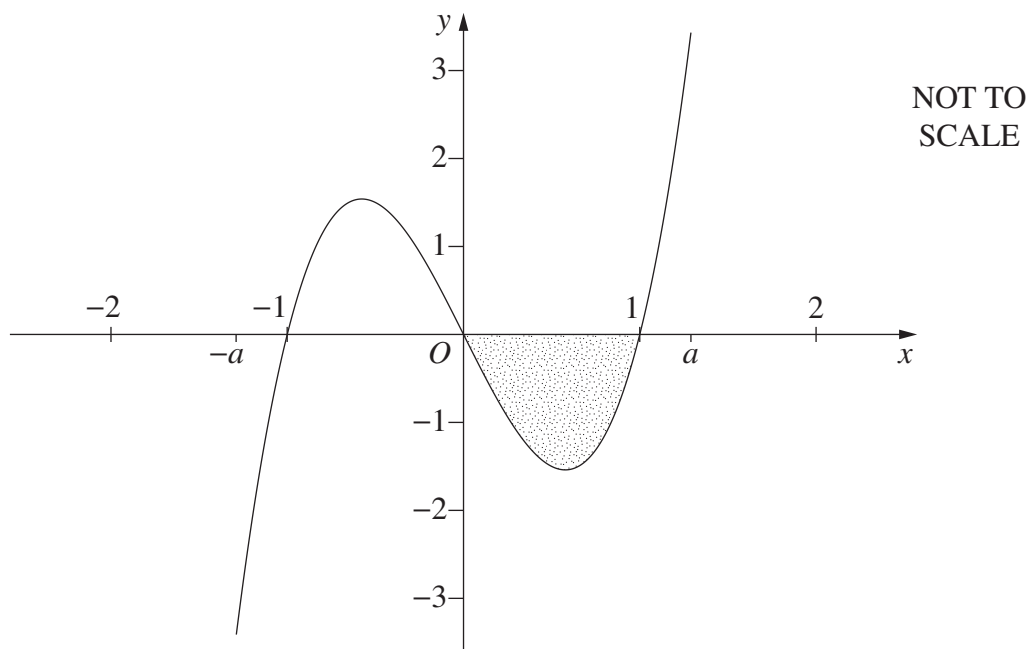
Which row of the table is correct for this polynomial?

	<i>Equation</i>	<i>Value of <math>b</math></i>	<i>Value of <math>c</math></i>
A.	$y = -(x - b)(x - c)^2$	$b > 0$	$c < 0$
B.	$y = -(x - b)(x - c)^2$	$b < 0$	$c > 0$
C.	$y = -x(x - b)(x - c)$	$b > 0$	$c < 0$
D.	$y = -x(x - b)(x - c)$	$b < 0$	$c > 0$

- 5 The diagram shows the graph  $y = f(x)$ , where  $f(x)$  is an odd function.

The shaded area is 1 square unit.

The number  $a$ , where  $a > 1$ , is chosen so that  $\int_0^a f(x) dx = 0$ .



What is the value of  $\int_{-a}^1 f(x) dx$ ?

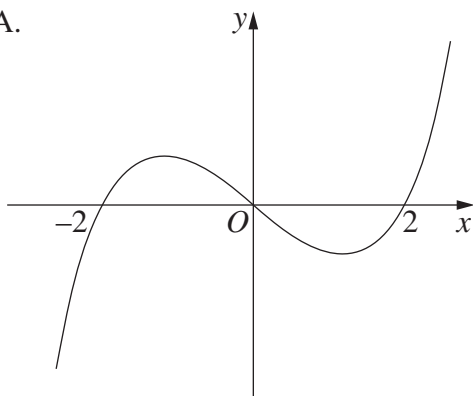
- A. -1
- B. 0
- C. 1
- D. 3

- 6 The following table gives the signs of the first and second derivatives of a function  $y = f(x)$  for different values of  $x$ .

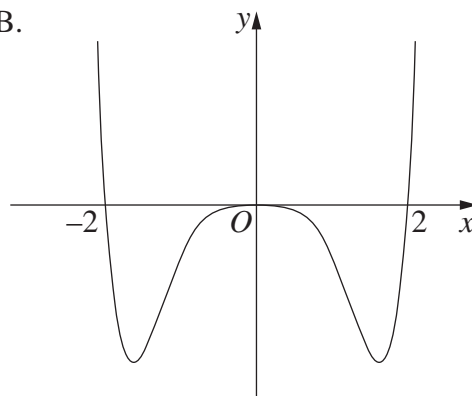
$x$	$-2$	$0$	$2$
$f'(x)$	$+$	$0$	$+$
$f''(x)$	$-$	$0$	$+$

Which of the following is a possible sketch of  $y = f(x)$ ?

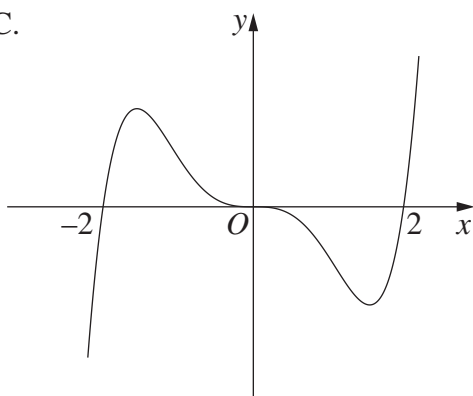
A.



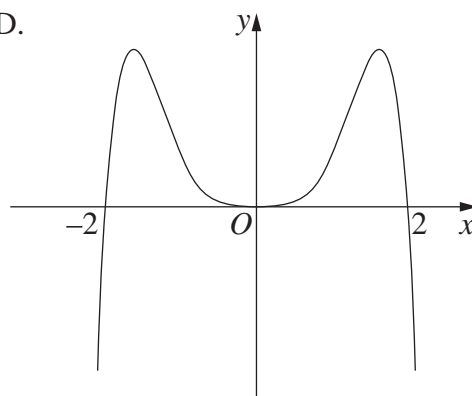
B.



C.



D.



- 7 It is given that  $y = f(g(x))$ , where  $f(1) = 3$ ,  $f'(1) = -4$ ,  $g(5) = 1$  and  $g'(5) = 2$ .

What is the value of  $y'$  at  $x = 5$ ?

- A.  $-8$
- B.  $-4$
- C.  $3$
- D.  $6$

- 8 What is the solution of the equation  $\log_a x^3 = b$ , where  $a$  and  $b$  are positive constants?

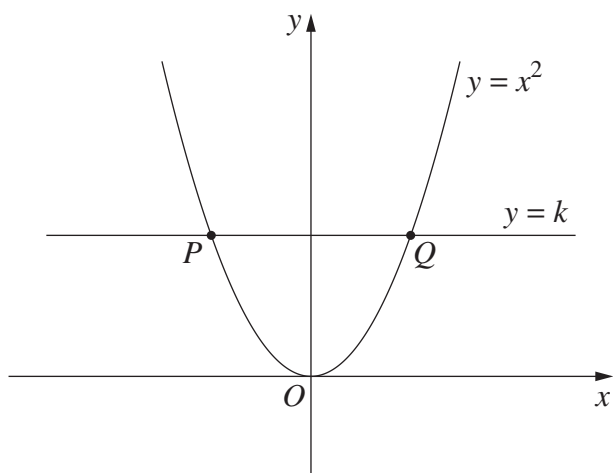
- A.  $x = b^{\frac{a}{3}}$
- B.  $x = a^{\frac{b}{3}}$
- C.  $x = \frac{b^a}{3}$
- D.  $x = \frac{a^b}{3}$

- 9 Let  $f(x)$  be any function with domain all real numbers.

Which of the following is an even function, regardless of the choice of  $f(x)$ ?

- A.  $2f(x)$
- B.  $f(f(x))$
- C.  $(f(-x))^2$
- D.  $f(x)f(-x)$

- 10 The graph  $y = x^2$  meets the line  $y = k$  (where  $k > 0$ ) at points  $P$  and  $Q$  as shown in the diagram. The length of the interval  $PQ$  is  $L$ .



Let  $a$  be a positive number. The graph  $y = \frac{x^2}{a^2}$  meets the line  $y = k$  at points  $S$  and  $T$ .

What is the length of  $ST$ ?

- A.  $\frac{L}{a}$
- B.  $\frac{L}{a^2}$
- C.  $aL$
- D.  $a^2L$



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Centre Number

# Mathematics Advanced

## Section II Answer Booklet

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Student Number

90 marks

Attempt Questions 11–32

Allow about 2 hours and 45 minutes for this section

### Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

**Question 11** (2 marks)

The first three terms of an arithmetic sequence are 3, 7 and 11.

2

Find the 15th term.

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Do NOT write in this area.

**Question 12** (3 marks)

The table shows the probability distribution of a discrete random variable.

$x$	0	1	2	3	4
$P(X = x)$	0	0.3	0.5	0.1	0.1

- (a) Show that the expected value  $E(X) = 2$ . **1**

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- (b) Calculate the standard deviation, correct to one decimal place. **2**

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**Question 13** (2 marks)

Let  $P(t)$  be a function such that  $\frac{dP}{dt} = 3000e^{2t}$ .

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When  $t = 0$ ,  $P = 4000$ .

Find an expression for  $P(t)$ .

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**Question 14** (3 marks)

Find the equation of the tangent to the curve  $y = (2x + 1)^3$  at the point  $(0, 1)$ .

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**Question 15** (5 marks)

A table of future value interest factors for an annuity of \$1 is shown.

<i>Rate</i> <i>Period</i>	1.5%	3%	4.5%	6%
5	5.152	5.309	5.471	5.637
10	10.703	11.464	12.288	13.181
20	23.124	26.870	31.371	36.786
40	54.268	75.401	107.030	154.762

- (a) Micky wants to save \$450 000 over the next 10 years.

**2**

If the interest rate is 6% per annum compounding annually, how much should Micky contribute each year? Give your answer to the nearest dollar.

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- (b) Instead, Micky decides to contribute \$8535 every three months for 10 years to an annuity paying 6% per annum, compounding quarterly.

**3**

How much will Micky have at the end of 10 years?

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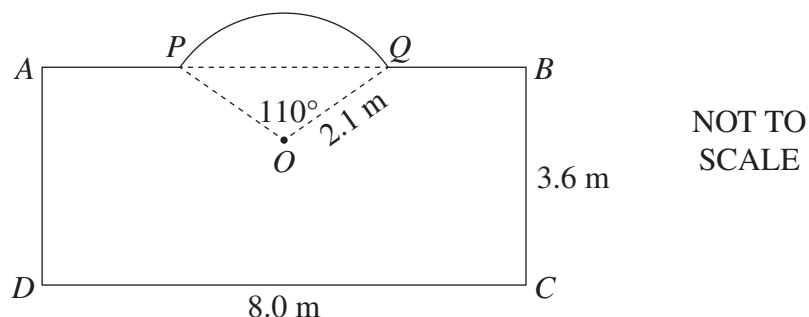
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**Question 16** (4 marks)

The diagram shows a shape  $APQBCD$ . The shape consists of a rectangle  $ABCD$  with an arc  $PQ$  on side  $AB$  and with side lengths  $BC = 3.6$  m and  $CD = 8.0$  m.

4

The arc  $PQ$  is an arc of a circle with centre  $O$  and radius  $2.1$  m and  $\angle POQ = 110^\circ$ .



What is the perimeter of the shape  $APQBCD$ ? Give your answer correct to one decimal place.

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**Question 17** (2 marks)

Find  $\int x\sqrt{x^2 + 1} \, dx$ .

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**Please turn over**

**Question 18** (6 marks)

A university uses gas to heat its buildings. Over a period of 10 weekdays during winter, the gas used each day was measured in megawatts (MW) and the average outside temperature each day was recorded in degrees Celsius ( $^{\circ}\text{C}$ ).

Using  $x$  as the average daily outside temperature and  $y$  as the total daily gas usage, the equation of the least-squares regression line was found.

The equation of the regression line predicts that when the temperature is  $0^{\circ}\text{C}$ , the daily gas usage is 236 MW.

The ten temperatures measured were:  $0^{\circ}$ ,  $0^{\circ}$ ,  $0^{\circ}$ ,  $2^{\circ}$ ,  $5^{\circ}$ ,  $7^{\circ}$ ,  $8^{\circ}$ ,  $9^{\circ}$ ,  $9^{\circ}$ ,  $10^{\circ}$ .

The total gas usage for the ten weekdays was 1840 MW.

In any bivariate dataset, the least-squares regression line passes through the point  $(\bar{x}, \bar{y})$ , where  $\bar{x}$  is the sample mean of the  $x$ -values and  $\bar{y}$  is the sample mean of the  $y$ -values.

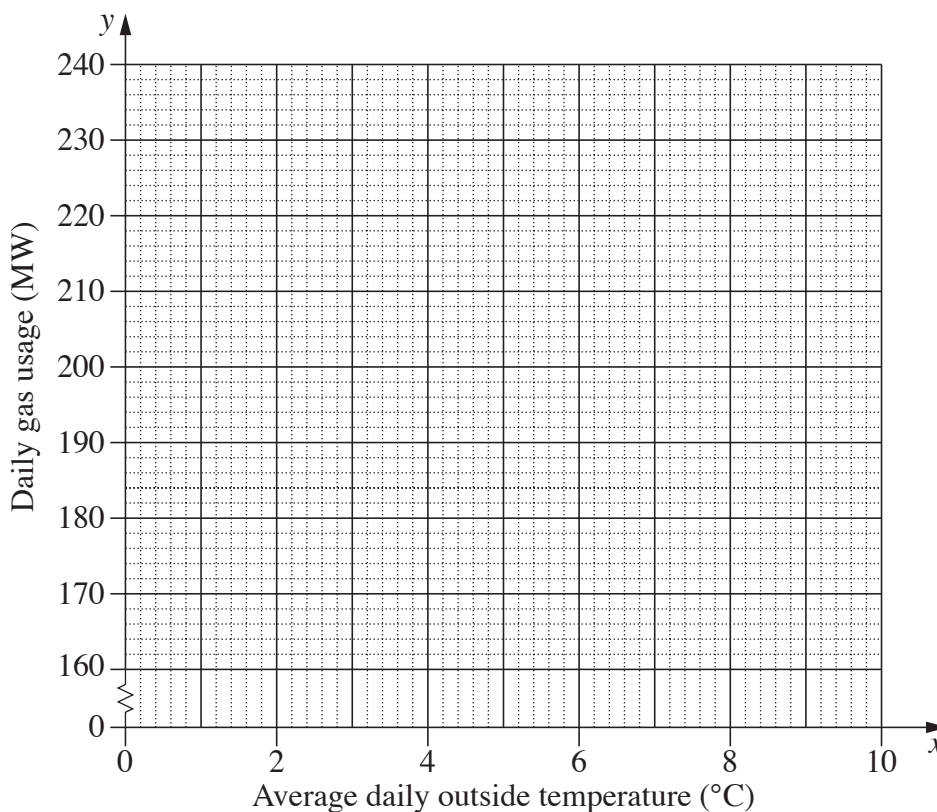
- (a) Using the information provided, plot the point  $(\bar{x}, \bar{y})$  and the  $y$ -intercept of the least-squares regression line on the grid.

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**Question 18 continues on page 17**



Question 18 (continued)

(b) What is the equation of the regression line?

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(c) In the context of the dataset, identify ONE problem with using the regression line to predict gas usage when the average outside temperature is 23°C.

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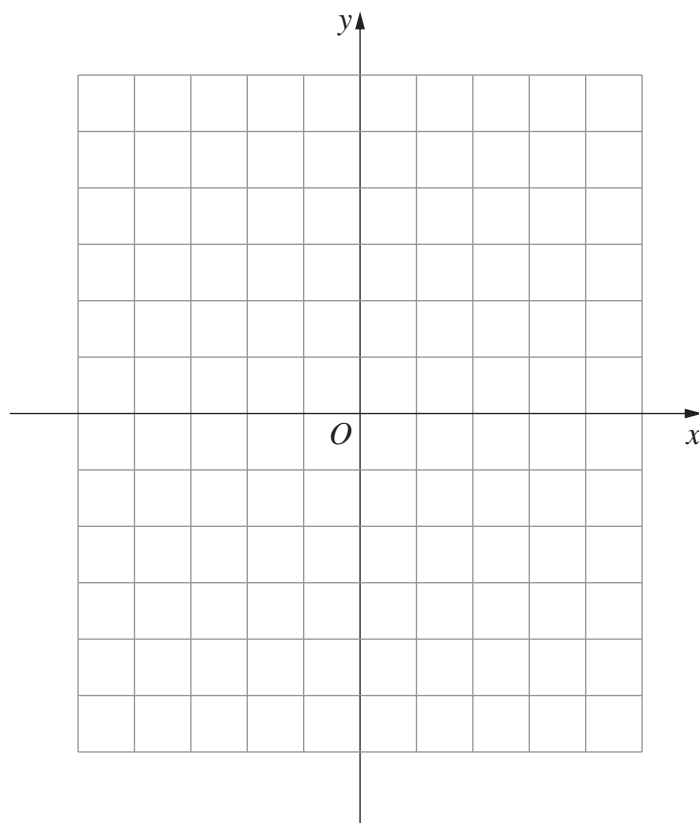
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**End of Question 18**

**Please turn over**

**Question 19** (4 marks)

- (a) Sketch the graphs of the functions  $f(x) = x - 1$  and  $g(x) = (1 - x)(3 + x)$  showing the  $x$ -intercepts. 2



- (b) Hence, or otherwise, solve the inequality  $x - 1 < (1 - x)(3 + x)$ . 2

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**Question 20** (3 marks)

Find all the values of  $\theta$ , where  $0^\circ \leq \theta \leq 360^\circ$ , such that

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$$\sin(\theta - 60^\circ) = -\frac{\sqrt{3}}{2}.$$

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**Please turn over**

**Question 21** (3 marks)

The fourth term of a geometric sequence is 48.

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The eighth term of the same sequence is  $\frac{3}{16}$ .

Find the possible value(s) of the common ratio and the corresponding first term(s).

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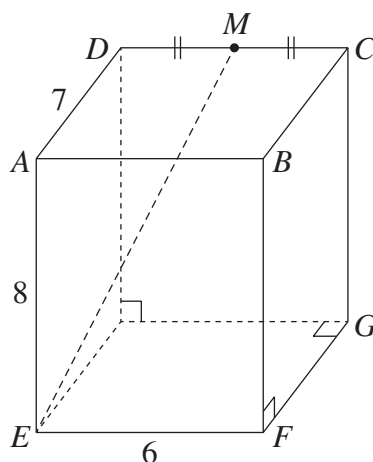
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**Question 22** (3 marks)

In the rectangular prism shown,  $AD = 7$  cm,  $AE = 8$  cm,  $EF = 6$  cm. Point  $M$  is the midpoint of  $CD$ .

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SCALE

Find  $\angle AEM$ , to the nearest degree.

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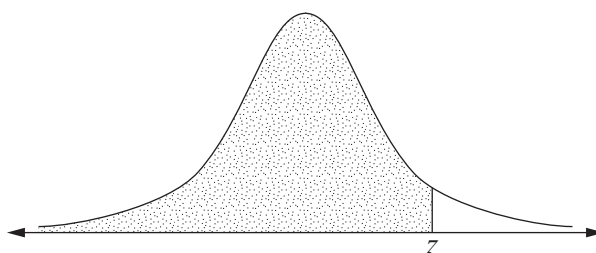
**Question 23** (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below  $z$  for some positive values of  $z$ .

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$z$	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

The probability values given in the table are represented by the shaded area in the following diagram.



The weights of adult male koalas form a normal distribution with mean  $\mu = 10.40$  kg, and standard deviation  $\sigma = 1.15$  kg.

In a group of 400 adult male koalas, how many would be expected to weigh more than 11.93 kg?

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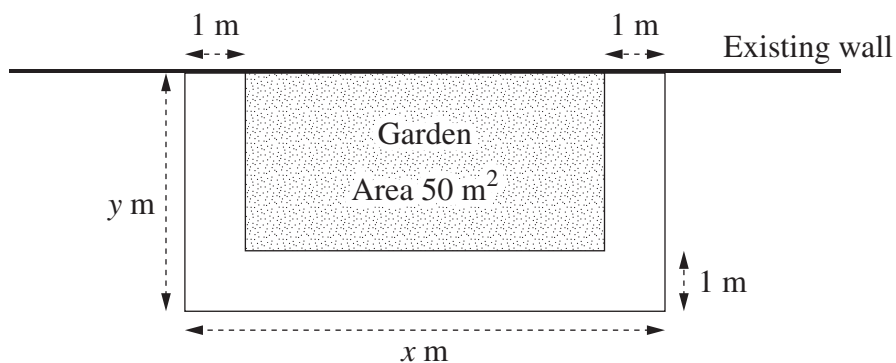
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**Questions 11–23 are worth 44 marks in total**

**Please turn over**

**Question 24** (5 marks)

A gardener wants to build a rectangular garden of area  $50 \text{ m}^2$  against an existing wall as shown in the diagram. A concrete path of width 1 metre is to be built around the other three sides of the garden.



Let  $x$  and  $y$  be the dimensions, in metres, of the outer rectangle as shown.

- (a) Show that  $y = \frac{50}{x-2} + 1$ .

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**Question 24 continues on page 25**



Question 24 (continued)

- (b) Find the value of  $x$  such that the area of the concrete path is a minimum. Show that your answer gives a minimum area.

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End of Question 24

**Question 25** (6 marks)

On the first day of November, Jia deposits \$10 000 into a new account which earns 0.4% interest per month, compounded monthly. At the end of each month, after the interest is added to the account, Jia intends to withdraw \$ $M$  from the account.

Let  $A_n$  be the amount (in dollars) in Jia's account at the end of  $n$  months.

- (a) Show that  $A_2 = 10\,000(1.004)^2 - M(1.004) - M$ .

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- (b) Show that  $A_n = (10\,000 - 250M)(1.004)^n + 250M$ .

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**Question 25 continues on page 27**

Question 25 (continued)

- (c) Jia wants to be able to make at least 100 withdrawals.

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What is the largest value of  $M$  that will enable Jia to do this?

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**End of Question 25**

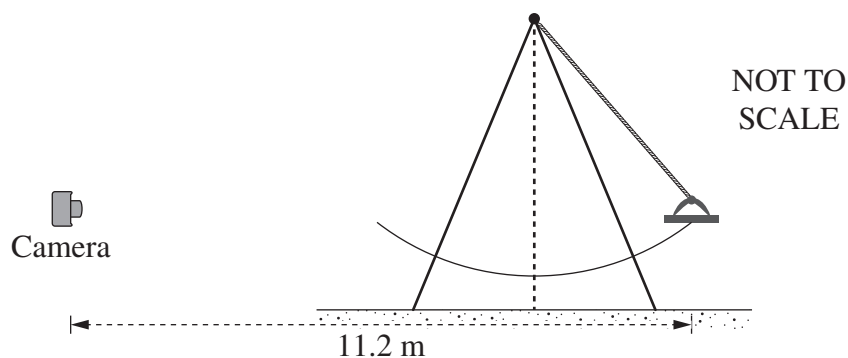
**Please turn over**

**Question 26** (4 marks)

A camera films the motion of a swing in a park.

Let  $x(t)$  be the horizontal distance, in metres, from the camera to the seat of the swing at  $t$  seconds.

The seat is released from rest at a horizontal distance of 11.2 m from the camera.



- (a) The rate of change of  $x$  can be modelled by the equation

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$$\frac{dx}{dt} = -1.5\pi \sin\left(\frac{5\pi}{4}t\right).$$

Find an expression for  $x(t)$ .

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- (b) How many times does the swing reach the closest point to the camera during the first 10 seconds?

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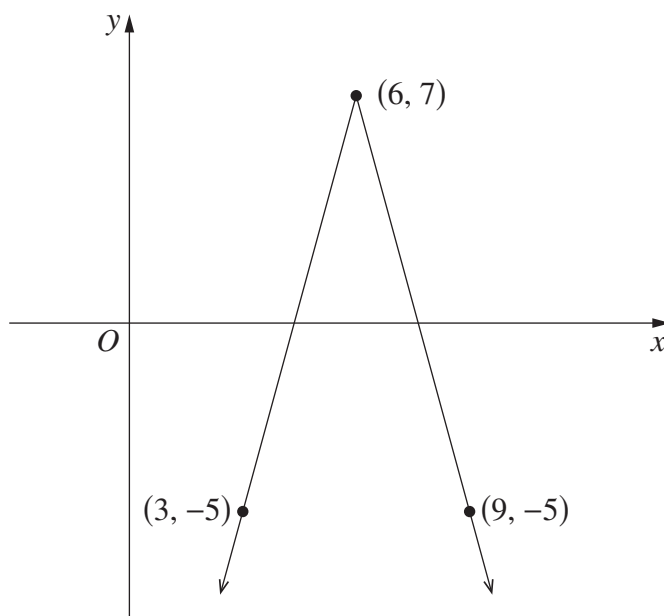
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**Question 27** (5 marks)

The graph of  $y = f(x)$ , where  $f(x) = a|x - b| + c$ , passes through the points  $(3, -5)$ ,  $(6, 7)$  and  $(9, -5)$  as shown in the diagram.



- (a) Find the values of  $a$ ,  $b$  and  $c$ .

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- (b) The line  $y = mx$  cuts the graph of  $y = f(x)$  in two distinct places.

**2**

Find all possible values of  $m$ .

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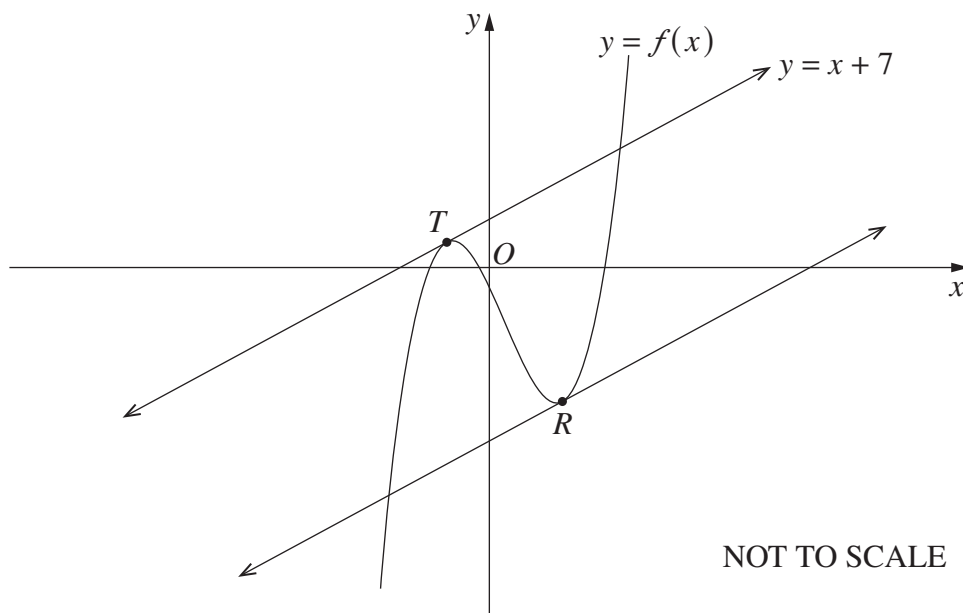
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**Question 28** (4 marks)

The curve  $y = f(x)$  is shown on the diagram. The equation of the tangent to the curve at point  $T (-1, 6)$  is  $y = x + 7$ . At a point  $R$ , another tangent parallel to the tangent at  $T$  is drawn.

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The gradient function of the curve is given by  $\frac{dy}{dx} = 3x^2 - 6x - 8$ .

Find the coordinates of  $R$ .

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**Question 29** (6 marks)

A continuous random variable  $X$  has probability density function  $f(x)$  given by

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \leq x \leq 1 \\ 0, & \text{for all other values of } x \end{cases}$$

- (a) Find the mode of  $X$ .

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- (b) Find the cumulative distribution function for the given probability density function.

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- (c) Without calculating the median, show that the mode is greater than the median.

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**Question 30** (5 marks)

Let  $f(x) = e^{-x} \sin x$ .

- (a) Find the coordinates of the stationary points of  $f(x)$  for  $0 \leq x \leq 2\pi$ .  
You do NOT need to check the nature of the stationary points.

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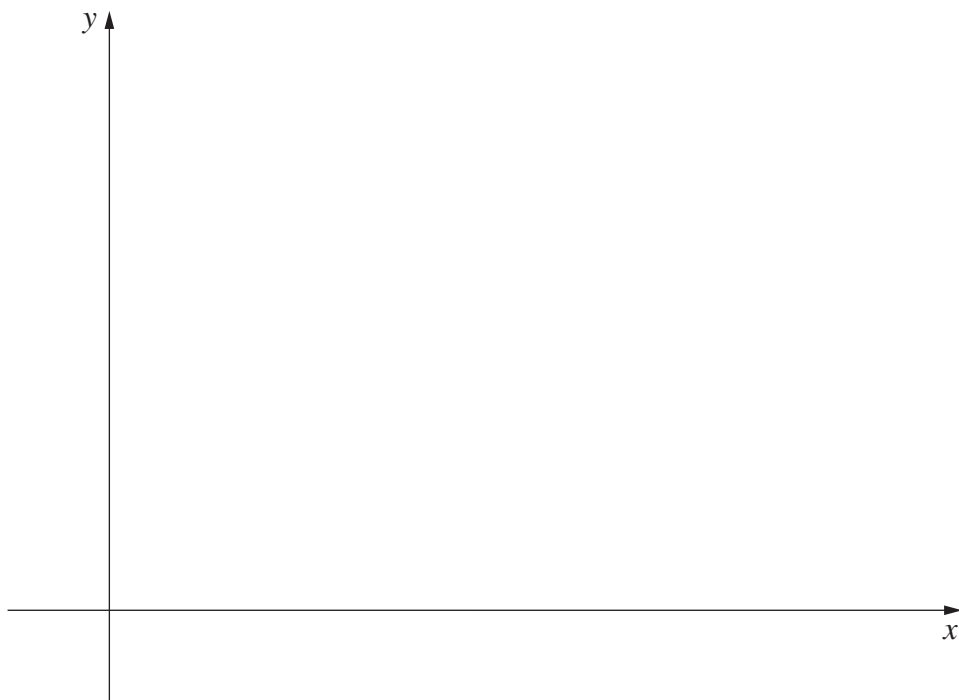
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**Question 30 continues on page 33**



Question 30 (continued)

- (b) Without using any further calculus, sketch the graph of  $y = f(x)$ , for  $0 \leq x \leq 2\pi$ , showing stationary points and intercepts. 2



**End of Question 30**

**Please turn over**

**Question 31** (5 marks)

Four Year 12 students want to organise a graduation party. All four students have the same probability,  $P(F)$ , of being available next Friday. All four students have the same probability,  $P(S)$ , of being available next Saturday.

It is given that  $P(F) = \frac{3}{10}$ ,  $P(S|F) = \frac{1}{3}$ , and  $P(F|S) = \frac{1}{8}$ .

Kim is one of the four students.

- (a) Is Kim's availability next Friday independent from his availability next Saturday? Justify your answer. 1

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- (b) Show that the probability that Kim is available next Saturday is  $\frac{4}{5}$ . 2

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**Question 31 continues on page 35**

Question 31 (continued)

- (c) What is the probability that at least one of the four students is NOT available next Saturday? **2**

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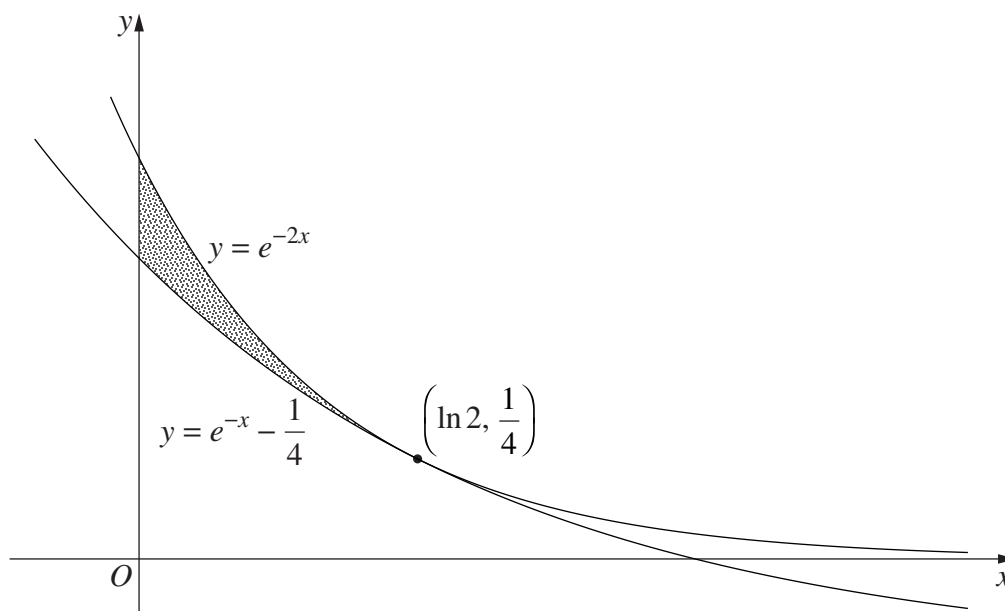
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**End of Question 31**

**Please turn over**

**Question 32** (6 marks)

The curves  $y = e^{-2x}$  and  $y = e^{-x} - \frac{1}{4}$  intersect at exactly one point as shown in the diagram. The point of intersection has coordinates  $\left(\ln 2, \frac{1}{4}\right)$ . (Do NOT prove this.)



- (a) Show that the area bounded by the two curves and the y-axis, as shaded in the diagram, is  $\frac{1}{4} \ln 2 - \frac{1}{8}$ .

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**Question 32 continues on page 37**

Question 32 (continued)

- (b) Find the values of  $k$  such that the curves  $y = e^{-2x}$  and  $y = e^{-x} + k$  intersect at two points.

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**If you use this space, clearly indicate which question you are answering.**

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**Section II extra writing space**

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Mathematics Advanced  
Mathematics Extension 1  
Mathematics Extension 2

**REFERENCE SHEET**

**Measurement**

**Length**

$$l = \frac{\theta}{360} \times 2\pi r$$

**Area**

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

**Surface area**

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

**Volume**

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

**Functions**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For  $ax^3 + bx^2 + cx + d = 0$ :

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

**Relations**

$$(x - h)^2 + (y - k)^2 = r^2$$

**Financial Mathematics**

$$A = P(1 + r)^n$$

**Sequences and series**

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

**Logarithmic and Exponential Functions**

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

## Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

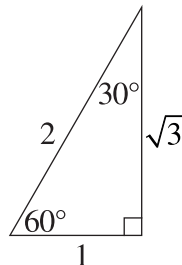
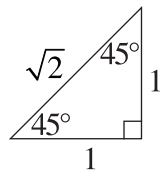
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



### Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

### Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

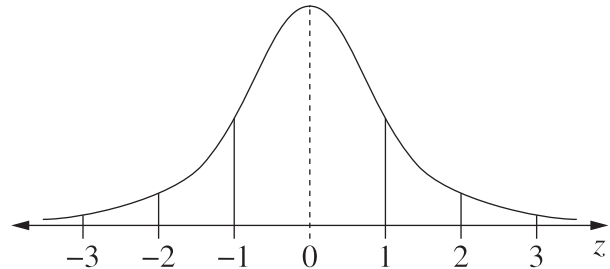
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

## Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score  
less than  $Q_1 - 1.5 \times IQR$   
or  
more than  $Q_3 + 1.5 \times IQR$

### Normal distribution



- approximately 68% of scores have  $z$ -scores between  $-1$  and  $1$
- approximately 95% of scores have  $z$ -scores between  $-2$  and  $2$
- approximately 99.7% of scores have  $z$ -scores between  $-3$  and  $3$

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

### Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

### Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

### Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

## Differential Calculus

### Function

### Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

## Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where  $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \cdots + f(x_{n-1})] \right\}$$

where  $a = x_0$  and  $b = x_n$

## Combinatorics

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

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## Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

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## Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

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## Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$