2012 HSC Notes from the Marking Centre – Mathematics

Contents

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics. It contains comments on candidate responses to the 2012 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2012 Higher School Certificate examination, the marking guidelines and other support documents developed by the Board of Studies to assist in the teaching and learning of Mathematics.

Candidates are reminded of the importance of reading the question carefully, looking for links between parts of a question, solving equations carefully, computing accurately, labelling tables and graphs appropriately, and setting out their work clearly.

General comments

The instructions indicate that relevant mathematical reasoning and/or calculations should be included in the responses for Questions 11–14. Candidates are reminded that where a question is worth several marks, full marks may not be awarded for an answer, even if the answer given is correct, if no working is shown.

This is because mathematical communication and reasoning are included in the objectives and outcomes assessed by the examination.

Candidates are advised to show all their working so that marks can be awarded for some correct steps towards their answer. A simple example is when candidates have to round their answer to a certain degree of accuracy. Candidates should always write their calculator display before rounding their answer. They should only round their answer in the last step of working, not in an earlier step. Markers can then see that candidates have rounded correctly, even if the answer is not correct.

Question 11

(a) In most responses, candidates correctly factorised the quadratic expression. In weaker responses, the most common error was setting the expression to zero, and then solving to give values of x. Candidates could have checked their answers by expansion. In a few responses, candidates differentiated the expression, but did not proceed to any meaningful conclusion. In many responses, candidates wrote the correct answer without showing any working.

(b) Only a handful of candidates achieved full marks. In better responses, candidates solved the inequality in one line by stating $-2 < 3x - 1 < 2$. In many responses, candidates wrote separate inequalities but made errors with the direction of the inequality or with the negative sign. The inequality was actually an intersection rather than a union, which was ignored by many candidates.

A common approach was to create an equation and solve to produce $x = 1$ and $x = -\frac{1}{3}$ then test the three regions created by these critical values. Another problem was the failure to divide by 3 on the last line of one inequality; for example, $3x < 3$ followed by $x < 3$.

(c) Most candidates earned full marks for this part. A common mistake was to use $2x$ as the gradient rather than evaluating to find a numerical answer, resulting in a quadratic rather than a line.

(d) In most responses, candidates differentiated the expression using the chain rule and arrived at the correct answer. In weaker responses, the most common mistakes were failing to differentiate the expression inside the bracket and failing to write down the power in the answer to obtain $10e^{2x}\left(3+e^{2x}\right)$.

(e) A significant number of candidates had difficulty with this part. In better responses, candidates wrote down the focal length $a = 4$ and vertex $(0,2)$ of the parabola and then found the coordinates of the focus $(0,6)$. In weaker responses, the most common error was to assume that the focus of every parabola is $(0,a)$. In better responses, candidates usually included a diagram rather than relying on a formula such as $S(h,a+k)$.

(f) This part was well done and correctly set out by most candidates and a variety of solutions were presented. In some responses, candidates found the angle using a formula for the area and then applying the arc length formula. In other responses, candidates found the percentage of the area of the sector compared to that of the circle and applied this result to the circumference of the circle. Common errors included an inability to recall formulae, confusion between radians and degrees and substitution of the angle to find the arc length.

(g) In better responses, candidates used the table of standard integrals to arrive at the primitive $2\tan\frac{x}{2}$ and then substituted the limits correctly. In weaker responses, candidates arrived at a primitive of the form $a\tan(bx)$ where $a \neq 2$ and $b \neq \frac{1}{2}$.

Question 12

Overall the standard of responses was high with many candidates gaining almost full marks. Solutions were generally well set out, with logical steps allowing markers to award part marks when errors were made. The exception to this was part (c) (iii).

(a) (i) In most responses, candidates recognised they were required to use the product rule. In many responses, candidates wrote u, u', v and v' to assist them. In a small number of responses, candidates quoted the product rule incorrectly, often using subtraction instead of addition and some candidates differentiated $\log_e x$ incorrectly.

(ii)In most responses, candidates recognised they were required to use the quotient rule. In some weaker responses, candidates differentiated $\cos x$ incorrectly to obtain $\sin x$ although they could have used the table of standard integrals available on the back of the examination paper to find the derivative of $\cos x$.

(b) In many responses, candidates recognised that the primitive involved a log function, but in a significant number of responses, candidates obtained an incorrect constant multiplier or multiplied by a multiple of x. In other weaker responses, candidates did not recognise that the primitive was a log function and attempted to integrate each term separately. In a few weaker responses, candidates attempted to give the solution as an inverse trig function.

(c) (i) In most responses, candidates recognised that this question involved an arithmetic series. In better responses, candidates either substituted into the correct formula or listed the sequence of numbers. In a small number of responses, candidates used an incorrect formula but provided the correct values for the first term and the common difference.

(ii) In better responses, candidates stated the correct formula, either $S_n = \frac{n}{2}(a+l)$ or

$S_n = \frac{n}{2}(2a+(n-1)d)$ and showed their substitution before any calculation was performed. This allowed markers to allocate marks to those candidates who made numerical errors. In some responses, candidates wrote down the arithmetic series and calculated the correct sum. In a small number of weaker responses, candidates attempted to use an incorrect formula.

(iii) In most responses, candidates recognised the need to equate the 200 tiles to their formula for the sum of n terms, although there were some who equated this to the formula for individual terms. In many responses, candidates arrived at the correct quadratic equation or inequation, and then attempted to factorise to find the solution. If unsuccessful, they then attempted, often several times, to apply the quadratic formula to obtain a solution. In many responses, elementary errors were made when attempting to solve the quadratic equation or inequation, but most candidates correctly interpreted their decimal solution to obtain an integer answer. After unsuccessfully attempting to solve a quadratic equation or inequation many candidates used a numerical approach and were usually successful using this method, probably at the expense of time. Success from guess and check was common; however candidates who use this approach should show any workings in their response.

(d) (i) In the majority of responses, candidates correctly used the five function values, often in a table, to find the approximate area. In weaker responses, common errors included the use of function notation in the quoted formula with incorrect substitution, incorrect weightings and missing brackets. Numerical errors were also common, highlighting the need to show full substitution before any numerical evaluation. A small number of candidates applied the Trapezoidal rule rather than Simpson's rule.

(ii) In better responses, candidates who recognised that $V = Ah$ usually succeeded with this part. In many responses, candidates attempted to answer this question using calculus and were unsuccessful.

Question 13

(a) (i) In most responses, candidates used the distance formula or Pythagoras' theorem to find the distance AB. In weaker responses, errors included carelessly swapping the coordinates of A and B or incorrectly calculating the x and y intercepts of the line AB.

(ii) In most responses, candidates recognised that this part required the use of the cosine rule to find $\angle ABC$. In better responses, candidates correctly substituted into the formula $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ and rounded their answer to the nearest degree. In responses where candidates stated the formula as $b^2 = a^2 + c^2 - 2ac\cos B$ errors were made when changing the subject to In other responses, common errors included not rounding to the nearest degree, inconsistent substitution into the cosine rule, finding the angle in radians or assuming that ΔABC was right-angled. In a few responses, candidates successfully found the size of $\angle ABC$ by finding the perpendicular distance CN and then using the sine rule or right-angled trigonometry.

(iii) In better responses, candidates found the equation CN and then solved it with the equation for AB to find the coordinates of N. In weaker responses, candidates provided as incorrect equation for CN often resulting from using the reciprocal gradient of AB instead of the negative reciprocal. Algebraic and arithmetic errors were common.

In a few responses, candidates used the perpendicular distance formula to find the distance CN, then solved a distance equation with the equation of AB to find N. This method required many algebraic steps and often proved too difficult to complete.

(b) (i) In the majority of responses, candidates equated the quadratic equations to find the x-coordinate of A. In weaker responses, candidates divided through by x thus finding only one of the two solutions for a quadratic equation.

(ii) In better responses, candidates recognised that this question involved the area enclosed between two curves and simplified the definite integral of the form $\int_0^4 \left[\left(5x - x^2\right) - \left(x^2 - 3x\right)\right] dx$ before evaluating. In many weaker responses, candidates made the question more difficult by dividing the area into several parts and then evaluating them separately. Other common errors included incorrect limits and the use of absolute value signs. In weaker responses, candidates differentiated instead of integrating.

(c) (i) This part was answered well.

(ii) In better responses, candidates recognised that this question followed from part (i) and correctly calculated the complement of $P(RR)$. In responses that required $P(WR)$, $P(RW)$ and $P(WW)$ to be added together, many candidates misinterpreted 'at least one' and omitted $P(WW)$ from their addition.

(iii) This part was generally done well. The most common error was to calculate $P(RR)\times P(WW)$ rather than $P(RR)+P(WW)$.

Question 14

(a) (i) Most responses were well set out, allowing part marks to be awarded when small errors were made. In most responses, candidates specified that $y'=0$ but a significant number failed to correctly factorise to find the solutions. A common error was to recognise $12x$ as a common factor, but leave the constant term as 12 in the remaining factor. This led to three stationary points that could not be graphed consistently.

In most responses, candidates used the sign of y'' to ascertain the nature of the stationary points. However, in some responses candidates used the gradient to the left of and right of the stationary points to determine their nature.

(ii) Most candidates who correctly answered the first part produced suitable sketches in their response. It was necessary to either label the stationary points, or to clearly label the axes to identify their location.

Candidates who made errors in the first part were presented with data that could not be graphed consistently. In many responses, candidates, rather than finding and correcting these error(s), chose to draw extremely elaborate relations in an attempt to satisfy the information they had found.

(iii) Candidates who did not solve (i) but had made an attempt at sketching (ii) could obtain the mark, subject to use of correct inequalities.

(iv) Few candidates apprehended the effect of the constant k. Despite the function being a quartic equation, a majority of candidates attempted to use quadratic discriminant to resolve the question. In many responses, candidates achieved the mark by finding a correct k for their incorrect graph from part (ii).

(b) Many candidates correctly stated $V=\pi\int_0^1\left\{\dfrac{3}{(x+2)^2}\right\}^2 dx$, but most omitted the dx.

Omitting π was not a common error.

In many responses, candidates were unable to find the primitive. The most common error involved attempting to use logarithms in the primitive or to obtain a primitive involving $(x+5)^{-5}$.

(c) (i) In most responses, candidates provided the correct answer, and showed clear working-out. In most responses, candidates derived the value of k from the given condition. However, in some responses, candidates demonstrated that the given k could be used to produce the required conditions.

(ii) In almost all responses, candidates correctly substituted the given values to produce the correct answer. In better responses, candidates used the exact value of k, namely $k = \frac{\ln^2}{20}$, to produce a more precise solution.

(iii) This part was answered poorly, or not attempted at all, by most candidates. In better responses, candidates either correctly differentiated the function or used $N' = kN$.

(iv) The vast majority of candidates scored full marks for this part. Responses were usually well presented.

Question 15

(a) (i) Responses that correctly identified the geometric progression usually obtained full marks. Simple addition of 10 terms in geometric progression was a successful technique, although this frequently led to rounding errors and took extra time. A common error was to find the tenth term rather than the sum of the first ten terms.

(ii) Candidates who correctly interpreted the need for a limiting sum were usually successful. In some responses, candidates did not realise that when an explanation is required, they need to supply some justification. Some candidates offered only a response with no calculations and did not come up with a limiting sum.

(b) (i) In the majority of responses, candidates understood that initial meant when $t = 0$ and then successfully substituted into the velocity formula to find the required velocity. A common error was an incorrect evaluation of $\cos(0)$.

(ii) In better responses, candidates approached this part by using a graph and obtained an answer quickly with efficient working. This part was only worth one mark and those candidates who employed calculus techniques had varying success but spent a lot of time to reach their, often incorrect, conclusion. In many responses, candidates gave the time when the particle was at its maximum velocity rather than the maximum velocity.

(iii) In most responses, candidates integrated the velocity equation and evaluated the resulting constant. Common errors included not finding the value of the constant and not integrating with respect to time. In many responses, candidates integrated 1 to become x rather than t.

(iv) In the majority of responses, candidates managed to get $t = \frac{\pi}{3}$ but did not substitute to find the position of the particle at this time. Candidates are reminded that when using calculus, angles are measured in radians and not degrees. Correct responses involved the substitution of $t = \frac{\pi}{3}$ rather than $t = 60$.

(c) (i) In most responses, candidates could form a correct expression for A_2. In some responses, candidates evaluated the rate incorrectly.

(ii) In a considerable number of responses, candidates found the correct expression for A_{300} and equated it to 0. In many weaker responses, candidates who did not gain the answer \$2319.50 used backtracking through their formula and working to try to get the correct answer – which often led to introduced errors. In particular, a number of candidates continued their sum to include 1.005^{300}, instead of finishing at 1.005^{299}.

(iii) In most responses, candidates substituted their value of M into A_n to form an inequality. However, a considerable number of candidates made mistakes throughout their subsequent working. Many candidates realised that they needed to use logarithms in this question but struggled to rearrange the formula $A_n < 1800$. In many responses, candidates applied the trial and error method on the inequality. However, candidates are reminded that they should show their guesses and checks rather than just the final answer. In a small number of responses, candidates failed to interpret their final answer correctly and rounded to 201 months rather than 202 months.

Question 16

(a) (i) In most responses, candidates displayed knowledge of the tests for similar triangles. However, many candidates did not present a logical argument using correct terminology. In many responses, candidates assumed parallel lines without explanation. Candidates needed to show that $\angle BFE = \angle EDA$ by linking it to an angle within the rhombus. Candidates also needed to state the properties of a rhombus, which are relevant to their proof, rather than merely claiming that those properties are given. Candidates are also reminded that if they are going to use additional constructions or name additional points in their proof, it is very important to redraw the diagram in their answer booklet with that information included.

(ii) In many responses, candidates made an incorrect ratio statement or did not solve the equation formed by a correct ratio statement. Candidates who drew the triangles in a similar orientation were, generally, more successful in establishing the correct ratio statement. In weaker responses, candidates failed to see the link between the similar triangles in part (i) and this part. As a result they did not use the ratios of corresponding sides to find x in terms of a and b but tried to use statements such as $a = x + (a - x)$ to try to obtain the result.

(b) (i) In better responses, candidates who found the coordinates of the points T or P were generally able to progress towards the desired expression. In a number of responses, candidates struggled with finding the gradient of the tangent. The most common error was to assume that the gradient of OT was 1, meaning that the gradient of the tangent was -1. In most responses, candidates generally used the point-gradient formula of a line to derive the desired result, with only a small number using the two-point form. Candidates using the second approach had problems with the algebraic manipulation required.

(ii) In most responses, candidates realised that they needed to substitute $y=1$ into the equation of the line PT to derive the co-ordinates of Q. In a number of responses, candidates first rearranged the equation PT in terms of y, and then used $y=1$ to achieve the result. In a significant number of responses, candidates failed to realise that because BQ was horizontal, with B lying on the y-axis, the x-coordinate of Q was also the length of BQ. In these responses, candidates tended to use the distance formula with the coordinates of B and Q, involving a great deal of unnecessary working that often resulted in errors.

(iii) In most responses, candidates found the length of OP and used their answer from part (ii) to get the correct expression for the area of the trapezium. In weaker responses, candidates who encountered difficulties either used an incorrect length for BQ or an incorrect formula for the area of the trapezium.

(iv) In the majority of responses, candidates recognised the need to use calculus in this question. The most common, and successful, method was to use the quotient rule and solve $\frac{dA}{d\theta}=0$. In many responses, candidates used the breakdown for u, u', v and v' but few wrote the quotient rule. While a substantial number of candidates wrote the derivative expressions correctly, errors occurred in the simplifications. Common errors included incorrect signs when expanding, uv' first rather than $u'v$, having an incorrect denominator or failing to include the denominator in the derivative. In many responses, candidates did not solve the derivative equal to zero correctly due to poor algebraic skills. Quite often the test for the nature of the stationary point was omitted. Of those who attempted to test for a minimum, the change of sign of the first derivative was the most common method. Very few candidates who attempted to use the 'second derivative test' obtained the correct answer. Often, the second derivative was not correctly determined. Candidates are reminded to ensure that their solution fits into the given domain and that they explicitly show that it is a minimum.

(c) (i) Those candidates who attempted this part realised the need to solve the two equations simultaneously. In many responses, candidates successfully substituted in for either y or x, but very few simplified correctly or used the discriminant to derive the required result. In a number of responses, candidates tried various substitutions in an attempt to gain the required result. In some responses, candidates tried to substitute x or y into the given result or work their way backwards from this given result.

(ii) In many responses, candidates attempted to manipulate the result given in part (i) with no success. The most common incorrect answer was $c>\frac{1}{4}$, obtained by using the result from part (i) and the fact that $r>0$. Those candidates who wrote down the formula solution in part (i), that is $y=\frac{-(1-2c)\pm\sqrt{(1-2c)^2-4\times1\times\left(c^2-r^2\right)}}{2}$ then realising that $y>0$ and $\Delta=0$, obtained the desired result.