NSW Education Standards Authority

2019 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Standard 2

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 15 marks (pages 2–8)

- Attempt Questions 1–15
- Allow about 25 minutes for this section

Section II – 85 marks (pages 9–32)

- Attempt Questions 16–42
- Allow about 2 hours and 5 minutes for this section

Section I

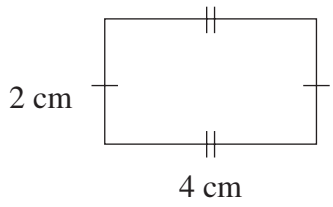
15 marks
Attempt Questions 1–15
Allow about 25 minutes for this section

Use the multiple-choice answer sheet for Questions 1–15.

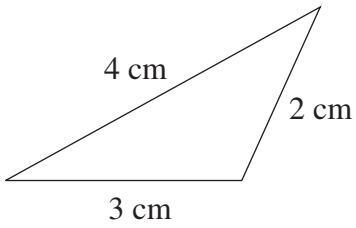
---

1 Which of the following shapes has a perimeter of 12 cm?

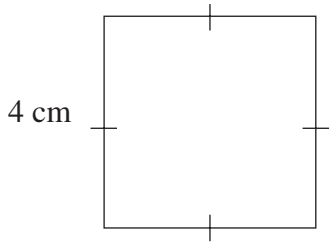
A.



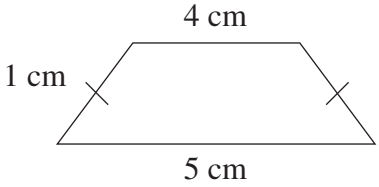
B.



C.



D.



NOT TO SCALE

2 Sugar is sold in four different sized packets.

Which is the best buy?

A. 100 g for $0.40

B. 500 g for $1.65

C. 1 kg for $3.50

D. 2 kg for $6.90

3 Chris opens a bank account and deposits $1000 into it. Interest is paid at 3.5% per annum, compounding annually.

Assuming no further deposits or withdrawals are made, what will be the balance in the account at the end of two years?

A. $1070.00

B. $1071.23

C. $1822.50

D. $2070.00

4 Which compass bearing is the same as a true bearing of 110°?

A. S20°E

B. S20°W

C. S70°E

D. S70°W

5 The Coordinated Universal Time (UTC) of Auckland is +12 hours and the UTC of Chicago is −5 hours.

When the time in Chicago is 2 pm, Thursday, what is the time in Auckland?

A. 9 pm, Wednesday

B. 7 am, Thursday

C. 9 pm, Thursday

D. 7 am, Friday

6 Mary is 18 years old and has just purchased comprehensive motor vehicle insurance. The following excesses apply to claims for at-fault motor vehicle accidents.

- Basic excess of $850 for each claim
- An additional age excess of $1600 for drivers under 25 years of age
- An additional age excess of $400 for drivers 25 years of age or over with no more than 2 years driving experience

How much would Mary be required to pay as excess if she made a claim as the driver at fault in a car accident?

A. $1600

B. $850 + $400

C. $850 + $1600

D. $850 + $1600 + $400

7 Julia earns $28 per hour. Her hourly pay rate increases by 2%.

How much will she earn for a 4-hour shift with this increase?

A. $2.24

B. $28.56

C. $112

D. $114.24

8 A person's weight is measured as 79.3 kg.

What is the absolute error of this measurement?

A. 10 grams

B. 50 grams

C. 100 grams

D. 500 grams

9 What is the interest earned, in dollars, if \$800 is invested for x months at a simple interest rate of 3% per annum?

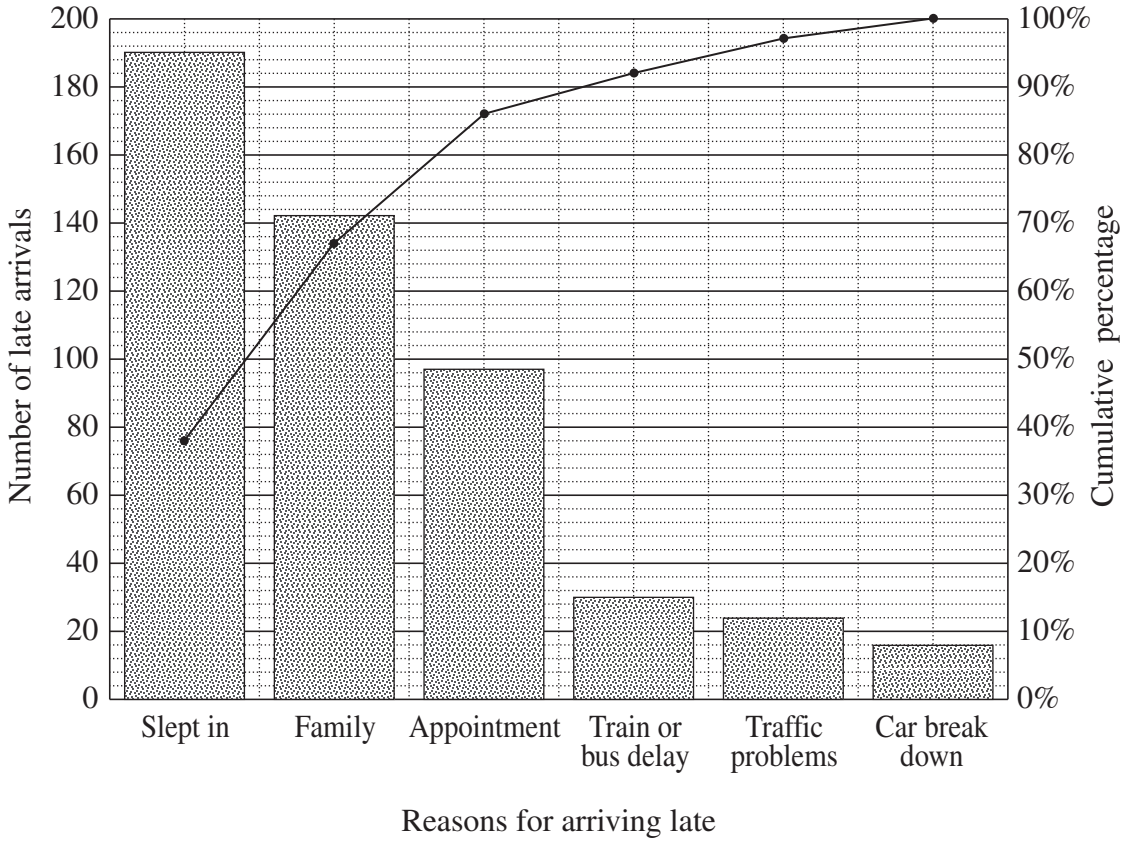
A. $2x$

B. $24x$

C. $200x$

D. $2400x$

10 A school collected data related to the reasons given by students for arriving late. The Pareto chart shows the data collected.



What percentage of students gave the reason 'Train or bus delay'?

A. 6%

B. 15%

C. 30%

D. 92%

11 Which of the following correctly expresses y as the subject of the formula $3x - 4y - 1 = 0$?

A. $y = \frac{3}{4}x - 1$

B. $y = \frac{3}{4}x + 1$

C. $y = \frac{3x - 1}{4}$

D. $y = \frac{3x + 1}{4}$

12 An owl is 7 metres above ground level, in a tree. The owl sees a mouse on the ground at an angle of depression of 32°.

How far must the owl fly in a straight line to catch the mouse, assuming the mouse does not move?

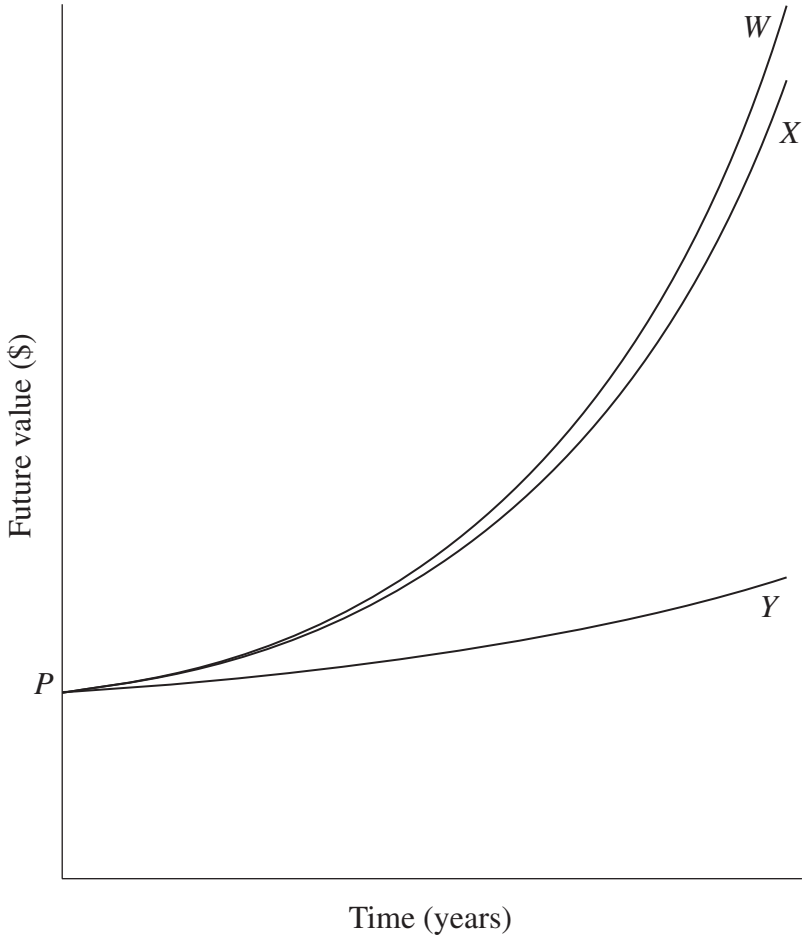
A. 3.7 m

B. 5.9 m

C. 8.3 m

D. 13.2 m

13 The graphs show the future values over time of $\$P$, invested at three different rates of compound interest.



Which of the following correctly identifies each graph?

A.

W	5% pa, compounding annually
X	10% pa, compounding annually
Y	10% pa, compounding quarterly

B.

W	5% pa, compounding annually
X	10% pa, compounding quarterly
Y	10% pa, compounding annually

C.

W	10% pa, compounding quarterly
X	10% pa, compounding annually
Y	5% pa, compounding annually

D.

W	10% pa, compounding annually
X	10% pa, compounding quarterly
Y	5% pa, compounding annually

14 Last Saturday, Luke had 165 followers on social media. Rhys had 537 followers. On average, Luke gains another 3 followers per day and Rhys loses 2 followers per day.

If x represents the number of days since last Saturday and y represents the number of followers, which pair of equations model this situation?

A. Luke: $y = 165x + 3$
Rhys: $y = 537x - 2$

B. Luke: $y = 165 + 3x$
Rhys: $y = 537 - 2x$

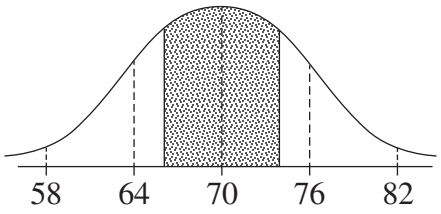
C. Luke: $y = 3x + 165$
Rhys: $y = 2x - 537$

D. Luke: $y = 3 + 165x$
Rhys: $y = 2 - 537x$

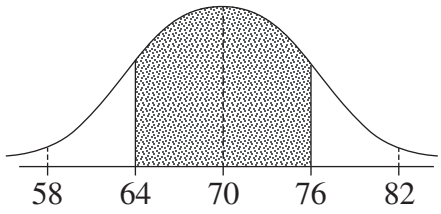
15 The scores on an examination are normally distributed with a mean of 70 and a standard deviation of 6. Michael received a score on the examination between the lower quartile and the upper quartile of the scores.

Which shaded region most accurately represents where Michael's score lies?

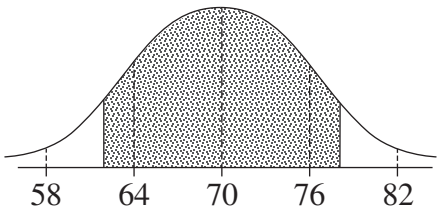
A.



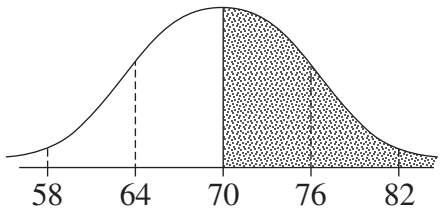
B.



C.



D.



2019 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Standard 2

Section II Answer Booklet

85 marks
Attempt Questions 16–42
Allow about 2 hours and 5 minutes for this section

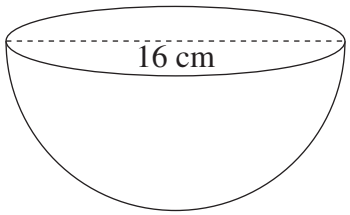
Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Please turn over

Question 16 (2 marks)

A bowl is in the shape of a hemisphere with a diameter of 16 cm. **2**



What is the volume of the bowl, correct to the nearest cubic centimetre?

.....

.....

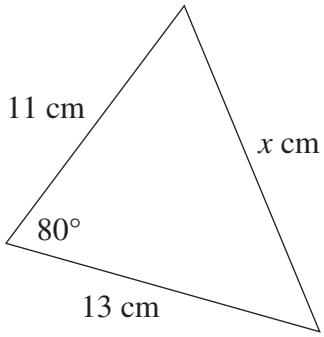
.....

.....

.....

Question 17 (3 marks)

The diagram shows a triangle with sides of length x cm, 11 cm and 13 cm and an angle of 80°. **3**



NOT TO SCALE

Use the cosine rule to calculate the value of x, correct to two significant figures.

.....

.....

.....

.....

.....

.....

Question 18 (4 marks)

Andrew, Brandon and Cosmo are the first three batters in the school cricket team. In a recent match, Andrew scored 30 runs, Brandon scored 25 runs and Cosmo scored 40 runs.

(a) What is the ratio of Andrew's to Brandon's to Cosmo's runs scored, in simplest form? **2**

.....

.....

(b) In this match, the ratio of the total number of runs scored by Andrew, Brandon and Cosmo to the total number of runs scored by the whole team is 19 : 36. **2**

How many runs were scored by the whole team?

.....

.....

.....

.....

.....

.....

Question 19 (3 marks)

The heights, in centimetres, of 10 players on a basketball team are shown. **3**

170, 180, 185, 188, 192, 193, 193, 194, 196, 202

Is the height of the shortest player on the team considered an outlier? Justify your answer with calculations.

.....

.....

.....

.....

.....

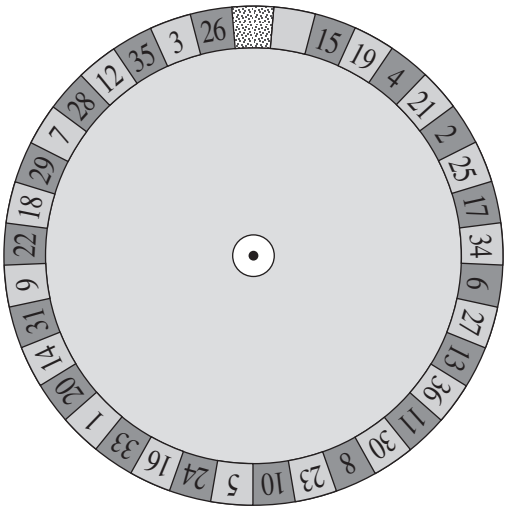
.....

.....

.....

Question 20 (2 marks)

A roulette wheel has the numbers 0, 1, 2, …, 36 where each of the 37 numbers is equally likely to be spun. **2**



If the wheel is spun 18 500 times, calculate the expected frequency of spinning the number 8.

.....

.....

.....

.....

.....

.....

Question 21 (2 marks)

A person owns 1526 shares with a market value of $8.75 per share. The total dividend received for these shares is $1068.20. **2**

Calculate the percentage dividend yield.

.....

.....

.....

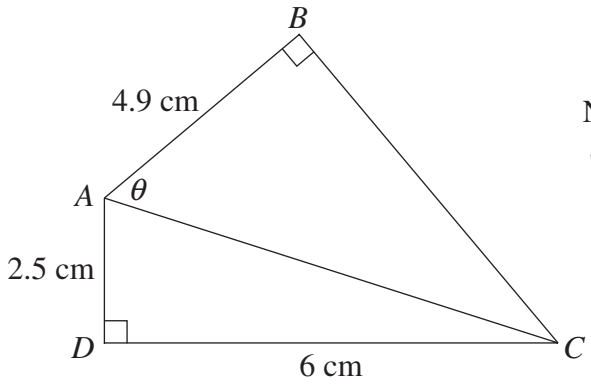
.....

.....

.....

Question 22 (3 marks)

Two right-angled triangles, *ABC* and *ADC*, are shown. **3**



Calculate the size of angle θ, correct to the nearest minute.

.....

.....

.....

.....

.....

.....

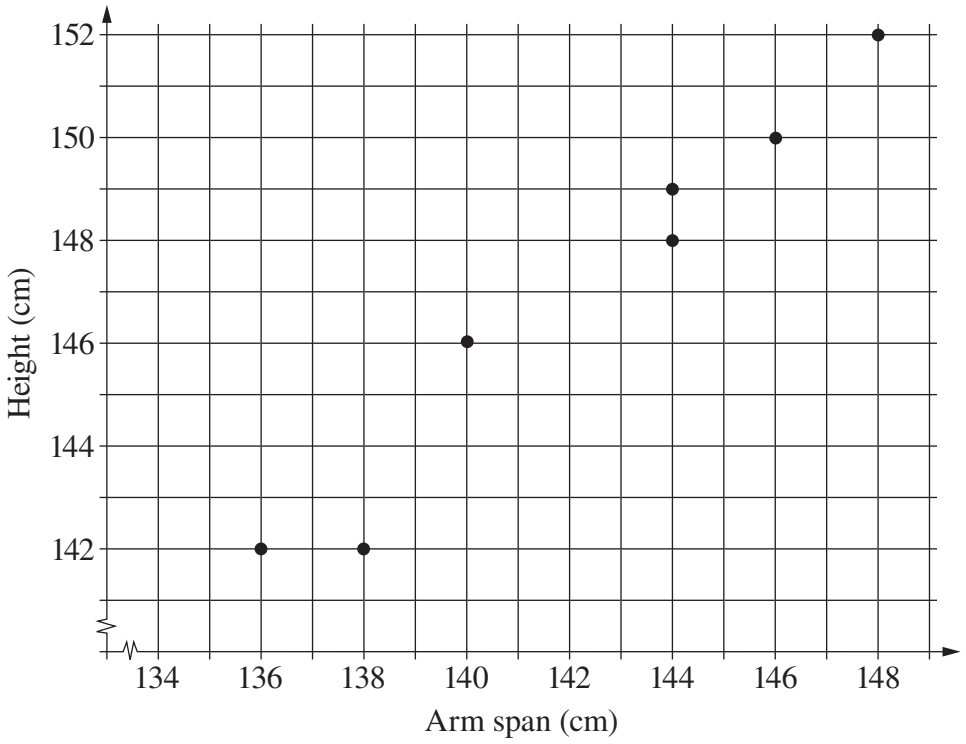
.....

.....

.....

Question 23 (3 marks)

A set of bivariate data is collected by measuring the height and arm span of seven children. The graph shows a scatterplot of these measurements.



(a) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. **1**

.....

(b) Identify the direction and the strength of the linear association between height and arm span. **1**

.....

(c) The equation of the least-squares regression line is shown. **1**

$$\text{Height} = 0.866 \times (\text{arm span}) + 23.7$$

A child has an arm span of 143 cm.

Calculate the predicted height for this child using the equation of the least-squares regression line.

.....

.....

Question 24 (2 marks)

Amanda uses 80 kilocalories of energy per kilometre while she is running. **2**

She eats a burger that contains 2180 kilojoules of energy. How many kilometres will she need to run to use up all the energy from the burger? Give your answer correct to one decimal place. (1 kilocalorie = 4.184 kilojoules)

.....

.....

.....

.....

.....

Question 25 (3 marks)

A bowl of fruit contains 17 apples of which 9 are red and 8 are green. **3**

Dennis takes one apple at random and eats it. Margaret also takes an apple at random and eats it.

By drawing a probability tree diagram, or otherwise, find the probability that Dennis and Margaret eat apples of the same colour.

.....

.....

.....

.....

.....

Question 26 (4 marks)

A project requires activities *A* to *F* to be completed. The activity chart shows the immediate prerequisite(s) and duration for each activity.

Activity	*Immediate prerequisite(s)*	*Duration in hours*
A	—	2
B	*A*	6
C	*A*	5
D	*B*	2
E	*C*, *D*	4
F	*E*	1

(a) By drawing a network diagram, determine the minimum time for the project to be completed. **3**

Minimum time = ...

(b) Determine the float time of the non-critical activity. **1**

.....

.....

Question 27 (3 marks)

Ashley has a credit card with the following conditions: **3**

- There is no interest-free period.
- Interest is charged at the end of each month at 18.25% per annum, compounding daily, from the purchase date (included) to the last day of the month (included).

Ashley's credit card statement for April is shown, with some figures missing.

Statement period: 1 April to 30 April

Date	**Details**	**Amount ($)**
1 April	Opening balance	0
20 April	Furniture	3700
30 April	Interest charged	***
30 April	Closing balance	***

Minimum payment: ☐

The minimum payment is calculated as 2% of the closing balance on 30 April.

Calculate the minimum payment.

.....

.....

.....

.....

.....

.....

Question 28 (4 marks)

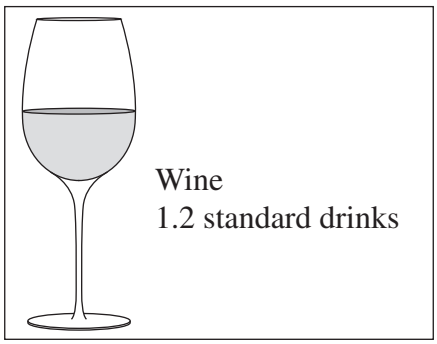
The formula below is used to calculate an estimate for blood alcohol content (*BAC*) for females. **4**

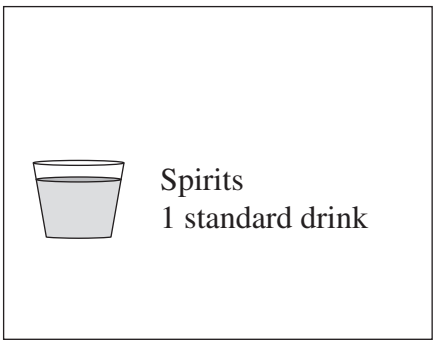
$$BAC_{\text{Female}} = \frac{10N - 7.5H}{5.5M}$$

The number of hours required for a person to reach zero *BAC* after they stop consuming alcohol is given by the following formula.

$$\text{Time} = \frac{BAC}{0.015}$$

The number of standard drinks in a glass of wine and a glass of spirits is shown.





Hannah weighs 60 kg. She consumed 3 glasses of wine and 4 glasses of spirits between 6:15 pm and 12:30 am the following day. She then stopped drinking alcohol.

Using the given formulae, calculate the time in the morning when Hannah's *BAC* should reach zero.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 29 (2 marks)

Part of a supermarket receipt is shown. **2**

SUPERMARKET

RECEIPT

Date: 22/09/2019

Description	$
*Chocolates 300 g	*A*
Tomatoes 1 kg	5.00
Natural almonds 400 g	*B*
Cheese slices 500 g	8.50
Milk 2 L	3.20
Bananas 570 g	2.85
Total for 6 items	**36.25**
GST included in total	0.70

*GST of 10% is included in the price of item.

Determine the missing values, *A* and *B*, to complete the receipt.

.....

.....

.....

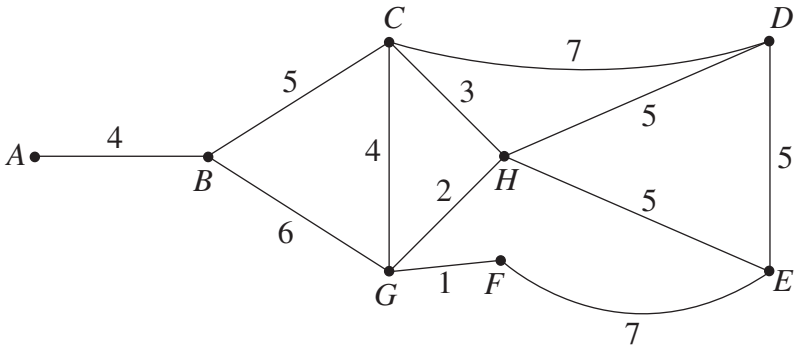
.....

.....

Do NOT write in this area.

Question 30 (3 marks)

The network diagram shows the tracks connecting 8 picnic sites in a nature park. The vertices A to H represent the picnic sites. The weights on the edges represent the distances along the tracks between the picnic sites, in kilometres.



(a) Each picnic site needs to provide drinking water. The main water source is at site A. **2**

By drawing a minimum spanning tree in the space below, calculate the minimum length of water pipes required to supply water to all the sites if the water pipes can only be laid along the tracks.

Minimum length = ...

(b) One day, the track between C and H is closed. State the vertices that identify the shortest path from C to E that avoids the closed track. **1**

.....

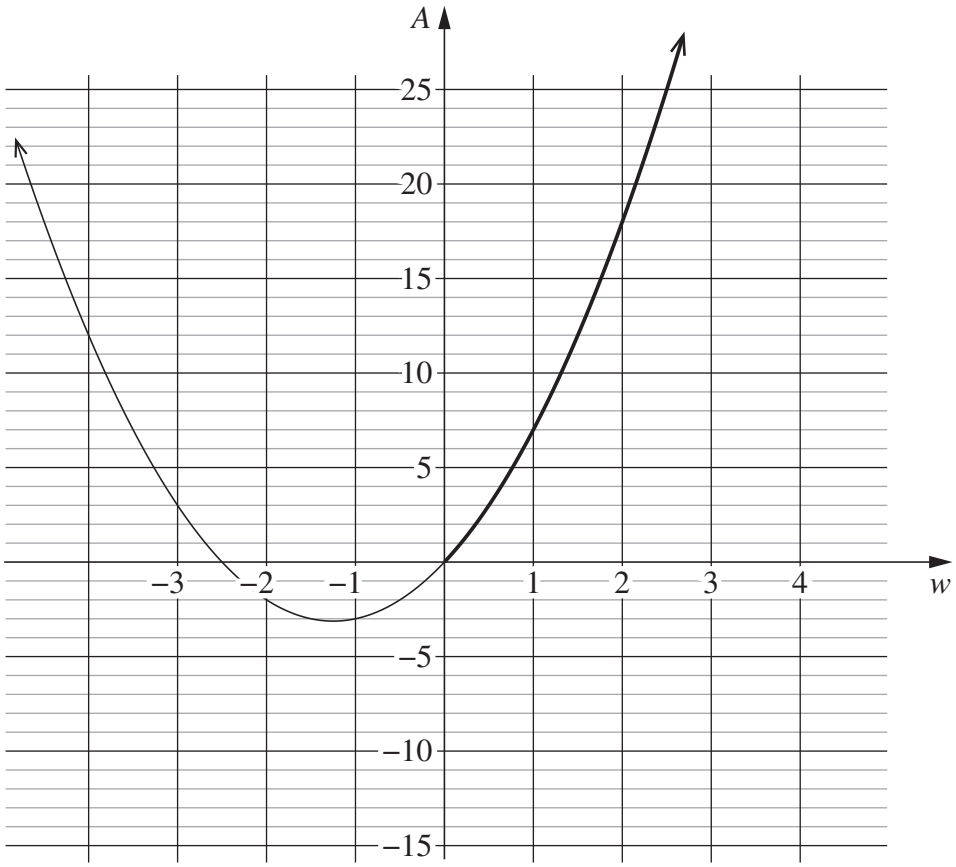
.....

Questions 16–30 are worth 43 marks in total

Question 31 (3 marks)

A rectangle has width w centimetres. The area of the rectangle, A, in square centimetres, is $A = 2w^2 + 5w$.

The graph of $A = 2w^2 + 5w$ is shown.



(a) Explain why, in this context, the model $A = 2w^2 + 5w$ only makes sense for the bold section of the graph. **1**

.....

.....

(b) The area of the rectangle is 18 cm^2. Calculate the perimeter of the rectangle. **2**

.....

.....

.....

.....

.....

Question 32 (3 marks)

The table shows the income tax rates for the 2018–2019 financial year. **3**

Taxable income	*Tax payable on this income*
\$0 to \$18 200	Nil
\$18 201 to \$37 000	19c for each \$1 over \$18 200
\$37 001 to \$90 000	\$3 572 plus 32.5c for each \$1 over \$37 000
\$90 001 to \$180 000	\$20 797 plus 37c for each \$1 over \$90 000
\$180 001 and over	\$54 097 plus 45c for each \$1 over \$180 000

The Medicare levy is calculated as 2% of taxable income.

For the 2018–2019 financial year, Charlie pays a Medicare levy of \$1934.80.

Calculate the tax payable on Charlie's taxable income.

.....

.....

.....

.....

.....

.....

Question 33 (4 marks)

The time taken for a car to travel between two towns at a constant speed varies inversely with its speed.

It takes 1.5 hours for the car to travel between the two towns at a constant speed of 80 km/h.

(a) Calculate the distance between the two towns. **1**

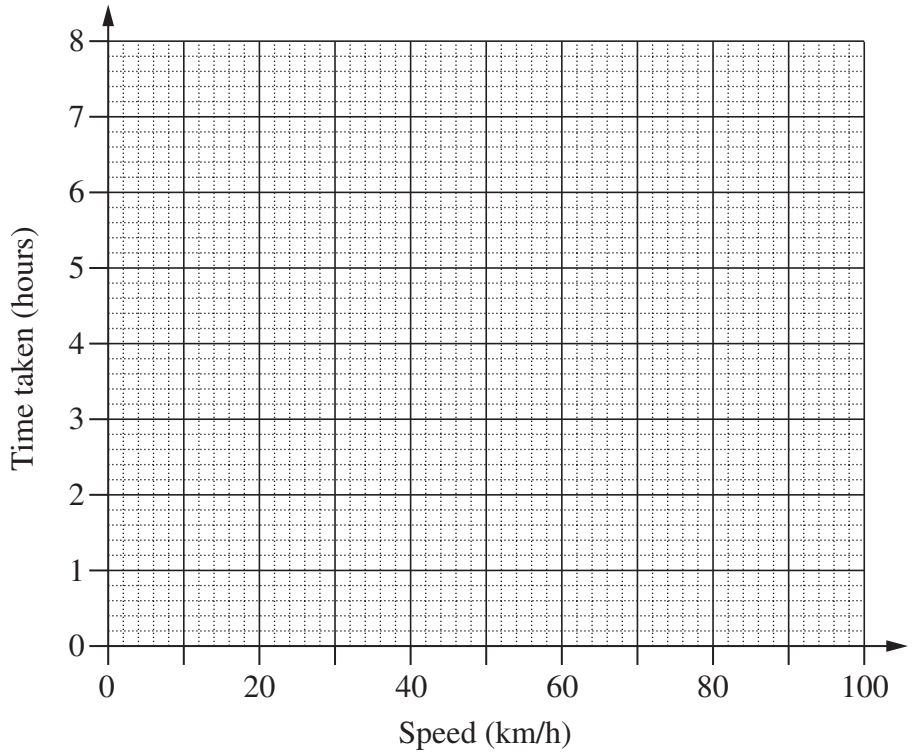
.....

.....

.....

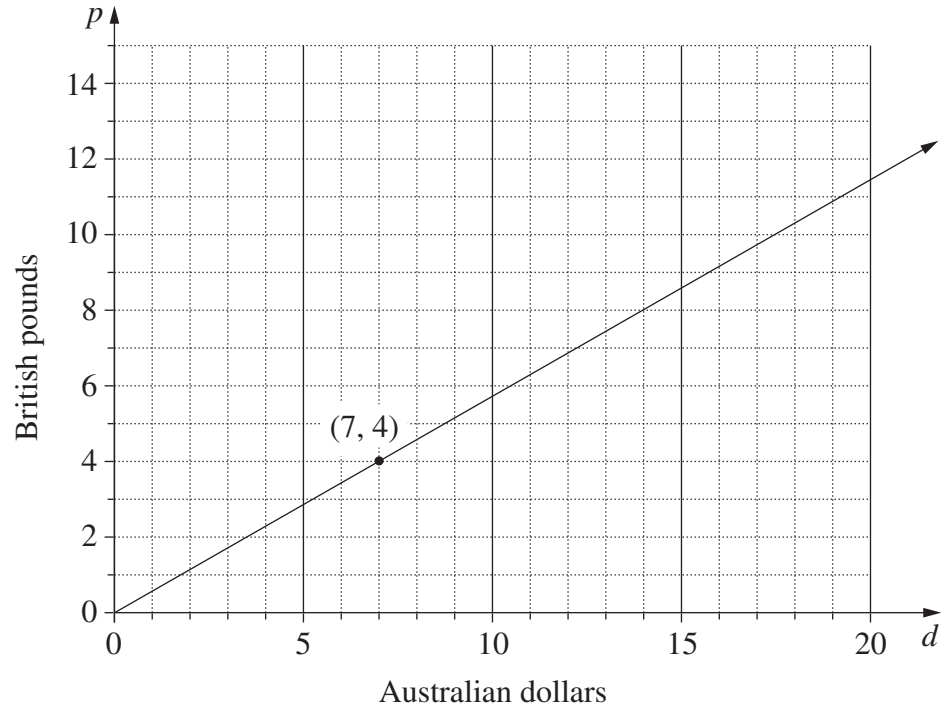
.....

(b) By first plotting four points, draw the curve that shows the time taken to travel between the two towns at different constant speeds. **3**



Question 34 (3 marks)

The relationship between British pounds (p) and Australian dollars (d) on a particular day is shown in the graph.



(a) Write the direct variation equation relating British pounds to Australian dollars in the form $p = md$. Leave m as a fraction. **1**

.....

.....

(b) The relationship between Japanese yen (y) and Australian dollars (d) on the same day is given by the equation $y = 76d$. **2**

Convert 93 100 Japanese yen to British pounds.

.....

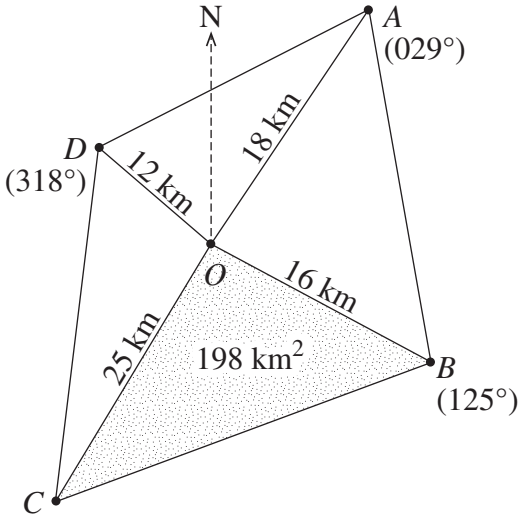
.....

.....

.....

Question 35 (3 marks)

A compass radial survey shows the positions of four towns A, B, C and D relative to point O. **3**



NOT TO SCALE

The area of triangle BOC is 198 km^2.

Calculate the bearing of town C from point O, correct to the nearest degree.

.....

.....

.....

.....

.....

.....

.....

.....

.....

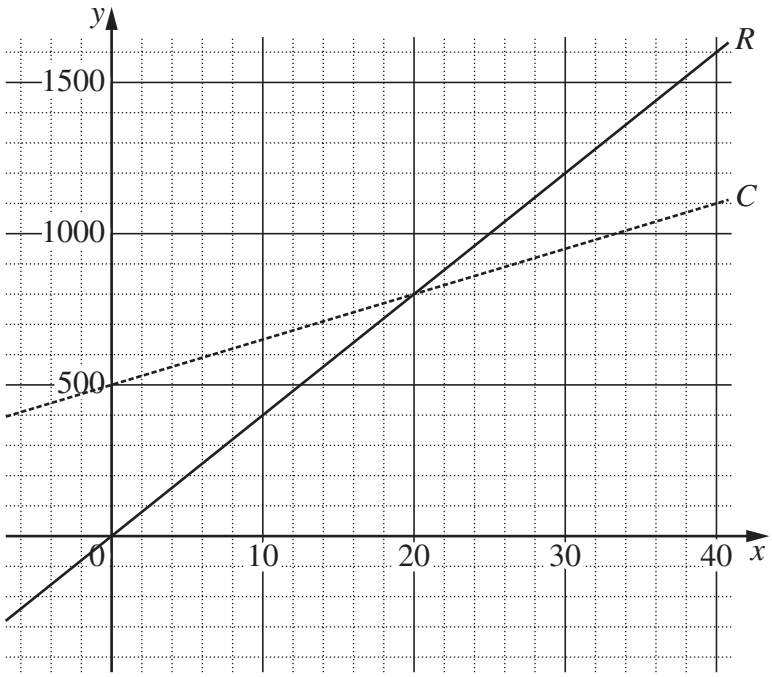
.....

Do NOT write in this area.

Question 36 (4 marks)

A small business makes and sells bird houses.

Technology was used to draw straight-line graphs to represent the cost of making bird houses (C) and the revenue from selling bird houses (R). The x-axis displays the number of bird houses and the y-axis displays the cost/revenue in dollars.



(a) How many bird houses need to be sold to break even? **1**

.....

(b) By first forming equations for cost (C) and revenue (R), determine how many bird houses need to be sold to earn a profit of $1900. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 37 (3 marks)

A new car is bought for $24 950. Each year the value of the car is depreciated by the same percentage. **3**

The table shows the value of the car, based on the declining-balance method of depreciation, for the first three years.

End of year	*Value*
1	$21 457.00
2	$18 453.02
3	$15 869.60

What is the value of the car at the end of 10 years?

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 38 (2 marks)

In a particular country, the birth weight of babies is normally distributed with a mean of 3000 grams. It is known that 95% of these babies have a birth weight between 1600 grams and 4400 grams. **2**

One of these babies has a birth weight of 3497 grams. What is the z-score of this baby's birth weight?

.....

.....

.....

.....

.....

.....

.....

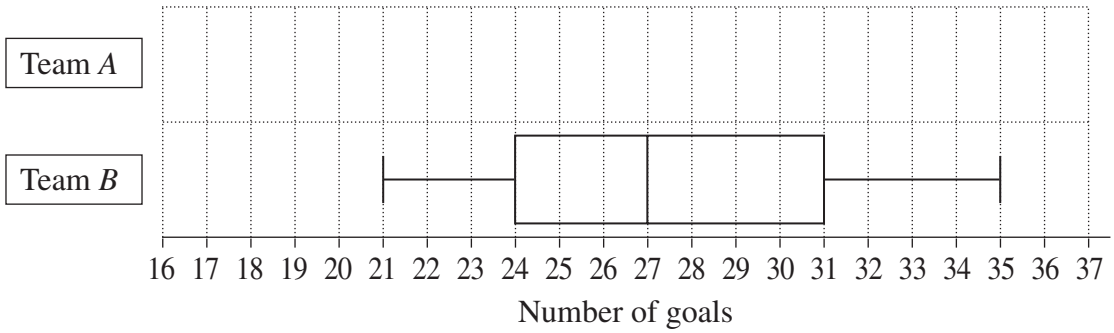
Question 39 (5 marks)

Two netball teams, Team *A* and Team *B*, each played 15 games in a tournament. For each team, the number of goals scored in each game was recorded. **5**

The frequency table shows the data for Team *A*.

Number of goals	*Frequency*
19	1
20	0
21	1
22	1
23	1
24	3
25	0
26	4
27	3
28	1

The data for Team *B* was analysed to create the box-plot shown.



Compare the distributions of the number of goals scored by the two teams. Support your answer with the construction of a box-plot for the data for Team *A*.

.....

.....

.....

.....

.....

.....

.....

.....

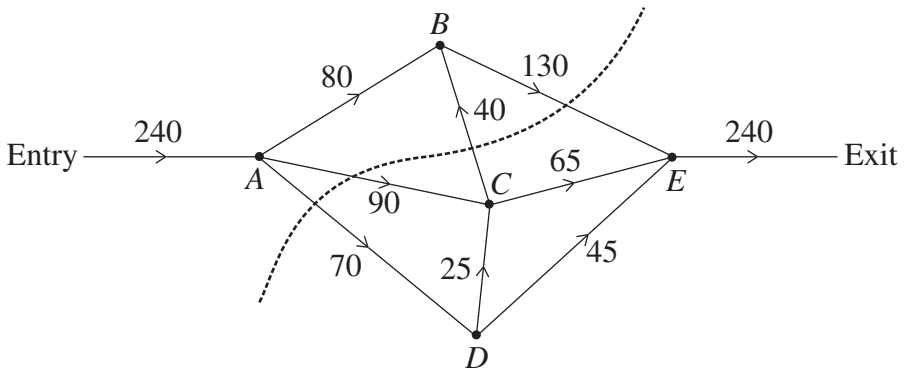
.....

.....

Question 40 (3 marks)

A museum is planning an exhibition using five rooms.

The museum manager draws a network to help plan the exhibition. The vertices *A*, *B*, *C*, *D* and *E* represent the five rooms. The numbers on the edges represent the maximum number of people per hour who can pass through the security checkpoints between the rooms.



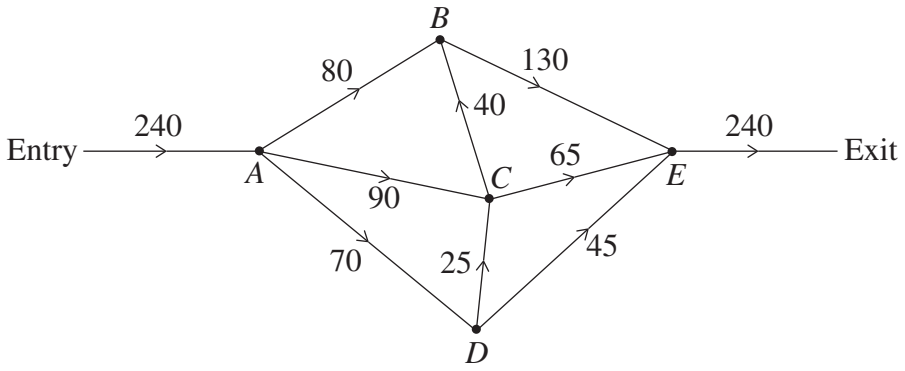
(a) What is the capacity of the cut shown? **1**

.....

.....

(b) The museum manager is planning for a maximum of 240 visitors to pass through the exhibition each hour. By using the 'minimum cut – maximum flow' theorem, the manager determines that the plan does not provide sufficient flow capacity. **2**

Draw the minimum cut onto the network below and recommend a change that the manager could make to one or more security checkpoints to increase the flow capacity to 240 visitors per hour.

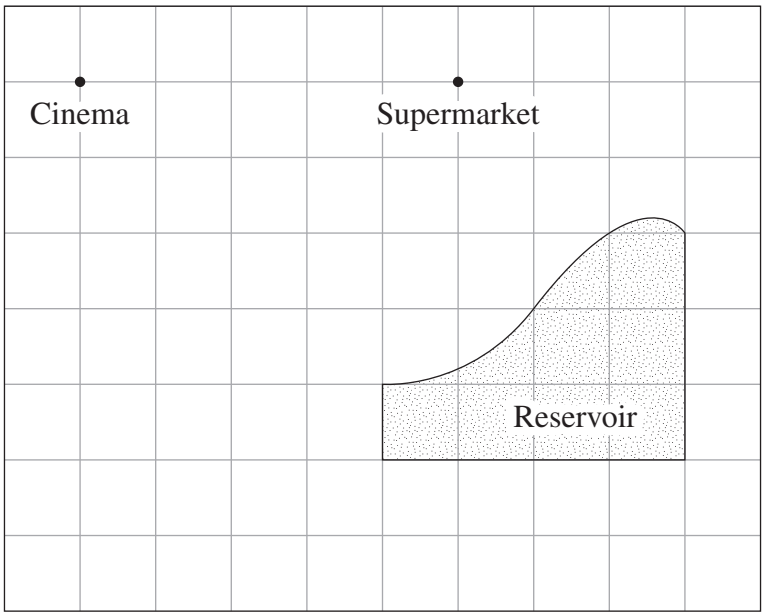


Recommended change: .....

.....

Question 41 (6 marks)

A map is drawn to scale, on 1-cm grid paper, showing the positions of a supermarket and a cinema. A reservoir is also shown.



(a) It takes 10 minutes to walk in a straight line from the cinema to the supermarket at a constant speed of 3 km/h. Show that the scale of the map is 1 cm = 100 m. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 41 continues on page 31

Question 41 (continued)

(b) The reservoir is initially empty. During a storm 20 mm of rain falls on the reservoir. **3**

With the aid of one application of the trapezoidal rule, estimate the amount of water in the reservoir immediately after the storm. Assume that all rain which falls over the reservoir is stored. Give your answer in cubic metres.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

End of Question 41

Please turn over

Question 42 (3 marks)

The table shows the future values of an annuity of $1 for different interest rates for 4, 5 and 6 years. The contributions are made at the end of each year. **3**

Future value of an annuity of $1

Years	*Interest rate per annum*			
	1%	*2%*	*3%*	*4%*
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

An annuity account is opened and contributions of $2000 are made at the end of each year for 7 years.

For the first 6 years, the interest rate is 4% per annum, compounding annually.
For the 7th year, the interest rate increases to 5% per annum, compounding annually.

Calculate the amount in the account immediately after the 7th contribution is made.

.....

.....

.....

.....

.....

.....

.....

.....

End of paper

NSW Education Standards Authority

2019 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Standard 1
Mathematics Standard 2

REFERENCE SHEET

Measurement

Limits of accuracy

Absolute error = $\frac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement – absolute error

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV\left(1 + r\right)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0\left(1 - r\right)^n$$

Statistical Analysis

An outlier is a score

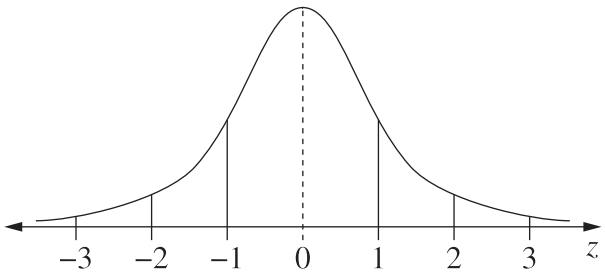
less than $Q_1 - 1.5 \times IQR$

or

more than $Q_3 + 1.5 \times IQR$

$$z = \frac{x - \overline{x}}{s}$$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

BLANK PAGE