2021 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	C
3	D
4	C
5	A
6	D
7	A
8	C
9	B
10	B

Section II

Question 11

Criteria	Marks
• Provides correct solution	2
• Writes fraction using a common denominator, or equivalent merit	1

Sample answer:

$$\begin{aligned} x + \frac{x-1}{2} &= 9 \\ 2x + x - 1 &= 18 \\ 3x &= 19 \\ x &= \frac{19}{3} \end{aligned}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Uses correct trigonometric ratio, or equivalent merit	1

Sample answer:

In $\triangle XYZ$

$$\cos 30° = \frac{XY}{16}$$

$XY = 13.86$ cm (two decimal places)

Question 12 (b)

Criteria	Marks
• Provides correct solution	3
• Calculates the area of $\triangle XYZ$ and the semicircle, or equivalent merit	2
• Calculates the area of a semicircle, or equivalent merit	1

Sample answer:

$$\text{Shaded area} = \frac{1}{2}\pi(8)^2 - \frac{1}{2} \times 13.86 \times 16 \times \sin 30°$$

$= 45.1$ cm^2 (one decimal place)

Question 13

Criteria	Marks
• Provides correct solution	3
• Finds the derivative and attempts to calculate gradient	2
• Applies the product rule, or equivalent merit	1

Sample answer:

$$y = x\tan x$$

$$\frac{dy}{dx} = x\sec^2 x + \tan x$$

$$\text{Gradient of tangent} = \frac{\pi}{3}\sec^2\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3}\frac{1}{\left(\frac{1}{2}\right)^2} + \sqrt{3}$$

$$= \frac{4\pi}{3} + \sqrt{3}$$

Question 14

Criteria	Marks
• Provides correct solution	2
• Substitutes into sum of an arithmetic series formula	1

Sample answer:

$$a = 5,\ S_{43} = 2021$$

$$\therefore\ 2021 = \frac{43}{2}\left(2\times 5 + (43-1)d\right)$$

$$94 = 10 + 42d$$

$$d = 2$$

Question 15

Criteria	Marks
• Provides correct solution	2
• Finds the anti-derivative of $\sqrt{2x+4}$, or equivalent merit	1

Sample answer:

$$\begin{aligned}\int_{-2}^{0}\sqrt{2x+4}\,dx &= \int_{-2}^{0}(2x+4)^{\frac{1}{2}}\,dx \\ &= \frac{1}{2}\cdot\frac{2}{3}\left[(2x+4)^{\frac{3}{2}}\right]_{-2}^{0} \\ &= \frac{1}{3}\left(4^{\frac{3}{2}}-0\right) \\ &= \frac{8}{3}\end{aligned}$$

Question 16

Criteria	Marks
• Provides correct solution	3
• States $2x-6x^2>0$, or equivalent merit	2
• Finds the derivative, or equivalent merit	1

Sample answer:

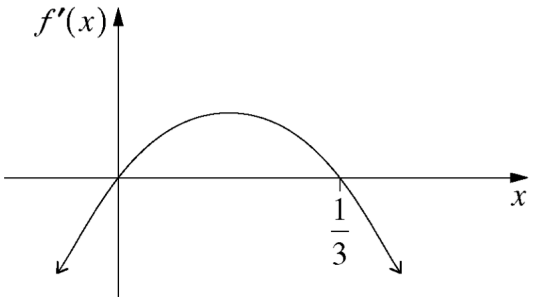
$f(x) = x^2 - 2x^3$

$f'(x) = 2x - 6x^2$

For $f(x)$ to be increasing, we need $f'(x) > 0$.

$2x - 6x^2 > 0$

$2x(1-3x) > 0$



$\therefore\ 0 < x < \frac{1}{3}$

Question 17 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$y = 29.2 - 0.011 \times 540$

$= 23.3°\text{C}$

Question 17 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Provides a correct relevant statement relating temperature to height above sea level, or equivalent merit	1

Sample answer:

For a one metre increase in the height above sea level, we expect a $0.011°\text{C}$ drop in average maximum daily temperature.

Question 17 (b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

It would be better to use the latitude. From the information given, the average maximum daily temperature has a stronger correlation with latitude than with height above sea level.

Question 18

Criteria	Marks
• Provides correct solution	3
• Calculates $47.18°$, or equivalent merit	2
• Substitutes into the sine rule correctly, or equivalent merit	1

Sample answer:

$$\frac{\sin A\hat{B}C}{25} = \frac{\sin 28°}{16}$$

$$\therefore \sin A\hat{B}C = 0.733\ldots$$

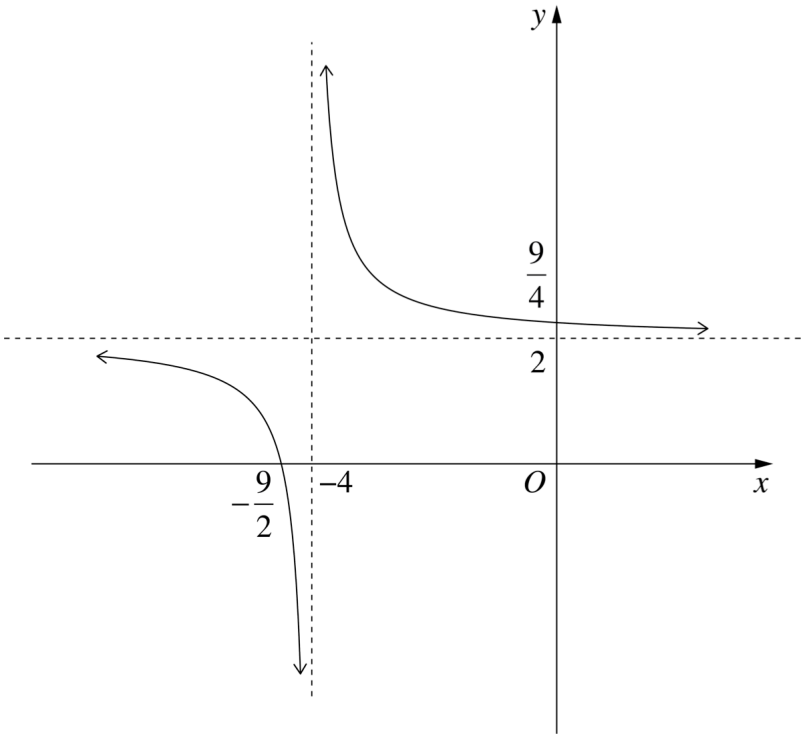
$$\therefore A\hat{B}C = 180° - 47.18\ldots°$$

$$= 133° \text{ (nearest degree)}$$

Question 19

Criteria	Marks
• Provides correct solution	3
• Provides correct graph without x and y intercepts, or equivalent merit	2
• Draws a hyperbola, or equivalent merit	1

Sample answer:



y-intercept: $y = 2 + \dfrac{1}{4}$

$= \dfrac{9}{4}$

x-intercept: $0 = 2 + \dfrac{1}{x+4}$

$x = -\dfrac{9}{2}$

Question 20

Criteria	Marks
• Provides correct solution	2
• Provides $\frac{\pi}{6}$, or equivalent merit	1

Sample answer:

$$0 \le x \le \frac{\pi}{4} \quad \Rightarrow \quad 0 \le 4x \le \pi$$

$$2\sin 4x = 1$$

$$\sin 4x = \frac{1}{2}$$

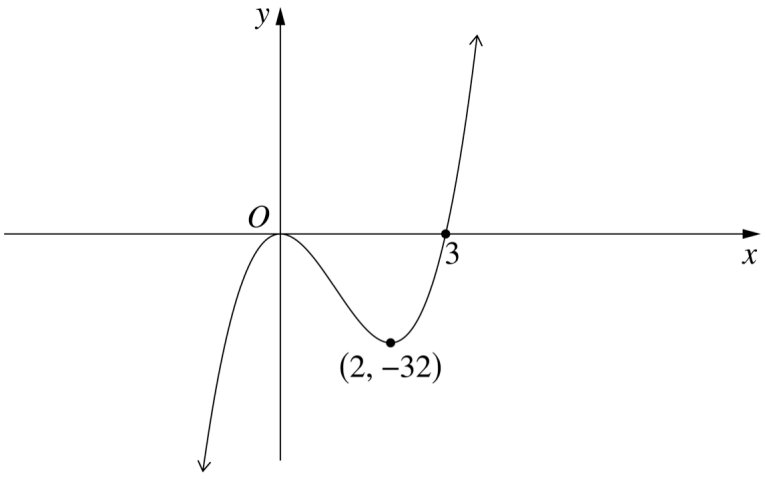
$$4x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{24}, \frac{5\pi}{24}$$

Question 21

Criteria	Marks
• Provides correct graph	2
• Provides graph with correct x intercepts, or equivalent merit	1

Sample answer:



Question 22 (a)

• Provides correct solution	1

Sample answer:

$$\text{Probability} = 0.1915 - 0.0398$$
$$= 0.1517$$

Question 22 (b)

Criteria	Marks
• Provides correct solution	3
• Finds associated probability, or equivalent merit	2
• Calculates *z*-score, or equivalent merit	1

Sample answer:

$$z = \frac{3528 - 3300}{570}$$
$$= 0.4$$

$$P(Z > 0.4) = 0.5 - 0.1554 = 0.3446$$

$\therefore$ Expect $1000 \times 0.3446 = 344.6$ babies born to have a birth weight greater than 3528 grams.

Question 23

Criteria	Marks
• Provides correct solution	4
• Equates –30 to a correct derivative, with $b = 2$, or equivalent merit	3
• Finds b and provides correct derivative, or equivalent merit	2
• Provides correct derivative, or equivalent merit	1

Sample answer:

$$P = 5000\,b^{-\frac{t}{10}}$$

$$t = 20,\ P = 1250$$

$$1250 = 5000\,b^{-2}$$

$$b^{-2} = \frac{1}{4}$$

$$b = 2\ (b > 0)$$

$$\therefore P = 5000(2)^{-\frac{t}{10}}$$

$$\frac{dP}{dt} = 5000 \times \ln 2 \times \frac{-1}{10} \times 2^{\frac{-t}{10}}$$

$$\frac{dP}{dt} = -30\ t = ?$$

$$-30 = -500 \times \ln 2 \times 2^{\frac{-t}{10}}$$

$$2^{\frac{t}{10}} = \frac{500 \ln 2}{30}$$

$$2^{\frac{t}{10}} = 11.55\ldots$$

$$\frac{t}{10} \ln 2 = \ln 11.55\ldots$$

$$t = 35.3$$

Question 24

Criteria	Marks
• Provides the correct solution	3
• Evaluates $\int_2^4 \frac{3}{x-1}\,dx$, or equivalent merit	2
• Calculates the area of the triangle, or equivalent merit	1

Sample answer:

$$\text{Area} = \frac{2\times 3}{2} + \int_2^4 \frac{3}{x-1}\,dx$$

$$= 3 + 3\left[\ln(x-1)\right]_2^4$$

$$= (3 + 3\ln 3) \text{ units}^2$$

Question 25

Criteria	Marks
• Provides correct solution	3
• Finds the correct future value of the annuity and attempts to apply the compound interest formula, or equivalent merit	2
• Identifies the correct interest factor from the table, or equivalent merit	1

Sample answer:

$$\text{Money in account after 8 years} = 1000 \times 8.2132$$

$$= \$8213.20$$

$$\text{Amount after 2 more years} = 8213.20\,(1.0125)^2$$

$$= \$8419.81$$

Question 26 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the time it takes to reach maximum height, or equivalent merit	1

Sample answer:

$y(t) = -5t^2 + 70t + 100$

$y'(t) = -10t + 70$

$y'(t) = 0 \quad \therefore -10t + 70 = 0$

$t = 7$

At $t = 7$, $\quad y = -5(7)^2 + 70(7) + 100$

$= 345$ m

Question 26 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the correct value of t, or equivalent merit	2
• Attempts to solve to find the time $y(t) = 0$, or equivalent merit	1

Sample answer:

To find when the particle hits the ground, we solve

$-5t^2 + 70t + 100 = 0$

$t^2 - 14t - 20 = 0$

$$t = \frac{14 \pm \sqrt{196 + 80}}{2}$$

$= 7 \pm \sqrt{69}$

$t = 7 + \sqrt{69}$, since $t > 0$

$y'(7 + \sqrt{69}) = -10(7 + \sqrt{69}) + 70$

$= -10\sqrt{69}$

$\therefore a = -10$ and $b = 69$

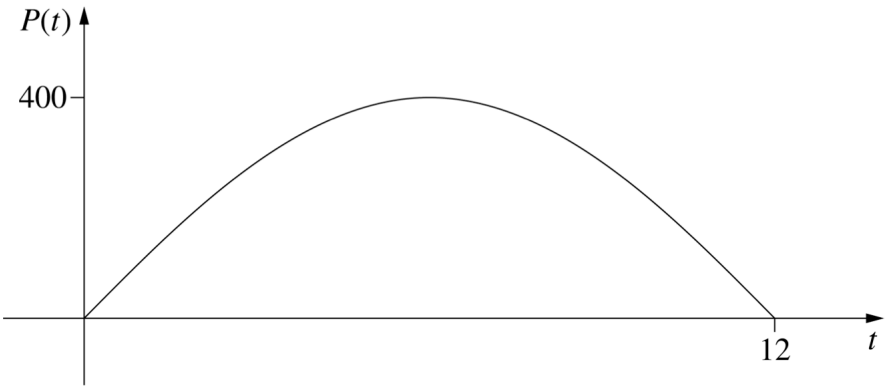
Question 27 (a)

Criteria	Marks
• Provides correct sketch	2
• Provides a sine graph with the correct amplitude, or equivalent merit	1

Sample answer:

$$\text{Period} = \frac{2\pi}{\frac{\pi}{12}}$$

$$= 24 \text{ hours}$$



Question 27 (b)

Criteria	Marks
• Provides correct solution	2
• Provides correct anti-derivative of $\sin\left(\frac{\pi}{12}t\right)$, or equivalent merit	1

Sample answer:

$$E = \int_a^b P(t)\,dt$$

$$= \int_a^b 400\sin\left(\frac{\pi}{12}t\right)dt$$

$$= -\frac{400(12)}{\pi}\left[\cos\left(\frac{\pi}{12}t\right)\right]_a^b$$

$$= -\frac{4800}{\pi}\left(\cos\frac{b\pi}{12} - \cos\frac{a\pi}{12}\right)$$

$$= \frac{4800}{\pi}\left(\cos\frac{a\pi}{12} - \cos\frac{b\pi}{12}\right)$$

Question 27 (c)

Criteria	Marks
• Provides correct solution	3
• Calculates the value of b, or equivalent merit	2
• Substitutes into E given in part (b), or equivalent merit	1

Sample answer:

$$300 = \frac{4800}{\pi}\left(\cos\frac{3\pi}{12} - \cos\frac{b\pi}{12}\right)$$

$$\frac{\pi}{16} = \frac{1}{\sqrt{2}} - \cos\frac{b\pi}{12}$$

$$\cos\frac{b\pi}{12} = \frac{1}{\sqrt{2}} - \frac{\pi}{16}$$

$$b = \frac{12}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}} - \frac{\pi}{16}\right)$$

$$b = 3.95$$

$\therefore$ He needs to wait $0.95 \times 60 = 57$ minutes (to the nearest minute)

Question 27 (d)

Criteria	Marks
• Provides correct solution	1

Sample answer:

It would take less time to charge his phone battery.

By starting to charge the battery six hours after sunrise, he is charging the phone at the time when the power is greatest.

Question 28 (a)

Criteria	Marks
• Provides correct solution	3
• Finds the correct anti-derivative, or equivalent merit	2
• Writes the correct definite integral, or equivalent merit	1

Sample answer:

$$\int_0^3 8 - 2^x \, dx$$

$$= \left[8x - \frac{2^x}{\ln 2} \right]_0^3$$

$$= \left(24 - \frac{8}{\ln 2} \right) - \left(0 - \frac{1}{\ln 2} \right)$$

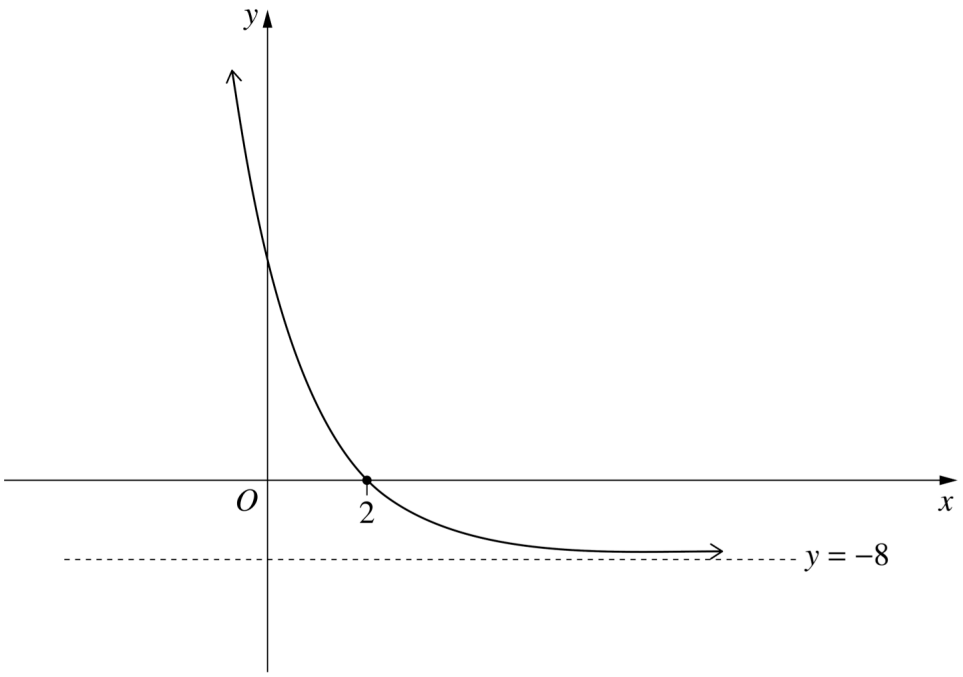
$$= 24 - \frac{8}{\ln 2} + \frac{1}{\ln 2}$$

$$= 24 - \frac{7}{\ln 2}$$

Question 28 (b)

Criteria	Marks
• Provides correct graph	2
• Draws the graph $y = f(-x)$, or equivalent merit	1

Sample answer:



Question 28 (c)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\int_2^5 g(x)\,dx = \frac{7}{\ln 2} - 24$$

Question 29 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the correct value of A_1, or equivalent merit	1

Sample answer:

$A_0 = \$5000$

$A_1 = 5000(1.03) + 1000$

$= \$6150$

$A_2 = 6150(1.03) + 1000$

$= \$7334.50$

$A_3 = 7334.50(1.03) + 1000$

$= \$8554.54$

Question 29 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the correct expression for the amount after n withdrawals using the sum of GP formula, or equivalent merit	2
• Attempts to find an expression for the amount after n withdrawals, or equivalent merit	1

Sample answer:

Let B_n = the amount in the account after n withdrawals

Then $B_0 = 30\,025.83$

$$B_1 = 30\,025.83(1.03) - 2000$$

$$B_2 = 30\,025.83(1.03)^2 - 2000(1.03) - 2000$$

$$B_n = 30\,025.83(1.03)^n - 2000\left[1 + (1.03) + \cdots + (1.03)^{n-1}\right]$$

We want $B_n < 0$.
Find n so that $B_n = 0$.

$$30\,025.83(1.03)^n = 2000\left[\frac{(1.03)^n - 1}{1.03 - 1}\right]$$

$$\frac{0.03 \times 30\,025.83}{2000}(1.03)^n = (1.03)^n - 1$$

$$0.45039(1.03)^n = (1.03)^n - 1$$

$$0.54961(1.03)^n = 1$$

$$(1.03)^n = 1.8194\ldots$$

$$n\ln(1.03) = \ln(1.8194)$$

$$n = \frac{\ln(1.8194)}{\ln(1.03)}$$

$$n \cong 20.25$$

$\therefore$ Megan makes 20 withdrawals of \$2000.

Question 30

Criteria	Marks
• Provides correct solution	2
• Writes $1 - e^{-0.01x} = 0.99$, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
F(x) &= 0.99 \\
1 - e^{-0.01x} &= 0.99 \\
e^{-0.01x} &= 0.01 \\
-0.01x &= \ln(0.01) \\
x &= 460.517\ldots \\
&= 460.52 \text{ hours (two decimal places)}
\end{aligned}
$$

Question 31

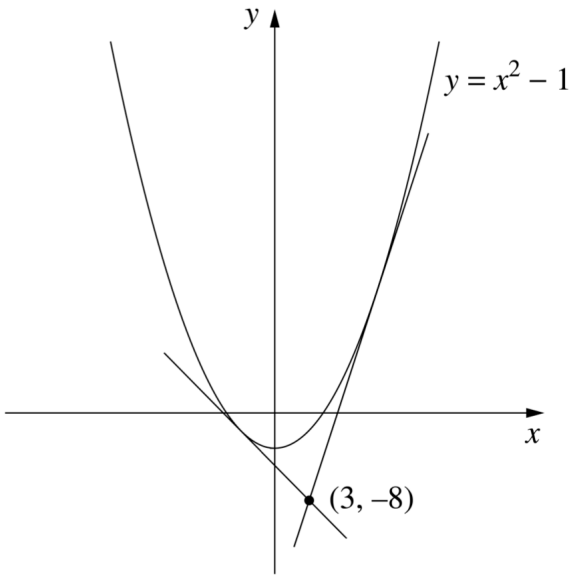
Criteria	Marks
• Provides correct solution	4
• Substitute $(3, -8)$ into the equation of the tangent at $x = a$, or equivalent merit	3
• Finds the equation of the tangent at $x = a$, or equivalent merit	2
• Finds the gradient of the tangent at $x = a$, or equivalent merit	1

Sample answer:

$y = x^2 - 1$

$y' = 2x$

Gradient of tangent at $x = a$ is $2a$.



$\therefore$ Equation of tangent at $(a, a^2 - 1)$ is

$$y - \left(a^2 - 1\right) = 2a\left(x - a\right)$$

$$y - a^2 + 1 = 2ax - 2a^2$$

$$y = 2ax - a^2 - 1$$

Since the tangent passes through $(3, -8)$ then

$$-8 = 6a - a^2 - 1$$

$$a^2 - 6a - 7 = 0$$

$$(a - 7)(a + 1) = 0$$

$$a = 7, \quad a = -1$$

$\therefore$ The equations of the two tangents are

When $a = 7$, $y = 2(7)x - 7^2 - 1$

$$y = 14x - 50$$

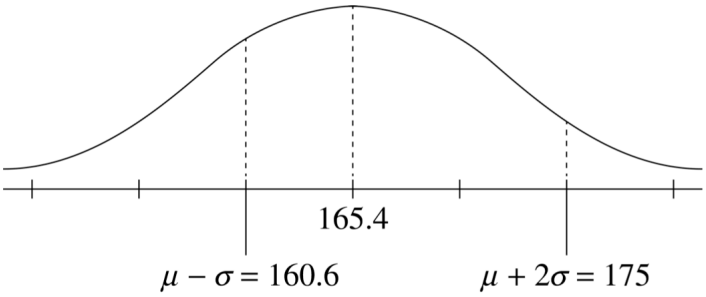
and when $a = -1$, $y = 2(-1)x - (-1)^2 - 1$

$$y = -2x - 2$$

Question 32

Criteria	Marks
• Provides correct solution	4
• Finds the mean and standard deviation for the heights of adult males, or equivalent merit	3
• Finds σ, or equivalent merit	2
• Labels the diagram, or equivalent merit	1

Sample answer:



$$\sigma = \frac{175 - 160.6}{3}$$

$$= 4.8$$

$$\mu = 160.6 + 4.8$$

$$= 165.4$$

Selected male is taller than 84% of population

$\therefore$ Selected male is 1 standard deviation above mean.

Mean height for males $= 1.05 \times 165.4 = 173.67$

Standard deviation of height for males $= 4.8 \times 1.1 = 5.28$

$\therefore$ Height $= 1.05\mu + 1.1\sigma$

$= 173.67 + 5.28$

$= 178.95$ cm

Question 33 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the anti-derivative of $\frac{Ax}{x^2+4}$, or equivalent merit	1

Sample answer:

$$\int_0^6 \frac{Ax}{x^2+4}\,dx = 1$$

$$\frac{A}{2}\left[\ln\left(x^2+4\right)\right]_0^6 = 1$$

$$\frac{A}{2}(\ln 40 - \ln 4) = 1$$

$$\frac{A}{2}\left(\ln\frac{40}{4}\right) = 1$$

$$\frac{A}{2}\ln 10 = 1$$

$$A = \frac{2}{\ln 10}$$

Question 33 (b)

Criteria	Marks
• Provides correct solution	2
• Finds the correct derivative of $f(x)$, or equivalent merit	1

Sample answer:

$$f'(x) = \frac{A\left(x^2+4\right) - Ax(2x)}{\left(x^2+4\right)^2}$$

$$= \frac{Ax^2 + 4A - 2Ax^2}{\left(x^2+4\right)^2}$$

$$= \frac{4A - Ax^2}{\left(x^2+4\right)^2} = 0$$

$$\therefore 4A - Ax^2 = 0$$

When $Ax^2 = 4A$

$$x^2 = 4$$

$$x = 2, \quad x > 0$$

$\therefore$ The mode of X is 2.

Question 33 (c)

Criteria	Marks
• Provides correct solution	2
• Provides correct definite integral, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
P(X<2) &= \int_0^2 \frac{Ax}{x^2+4}\,dx \\
&= \frac{A}{2}\left[\ln\left(x^2+4\right)\right]_0^2 \\
&= \frac{A}{2}(\ln 8 - \ln 4) \\
&= \frac{A}{2}\ln 2 \\
&= \frac{2}{\ln 10}\times\frac{1}{2}\times\ln 2 \\
&= \frac{\ln 2}{\ln 10} \\
&= \log_{10} 2
\end{aligned}
$$

Question 33 (d)

Criteria	Marks
• Provides correct solution	2
• Attempts to find the conditional probability, or equivalent merit	1

Sample answer:

$P(\text{IQ} > 130)$

$= P\left(\dfrac{\text{IQ} - 100}{15} > \dfrac{130 - 100}{15}\right)$

$= P(Z > 2)$

$= 2.5\%$

$$P\left(\text{IQ} > 130 \;\middle|\; \begin{matrix}\text{Time to} \\ \text{complete}\end{matrix} < 2\right)$$

$$= \frac{P\left(\text{IQ} > 130 \cap \begin{matrix}\text{Time to} \\ \text{complete}\end{matrix} < 2\right)}{P\left(\begin{matrix}\text{Time to} \\ \text{complete}\end{matrix} < 2\right)}$$

$$= \frac{2.5\% \times 80\%}{\log_{10} 2}$$

$= 0.06643\ldots$

$= 0.066$ (three decimal places)

Question 34

Criteria	Marks
• Provides correct solution	3
• Shows $E(X) = nr^{n+1}$ and attempts to find the sum of probabilities using sum of GP formula, or equivalent merit	2
• Finds correct expression for $E(X)$ or writes the sum of probabilities equal to 1	1

Sample answer:

$r + r^2 + \cdots + r^n = 1$

Sum of GP with $a = r$, common ratio $= r$, number of terms $= n$

$$\therefore \quad \frac{r\left(r^n - 1\right)}{r - 1} = 1$$

$$r^{n+1} - r = r - 1$$

$$r^{n+1} = 2r - 1 \qquad \text{————} \quad (*)$$

$$E(X) = \sum xP(X = x)$$

$$= r \times r^n + r^2 \times r^{n-1} + \cdots + r^n \times r$$

$$= r^{n+1} + r^{n+1} + \cdots + r^{n+1}$$

$$= r^{n+1} \times n$$

$$= nr^{n+1}$$

$$= n(2r - 1) \qquad \text{from } (*)$$

2021 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-T2 Trigonometric Functions and Identities	MA11-1
2	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
3	1	MA-E1 Logarithms and Exponentials	MA12-1
4	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
5	1	MA-E1 Logarithms and Exponentials	MA12-1
6	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
7	1	MA-C3 Applications of Differentiation	MA12-3
8	1	MA-F1 Working with Functions	MA11-1
9	1	MA-C1 Introduction to Differentiation	MA11-1
10	1	MA-C3 Applications of Differentiation	MA12-10

Section II

Question	Marks	Content	Syllabus outcomes
11	2	MA-F1 Working with Functions	MA11-1
12 (a)	2	MA-T1 Trigonometry and Measure of Angles	MA11-3
12 (b)	3	MA-T1 Trigonometry and Measure of Angles	MA11-3
13	3	MA-C2 Differential Calculus	MA12-3
14	2	MA-M1 Modelling Financial Situations	MA12-4
15	2	MA-C4 Integral Calculus	MA12-3
16	3	MA-C3 Applications of Differentiation	MA12-6
17 (a) (i)	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
17 (a) (ii)	2	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
17 (b)	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
18	3	MA-T1 Trigonometry and Measure of Angles	MA11-3
19	3	MA-F2 Graphing Techniques	MA12-1
20	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
21	2	MA-F2 Graphing Techniques	MA12-1

Question	Marks	Content	Syllabus outcomes
22 (a)	1	MA-S3 Random Variables	MA12-8
22 (b)	3	MA-S3 Random Variables	MA12-8
23	4	MA-C3 Applications of Differentiation	MA12-3
24	3	MA-C4 Integral Calculus	MA12-7
25	3	MA-M1 Modelling Financial Situations	MA12-2
26 (a)	2	MA-C3 Applications of Differentiation	MA12-6
26 (b)	3	MA-C3 Applications of Differentiation	MA12-6
27 (a)	2	MA-T3 Trigonometric Functions and Graphs	MA12-5
27 (b)	2	MA-C4 Integral Calculus	MA12-3
27 (c)	3	MA-T3 Trigonometric Functions and Graphs	MA12-5
27 (d)	1	MA-C4 Integral Calculus	MA12-10
28 (a)	3	MA-C4 Integral Calculus	MA12-7
28 (b)	2	MA-F2 Graphing Techniques	MA12-1
28 (c)	1	MA-C4 Integral Calculus	MA12-7
29 (a)	2	MA-M1 Modelling Financial Situations	MA12-4
29 (b)	3	MA-M1 Modelling Financial Situations	MA12-2
30	2	MA-S3 Random Variables	MA12-8
31	4	MA-C3 Applications of Differentiation	MA12-10
32	4	MA-S3 Random Variables	MA12-8
33 (a)	2	MA-S3 Random Variables	MA12-8
33 (b)	2	MA-S3 Random Variables	MA12-8
33 (c)	2	MA-S1 Random Variables	MA 12-8
33 (d)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
34	3	MA-S1 Probability and Discrete Probability Distributions	MA11-7