2024 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	B
3	A
4	C
5	A
6	D
7	C
8	D
9	B
10	B

Section II

Question 11

Criteria	Marks
• Provides 6 correct entries	3
• Provides 4 correct entries	2
• Provides 2 correct entries	1

Sample answer:

x-value	*First derivative at x*	*Second derivative at x*
$x = -3$	Positive	Negative
$x = 1$	Zero	Zero
$x = 5$	Positive	Positive

Question 12

Criteria	Marks
• Provides correct solution	3
• Finds the value of *n*, or equivalent merit	2
• Finds the correct values of *a* or *d*, or equivalent merit	1

Sample answer:

$T_1 = a = 50 \qquad d = 7$

$$\begin{aligned} T_n &= a + (n-1)d = 2024 \\ 50 + (n-1)7 &= 2024 \\ (n-1)7 &= 1974 \\ n - 1 &= 282 \\ n &= 283 \end{aligned}$$

$$\begin{aligned} S_{283} &= \frac{n}{2}(a + l) \\ &= \frac{283}{2}(50 + 2024) \\ &= 293\,471 \end{aligned}$$

Question 13

Criteria	Marks
• Completes the table correctly	3
• Provides two correct entries	2
• Provides one correct entry	1

Sample answer:

	Population W	*Population K*
Population in 1985	$A = 34$	$B = \mathbf{280}$
Percentage yearly change in the population	**5.5%**	**−3%**
Population when $x = 50$	**494**	61

$34(1.055)^{50}$

$= 494.426\ldots$

$= 494$

Question 14 (a)

Criteria	Marks
• Finds *x*-coordinates of the points of intersection	1

Sample answer:

Points of intersection:

$$(x-1)^2 = 5 - x^2$$
$$x^2 - 2x + 1 = 5 - x^2$$
$$2x^2 - 2x - 4 = 0$$
$$2(x-2)(x+1) = 0$$
$$x = 2, -1$$

Question 14 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the correct antiderivative	2
• Finds the correct integrand, or equivalent merit	1

Sample answer:

$$\text{Area} = \int_{-1}^{2} 5 - x^2 - (x-1)^2 \, dx$$
$$= \int_{-1}^{2} 4 - 2x^2 + 2x \, dx$$
$$= \left[4x - \frac{2x^3}{3} + x^2\right]_{-1}^{2}$$
$$= \left(8 - \frac{16}{3} + 4\right) - \left(-4 + \frac{2}{3} + 1\right)$$
$$= 9$$

Question 15

Criteria	Marks
• Provides correct solution	3
• Uses the initial volume to provide a correct expression for V	2
• Provides the correct anti-derivative, or equivalent merit	1

Sample answer:

$$\frac{dV}{dt} = 300 - 7.5t$$

$$V = 300t - \frac{7.5t^2}{2} + C$$

When $t = 0,\ V = 350$

$$350 = 0 - 0 + C$$

$$\therefore\ V = 300t - 3.75t^2 + 350$$

When $\frac{dV}{dt} = 0$

$$300 - 7.5t = 0$$

$$7.5t = 300$$

$$t = 40$$

When $t = 40$

$$V = 300 \times 40 - 3.75 \times 40^2 + 350$$

$$V = 6350 \text{ L}$$

Question 16

Criteria	Marks
• Compares the two datasets on skewness, central tendency and spread	3
• Compares two of the measures of centre and spread and skewness for the two gardens, or equivalent merit	2
• Compares skewness, measures of centre or spread for the two gardens, or equivalent merit	1

Sample answer:

The dataset for Garden A is negatively skewed while the dataset for Garden B is positively skewed.

The median for Garden A is higher than the median for Garden B.

The *IQR* of Garden A is larger than the *IQR* of Garden B.

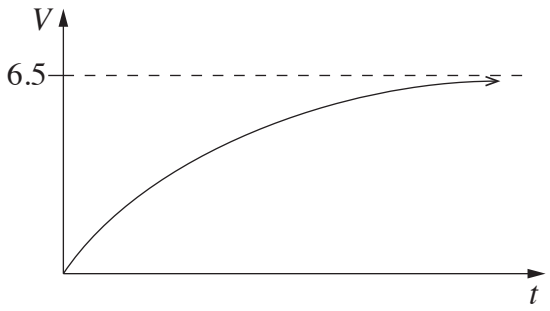
Answers could include:

The range of Garden A is larger than the range of Garden B.

Question 17 (a)

Criteria	Marks
• Provides correct graph, including labelled horizontal asymptote	2
• Provides a graph with correct shape, or equivalent merit	1

Sample answer:



Question 17 (b)

Criteria	Marks
• Provides correct solution	2
• Provides an expression with correct substitutions, or equivalent merit	1

Sample answer:

When $t = 1$, $2.6 = 6.5\left(1 - e^{-k}\right)$

$$1 - e^{-k} = \frac{2.6}{6.5}$$

$$1 - e^{-k} = 0.4$$

$$e^{-k} = 0.6$$

$$k = -\ln(0.6)$$

$$= 0.511 \quad \text{(3 decimal places)}$$

Question 17 (c)

Criteria	Marks
• Provides correct solution	2
• Finds the correct derivative, or equivalent merit	1

Sample answer:

$$\frac{dV}{dt} = 6.5ke^{-kt}$$

When $t = 2$, $\dfrac{dV}{dt} = 6.5ke^{-2k}$

$$= 1.195 \text{ volts/second}$$

Question 18 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Probability (player misses in 1 attempt)

$= 1 - 0.15 = 0.85$

Probability (player misses in both attempts)

$= (0.85)^2 = 0.7225$

Question 18 (b)

Criteria	Marks
• Provides correct solution	2
• Provides the correct inequality, $(0.85)^n < 0.2$, or equivalent merit	1

Sample answer:

Probability (player scores at least one goal)

$= 1 - \text{Prob}(\text{player misses in } n \text{ attempts})$

$= 1 - (0.85)^n$

When $1 - (0.85)^n > 0.8$

$(0.85)^n < 0.2$

Taking logs, $n \ln(0.85) < \ln(0.2)$

$$n > \frac{\ln(0.2)}{\ln(0.85)}$$

$\div 9.90$

The player needs 10 attempts.

Question 19

Criteria	Marks
• Provides correct graph	5
• Correctly identifies all points of inflection and stationary points	4
• Finds the nature of the stationary points	3
• Finds the x values of both stationary points, or equivalent merit	2
• Finds the first derivative, or equivalent merit	1

Sample answer:

$$y = x^4 - 2x^3 + 2$$
$$y' = 4x^3 - 6x^2$$
$$y'' = 12x^2 - 12x$$

Stationary points: $y' = 0$

$$4x^3 - 6x^2 = 0$$
$$2x^2(2x - 3) = 0$$
$$x = 0 \text{ or } x = \frac{3}{2}$$

When $x = 0$, $y = 2$ and $y'' = 0$

Check concavity:

x	-1	0	$\frac{1}{2}$
y''	24	0	-3

Concavity changes sign $\therefore$ Horizontal point of inflection at $(0, 2)$.

When $x = \frac{3}{2}$, $y = \frac{5}{16}$ and $y'' = 9$

$\therefore$ Local minimum at $\left(\frac{3}{2}, \frac{5}{16}\right)$

Points of inflection: $y'' = 0$

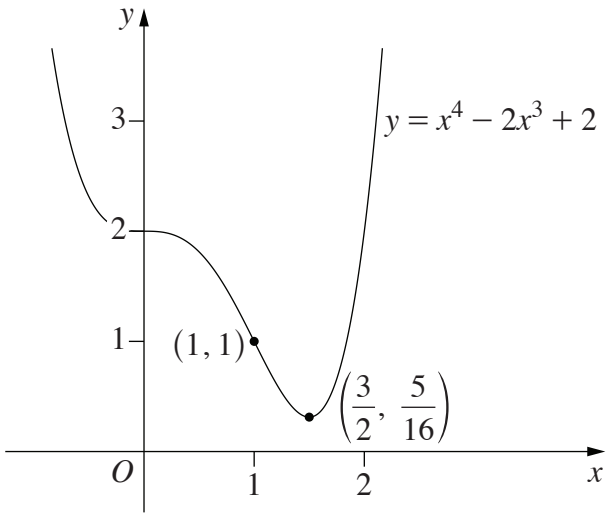
$$12x^2 - 12x = 0$$
$$12x(x - 1) = 0$$
$$x = 0 \text{ or } x = 1$$

Already found $(0, 2)$

Check concavity:

x	$\frac{1}{2}$	1	2
y''	-3	0	24

Concavity changes sign $\therefore$ Point of inflection at $(1, 1)$.



Question 20 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\tan 35° = \frac{40}{AC}$$

$$\therefore \quad AC = \frac{40}{\tan 35°}$$

$$= 57.1259\ldots$$

$$= 57.13 \text{ metres} \quad (2 \text{ decimal places})$$

Question 20 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the angle at C, or equivalent merit	2
• Finds the length of BC, or equivalent merit	1

Sample answer:

$$\tan 30° = \frac{40}{BC}$$

$$\therefore \quad BC = \frac{40}{\tan 30°}$$

$$= 69.28 \text{ metres} \quad (2 \text{ decimal places})$$

Using cosine rule for $\angle ACB$:

$$\cos \angle ACB = \frac{57.13^2 + 69.28^2 - 100^2}{2 \times (57.13) \times (69.28)}$$

$$= -0.24462$$

$$\therefore \quad \angle ACB = 104° \quad \text{(nearest degree)}$$

$\therefore$ Bearing of B from C

$$= 104° + 90°$$

$$= 194°$$

Question 21

Criteria	Marks
• Provides three correct observations	3
• Provides two correct observations	2
• Provides one correct observation	1

Sample answer:

- Female anacondas grow at a faster rate than males.
- Both females and males continue to grow after 4 years.
- Females are longer than males.

Question 22 (a)

Criteria	Marks
• Provides correct solution	3
• Finds the correct second derivative, or equivalent merit	2
• Finds the correct first derivative, or equivalent merit	1

Sample answer:

$$f(x) = \ln\left(1 + x^2\right)$$

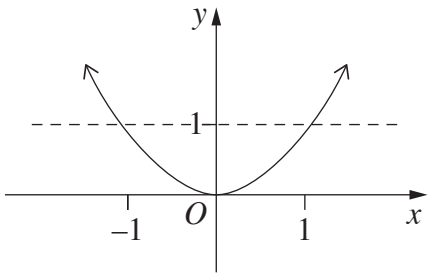
$$f'(x) = \frac{2x}{1 + x^2}$$

$$f''(x) = \frac{2\left(1 + x^2\right) - 2x(2x)}{\left(1 + x^2\right)^2}$$

$$= \frac{2 - 2x^2}{\left(1 + x^2\right)^2} > 0$$

When $2 - 2x^2 > 0$ (since denominator always positive)

ie when $x^2 < 1$



$\therefore$ when $-1 < x < 1$

Question 22 (b)

Criteria	Marks
• Provides correct solution	2
• Uses the values from the table correctly in the trapezoidal rule, or equivalent merit	1

Sample answer:

$$\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$$

$$\doteqdot 2 \times \frac{1-0}{2\times 4}\left[0 + 0.6931 + 2\left(0.0606 + 0.2231 + 0.4463\right)\right]$$

$$= 0.538275$$

Question 22 (c)

Criteria	Marks
• Provides correct answer with a reason	1

Sample answer:

From part (a) $f(x)$ is concave up when $-1 < x < 1$, so the straight line segments will lie above the curve.

So this is an overestimate.

Question 23 (a)

Criteria	Marks
• Provides correct solution, including a correct *z*-score	2
• Finds *z*-score corresponding to 70, or equivalent merit	1

Sample answer:

$$z = \frac{70 - 58}{15}$$

$$= 0.8$$

P from table $= 0.7881$

P required $= 0.7881 - 0.5 = 0.2881$

$\therefore$ 28.81%

Question 23 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

The z-score for 46 is -0.8.

By symmetry the area between 46 and 58 is equal to the area between 58 and 70.

So, the percentage of scores between 46 and 70 is twice 28.81%.

Question 23 (c)

Criteria	Marks
• Provides correct solution, with evidence of using the table	2
• Finds the *z*-score corresponding to 90th percentile, or equivalent merit	1

Sample answer:

$$1.3 = \frac{x - 58}{15}$$

$$x = 77.5$$

$\therefore$ Approximate minimum score $= 77$

Answers could include:

78

Question 24 (a)

Criteria	Marks
• Provides correct solution	3
• Sets up an appropriate series, or equivalent merit	2
• Finds a correct term in the series, or equivalent merit	1

Sample answer:

The first deposit accrues interest for 24 months

$\therefore$ Its future value is $80(1.005)^{24}$

The next deposit accrues interest for 23 months and so on

$\therefore$ Total amount after 24 months is

$$\begin{aligned}
&80(1.005)^{24} + 80(1.005)^{23} + \cdots + 80(1.005)^{1} \\
&= 80\left[(1.005)^{1} + (1.005)^{2} + \cdots + (1.005)^{24}\right] \\
&= 80 \times (1.005)\left[\frac{(1.005)^{24} - 1}{1.005 - 1}\right] \\
&= 2044.73
\end{aligned}$$

Question 24 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned}
80A &= 2044.73 \\
A &= \frac{2044.73}{80} \\
&= 25.559
\end{aligned}$$

Question 25 (a)

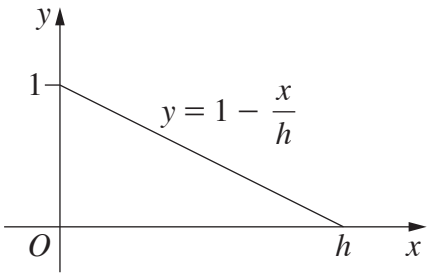
Criteria	Marks
• Provides correct solution	2
• Identifies that the area under the curve = 1, or equivalent merit	1

Sample answer:

We want the area of the triangle $= 1$

$$\therefore \quad \frac{1}{2} \times 1 \times h = 1$$

$$h = 2$$



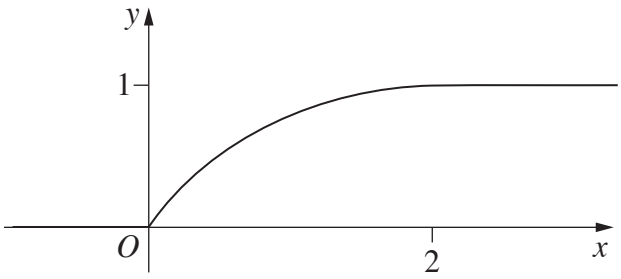
Question 25 (b)

Criteria	Marks
• Provides correct formula and sketch	2
• Finds the formula for the *CDF*, or equivalent merit	1

Sample answer:

$$\int_0^x 1 - \frac{t}{2}\, dx = \left[t - \frac{t^2}{4} \right]_0^x$$

$$= \begin{cases} 0, & \text{for } x < 0 \\ x - \frac{x^2}{4}, & \text{for } 0 \le x \le 2, \\ 1, & \text{for } x > 2 \end{cases}$$



Question 25 (c)

Criteria	Marks
• Provides correct solution	2
• Finds the correct quadratic equation, or equivalent merit	1

Sample answer:

$$t - \frac{t^2}{4} = \frac{1}{2}$$

$$\therefore\ t^2 - 4t + 2 = 0$$

$$t = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 2}}{2 \times 1}$$

$$t = \frac{4 \pm \sqrt{8}}{2}$$

$$t = 2 \pm \sqrt{2}$$

We need $t < 2$

$$\therefore\ t = 2 - \sqrt{2}$$

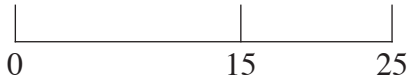
$$\approx 0.586$$

Question 26

Criteria	Marks
• Provides correct solution	4
• Makes substantial progress towards a solution	3
• Demonstrates in calculations understanding of importance of 15 years and 25 years	2
• Identifies the required interest rate, or equivalent merit	1

Sample answer:

$$r = \frac{2.4\%}{12} = 0.002$$



For 25 years need $1200 \times 225.430 = 270\ 516$

For 15 years need extra $800 \times 151.036 = 120\ 828.80$

Total $= \$391\ 344.80$

Question 27 (a)

Criteria	Marks
• Provides correct solution	2
• Applies the product rule, or equivalent merit	1

Sample answer:

$$y = x^2 \tan x$$
$$y' = x^2 \sec^2 x + 2x \tan x$$

Question 27 (b)

Criteria	Marks
• Provides correct solution	3
• Correctly uses the identity, $\tan^2 x = \sec^2 x - 1$	2
• Expands the integrand, or equivalent merit	1

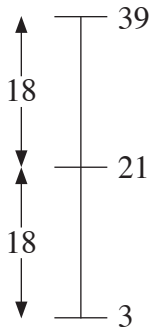
Sample answer:

$$\begin{aligned}\int (x\tan x + 1)^2 \, dx &= \int x^2 \tan^2 x + 2x \tan x + 1 \, dx \\ &= \int x^2(\sec^2 x - 1) + 2x \tan x + 1 \, dx \\ &= \int x^2 \sec^2 x + 2x \tan x - x^2 + 1 \, dx \\ &= x^2 \tan x - \frac{x^3}{3} + x + C\end{aligned}$$

Question 28 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the value of c or k, or equivalent merit	1

Sample answer:



$c = 21$

$k = 18$

Question 28 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\text{Period} = \frac{2\pi}{\frac{\pi}{24}} = 48$$

It takes 48 seconds.

Question 28 (c)

Criteria	Marks
• Provides a correct solution	4
• Finds $t = 3$, or equivalent merit	3
• Makes correct use of the periodic nature of cosine function	2
• Equates the appropriate trigonometric ratios, or equivalent merit	1

Sample answer:

We want $c - k\cos\left(\frac{\pi}{24}t\right) = c - k\cos\left(\frac{\pi}{24}(t-6)\right)$

$\therefore \quad \cos\left(\frac{\pi}{24}t\right) = \cos\left(\frac{\pi}{24}(t-6)\right)$

$\therefore \quad \frac{\pi}{24}t = \frac{\pi}{24}(t-6)$ which is impossible.

or $\quad -\frac{\pi}{24}t = \frac{\pi}{24}(t-6)$

$\therefore \quad -\frac{\pi}{24}t = \frac{\pi}{24}t - \frac{\pi}{4}$

$\therefore \quad \frac{\pi}{12}t = \frac{\pi}{4}$

$\therefore \quad t = 3$

or $\quad 2\pi - \frac{\pi}{24}t = \frac{\pi}{24}(t-6)$

$-\frac{2\pi}{24}t = -2\pi - \frac{\pi}{4}$

$\frac{\pi t}{12} = \frac{9\pi}{4}$

$\therefore \quad t = 27$

When $t = 3$ height is $21 - 18\cos\frac{3\pi}{24} = 4.37$ m

When $t = 27$ height is $21 - 18\cos\frac{27\pi}{24} = 37.63$ m

Question 29

Criteria	Marks
• Provides correct solution	4
• Finds the values of *a* and *b*	3
• Finds a relevant equation linking *a* and *b*, or equivalent merit	2
• Finds the *x*-ordinate of the point of intersection of the tangent and normal, or equivalent merit	1

Sample answer:

The tangent and normal meet where

$$2x + 3 = -\frac{1}{2}x - 2$$

$$\frac{5x}{2} = -5$$

$$x = -2$$

$$y = 2(-2) + 3$$

$$y = -1$$

$\therefore$ They meet at $(-2, -1)$.

If $y = ax^2 + bx + c$, then $y' = 2ax + b$

When $x = -4 \quad y' = 0$

$\therefore \quad -8a + b = 0$

$\therefore \quad b = 8a$

When $x = -2 \quad y' = 2$

$$-4a + b = 2$$

$\therefore \quad -4a + 8a = 2$

$\therefore \quad a = \frac{1}{2}$

$\therefore \quad b = 4$

When $x = -2 \quad y = -1$

$\therefore \quad a(-2)^2 + b(-2) + c = -1$

$$\frac{1}{2} \times 4 + 4 \times -2 + c = -1$$

$$-6 + c = -1$$

$\therefore \quad c = 5$

Question 30

Criteria	Marks
• Provides the correct solution	3
• Shows that the graph given is equal to S_∞, or equivalent merit	2
• Finds the limiting sum S, or equivalent merit	1

Sample answer:

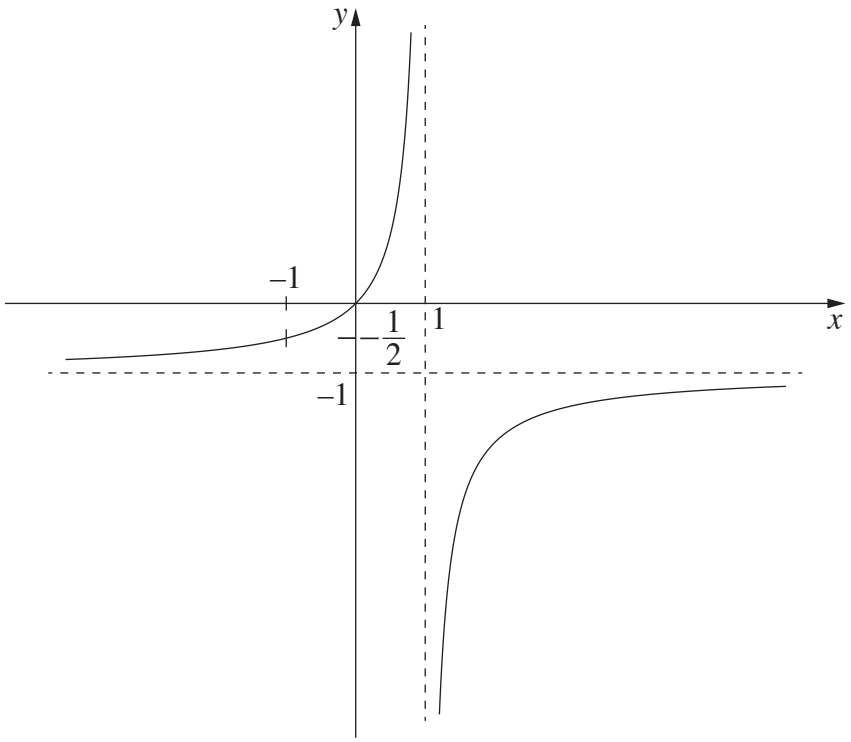
$$S = x + x^2 + x^3 + \cdots$$

$$= \frac{x}{1-x} \qquad \text{where } -1 < x < 1$$

$$y = -1 - \frac{1}{x-1} = \frac{-(x-1)-1}{x-1}$$

$$= \frac{-x}{x-1} = \frac{x}{1-x}$$

So, the graph of $y = \frac{x}{1-x}$ is as shown.



When $x = -1 \quad y = -\frac{1}{2}$

From the graph, when $-1 < x < 1$, $y > -\frac{1}{2}$

Hence, $S > -\frac{1}{2}$

Question 31 (a)

Criteria	Marks
• Provides correct solution	3
• Finds correct expressions for the perimeter and area, or equivalent merit	2
• Finds the correct expression for the perimeter or area, or equivalent merit	1

Sample answer:

$$\begin{aligned} P &= 1\theta + (1+x)\theta + 2x \\ &= (2+x)\theta + 2x \\ A &= \frac{1}{2}(1+x)^2\theta - \frac{1}{2}\cdot 1^2\theta \\ &= \frac{1}{2}\theta\left[1 + 2x + x^2 - 1\right] \\ &= \frac{1}{2}\theta\left(2x + x^2\right) \\ &= \frac{1}{2}\theta x(2+x) \qquad ① \end{aligned}$$

Since $P = (2+x)\theta + 2x$ and $(2+x)\theta = \dfrac{2A}{x}$

$\therefore \quad P(x) = 2x + \dfrac{2A}{x}$

Question 31 (b)

Criteria	Marks
• Provides correct solution	3
• Expresses θ in terms of A, or equivalent merit	2
• Shows $x = \sqrt{A}$ is a minimum, or equivalent merit	1

Sample answer:

$$P' = 2 - \frac{2A}{x^2} = 0 \text{ for minimum}$$

$\therefore \quad x^2 = A \qquad \therefore \quad x = \sqrt{A} \qquad (x > 0)$

$$P'' = \frac{4A}{x^3} > 0 \text{ when } x = \sqrt{A}$$

$\therefore$ This gives a minimum value.

Using ① $\quad A = \dfrac{1}{2}\theta\sqrt{A}\left(2 + \sqrt{A}\right)$

$$\therefore \quad \theta = \frac{2A}{\sqrt{A}\left(2+\sqrt{A}\right)} = \frac{2\sqrt{A}}{2+\sqrt{A}} < \frac{2\sqrt{A}}{\sqrt{A}}$$

$$< 2$$

2024 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
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2	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
3	1	MA-S3 Random Variables	MA12-8
4	1	MA-F2 Graphing Techniques	MA12-1
5	1	MA-C4 Integral Calculus	MA12-3
6	1	MA-F1 Working with Functions	MA11-2
7	1	MA-F2 Graphing Techniques	MA12-10
8	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
9	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
10	1	MA-C3 Applications of Differentiation	MA12-7

Section II

Question	Marks	Content	Syllabus outcomes
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12	3	MA-M1 Modelling Financial Situations	MA12-4
13	3	MA-E1 Logarithms and Exponentials	MA11-6
14 (a)	1	MA-F1 Linear, quadratic and cubic functions	MA11-1
14 (b)	3	MA-C4 Integral Calculus	MA12-3
15	3	MA-C4 Integral Calculus	MA12-3
16	3	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10
17 (a)	2	MA-F2 Graphing Techniques	MA12-1
17 (b)	2	MA-C3 Applications of Differentiation	MA12-3
17 (c)	2	MA-C3 Applications of Differentiation	MA12-3
18 (a)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
18 (b)	2	MA-E1 Logarithms and Exponentials	MA11-6
19	5	MA-C3 Applications of Differentiation	MA12-6
20 (a)	1	MA-T1 Trigonometry and Measure of Angles	MA11-3
20 (b)	3	MA-T1 Trigonometry and Measure of Angles	MA11-3
21	3	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10
22 (a)	3	MA-C2 Differential Calculus MA-C3 Applications of Differentiation	MA12-6
22 (b)	2	MA-C4 Integral Calculus	MA12-7
22 (c)	1	MA-C4 Integral Calculus	MA12-10
23 (a)	2	MA-S3 Random Variables	MA12-8
23 (b)	1	MA-S3 Random Variables	MA12-8
23 (c)	2	MA-S3 Random Variables	MA12-8

Question	Marks	Content	Syllabus outcomes
24 (a)	3	MA-M1 Modelling Financial Situations	MA12-4
24 (b)	1	MA-M1 Modelling Financial Situations	MA12-4
25 (a)	2	MA-S3 Random Variables	MA12-8
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25 (c)	2	MA-S3 Random Variables	MA12-8
26	4	MA-M1 Modelling Financial Situations	MA12-9
27 (a)	2	MA-C2 Differential Calculus	MA12-6
27 (b)	3	MA-C4 Integral Calculus	MA12-7
28 (a)	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
28 (b)	1	MA-T3 Trigonometric Functions and Graphs	MA12-5
28 (c)	4	MA-T3 Trigonometric Functions and Graphs	MA12-5
29	4	MA-C3 Applications of Differentiation	MA12-6
30	3	MA-F2 Graphing Techniques MA-M1 Modelling Financial Situations	MA12-9, MA12-10
31 (a)	3	MA-T1 Trigonometry and Measure of Angles	MA11-9
31 (b)	3	MA-C3 Applications of Differentiation	MA12-10