2020 HSC Mathematics Advanced Marking Guidelines

Section I

Multiple-choice Answer Key

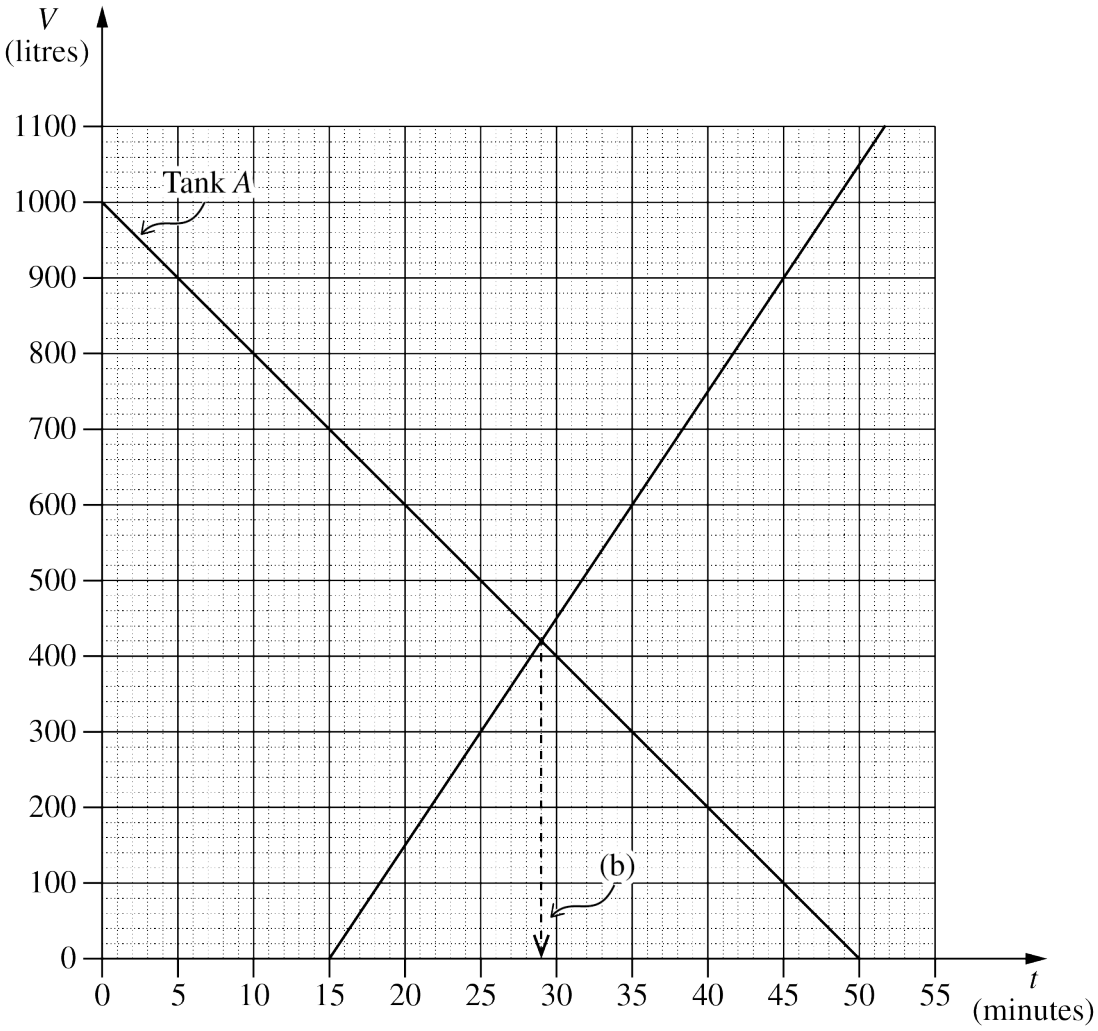
Question	Answer
1	D
2	B
3	A
4	B
5	C
6	B
7	A
8	A
9	C
10	D

Section II

Question 11 (a)

Criteria	Marks
• Provides the correct solution	1

Sample answer:



Question 11 (b)

Criteria	Marks
• Provides the correct solution	2
• Draws the graph of volume for tank *B*, or equivalent merit	1

Sample answer:

29 minutes

Question 11 (c)

Criteria	Marks
• Provides the correct answer	1

Sample answer:

45 minutes

Question 12

Criteria	Marks
• Provides the correct solution	3
• Attempts to use the sum of an arithmetic series, or equivalent merit OR • Finds the number of terms in the series	2
• Finds the values of a and d, or equivalent merit	1

Sample answer:

$4 + 10 + 16 + \cdots + 1354$

$d = 6 \qquad a = 4$

$1354 = 4 + (n-1) \times 6$

$\dfrac{1350}{6} = n - 1$

$n = 226$

$S_n = \dfrac{n}{2}(a + \ell)$

$= \dfrac{226}{2}(4 + 1354)$

$= 153\,454$

Question 13

Criteria	Marks
• Provides the correct solution	2
• Finds the anti-derivative of $\sec^2 x$, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$= \left[\tan x\right]_0^{\frac{\pi}{4}}$$

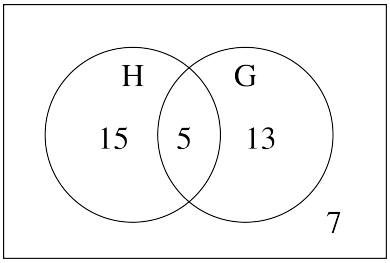
$$= \left[\tan\frac{\pi}{4} - \tan 0\right]$$

$= 1$

Question 14 (a)

Criteria	Marks
• Provides the correct solution	2
• Finds the number of students who study both History and Geography, or equivalent merit	1

Sample answer:



$40 - 7 = 33$
$20 + 18 - 33 = 5$

$$\frac{5}{40} = \frac{1}{8}$$

Question 14 (b)

Criteria	Marks
• Provides the correct answer	1

Sample answer:

$$P(\bar{H} \mid G) = \frac{13}{18}$$

Question 14 (c)

Criteria	Marks
• Provides the correct solution	2
• Obtains the probability of a student studying History as $\frac{20}{40}$, or equivalent merit	1

Sample answer:

$$\frac{20}{40} \times \frac{20}{39} = \frac{10}{39}$$

Question 15 (a)

Criteria	Marks
• Provides the correct solution	1

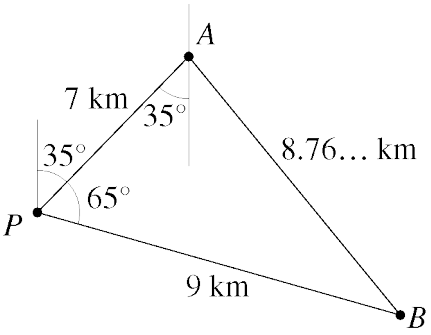
Sample answer:

$100° - 35° = 65°$

Question 15 (b)

Criteria	Marks
• Provides the correct solution	2
• Attempts to use the cosine rule to find the distance AB	1

Sample answer:



$AB^2 = 7^2 + 9^2 - 2 \times 7 \times 9 \cos 65°$

$= 76.75\ldots$

$AB = 8.76 \text{ km}$

Question 15 (c)

Criteria	Marks
• Provides the correct solution	2
• Finds another angle in $\triangle APB$, or equivalent merit	1

Sample answer:

$$\frac{\sin A}{9} = \frac{\sin 65°}{8.76\ldots}$$

$$A = 68.6\ldots$$

$$\div 69°$$

$\text{Bearing} = 180° - (69° - 35°)$

$= 146°$

Question 16

Criteria	Marks
• Provides the correct solution	4
• Finds the stationary points and the point of inflection, or equivalent merit	3
• Finds the stationary points, or equivalent merit	2
• Finds the derivative of the function, or equivalent merit	1

Sample answer:

$$y = -x^3 + 3x^2 - 1$$

$$\frac{dy}{dx} = -3x^2 + 6x$$

$$= 3x(2 - x)$$

When $\frac{dy}{dx} = 0$, we have $3x(2 - x) = 0$.

$x = 0$, $\quad x = 2$

$y = -1$ $\quad y = -2^3 + 3(2)^2 - 1$

$\quad = 3$

Stationary points are $(0, -1)$ and $(2, 3)$.

$$\frac{d^2y}{dx^2} = -6x + 6$$

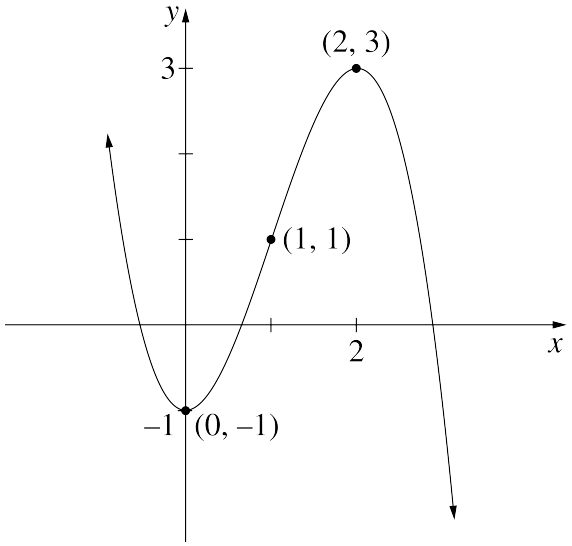
When $\frac{d^2y}{dx^2} = 0$, we have $6(1 - x) = 0$.

$$x = 1$$

$$y = -1^3 + 3 \times 1^2 - 1$$

$$= 1$$

The point of inflection is $(1, 1)$.



Question 17

Criteria	Marks
• Provides the correct solution	2
• Writes an anti-derivative involving the log function, or equivalent merit	1

Sample answer:

$$\int \frac{x}{4+x^2}\,dx = \frac{1}{2}\int \frac{2x}{4+x^2}\,dx$$

$$= \frac{1}{2}\ln\left(4+x^2\right)+c$$

Question 18 (a)

Criteria	Marks
• Provides the correct solution	2
• Attempts to use the product rule	1

Sample answer:

$$2e^{2x}(2x+1)+2e^{2x}$$

$$= 2e^{2x}(2x+1+1)$$

$$= 2e^{2x}(2x+2)$$

$$= 4e^{2x}(x+1)$$

Question 18 (b)

Criteria	Marks
• Provides the correct answer	1

Sample answer:

$$\int (x+1)e^{2x}\,dx$$

$$= \frac{1}{4}\int 4(x+1)e^{2x}\,dx$$

$$= \frac{1}{4}e^{2x}(2x+1)+c$$

Question 19

Criteria	Marks
• Provides the correct solution	2
• Shows that the LHS is equal to $\frac{1-\cos^2\theta}{\cos\theta}$, or equivalent merit	1

Sample answer:

$\sec\theta - \cos\theta = \sin\theta\tan\theta$

$$\text{LHS} = \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$= \sin\theta \times \frac{\sin\theta}{\cos\theta}$$

$$= \sin\theta \times \tan\theta$$

$$= \text{RHS}$$

Question 20

Criteria	Marks
• Provides the correct solution	2
• Attempts to use the trapezoidal rule	1

Sample answer:

$$\frac{\frac{5}{60}-0}{2(5)}\left[60+67+2(55+65+68+70)\right]$$

$$= \frac{643}{120}$$

$$\div 5.4 \text{ km}$$

Question 21 (a)

Criteria	Marks
• Provides the correct answer	1

Sample answer:

$T = 25 + 70(1.5)^{-0.4 \times 4}$

$= 61.5891\ldots$

$\div 61.6°$

Question 21 (b)

Criteria	Marks
• Provides the correct solution	2
• Finds the derivative of T, or equivalent merit	1

Sample answer:

$$\frac{dT}{dt} = 70(1.5)^{-0.4t}(-0.4)\ln(1.5)$$

When $t = 4$,

$$\frac{dT}{dt} = 70(1.5)^{-1.6}(-0.4)\ln(1.5)$$

$$= -5.934\ldots$$

Therefore at $t = 4$ the tea is cooling at 5.9°C per minute.

Question 21 (c)

Criteria	Marks
• Provides the correct solution	3
• Uses the log function to solve the equation	2
• Makes $1.5^{-0.4t}$ the subject of the equation, or equivalent merit	1

Sample answer:

$$55 = 25 + 70(1.5)^{-0.4t}$$

$$\frac{30}{70} = 1.5^{-0.4t}$$

$$\ln\left(\frac{3}{7}\right) = -0.4t\ln(1.5)$$

$$t = -\frac{\ln\left(\frac{3}{7}\right)}{0.4\ln(1.5)}$$

$$\doteq 5.22 \text{ minutes}$$

Question 22

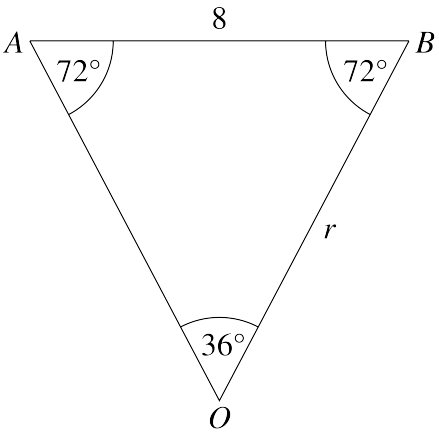
Criteria	Marks
• Provides a correct solution	4
• Calculates area of $\triangle AOB$, or equivalent merit	3
• Finds OA, or equivalent merit	2
• Finds AB, or equivalent merit	1

Sample answer:

$$\begin{aligned} AB &= 80 \div 10 \\ &= 8 \text{ cm} \end{aligned}$$

$$\angle AOB = 360° \div 10 = 36°$$

$$\begin{aligned} \angle OAB &= \frac{180° - 36°}{2} \\ &= 72° \end{aligned}$$



$$\frac{r}{\sin 72°} = \frac{8}{\sin 36°}$$

$$r = 12.944\ldots$$

$$\text{Area} = 10 \times \frac{1}{2} \times (12.944\ldots)^2 \times \sin 36°$$

$$\div 492.4 \text{ cm}^2$$

Question 23 (a)

Criteria	Marks
• Provides the correct solution	2
• Writes an equation involving a definite integral set equal to 1, or equivalent merit	1

Sample answer:

$$\int_0^k \sin x \, dx = 1$$

$$\left[-\cos x\right]_0^k = 1$$

$$-\cos k + \cos 0 = 1$$

$$\cos k = 0$$

$$k = \frac{\pi}{2}$$

Question 23 (b)

Criteria	Marks
• Provides the correct solution	2
• Writes the definite integral	1

Sample answer:

$$\int_0^1 \sin x \, dx$$

$$= \left[-\cos x\right]_0^1$$

$$= -\cos 1 + 1$$

$$\doteqdot 0.4597$$

Question 24

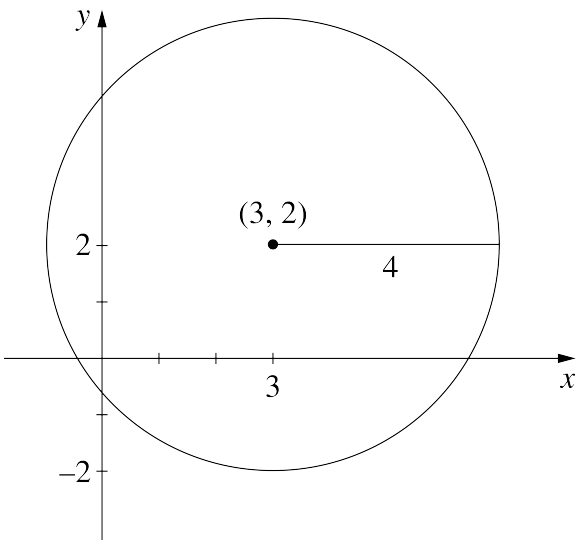
Criteria	Marks
• Provides the correct solution	3
• Finds the centre and the radius of the original circle or of the reflected circle, or equivalent merit	2
• Completes the square for the equation, or equivalent merit	1

Sample answer:

$$x^2 - 6x + y^2 + 4y - 3 = 0$$
$$x^2 - 6x + 9 + y^2 + 4y + 4 = 3 + 9 + 4$$
$$(x-3)^2 + (y+2)^2 = 16$$

The original circle has centre (3, –2) and radius 4.

The reflected circle has centre (3, 2) and radius 4.



Question 25 (a)

Criteria	Marks
• Provides the correct solution	3
• Writes an expression for the perimeter in terms of x and y and finds a correct expression for y in terms of x, or equivalent merit	2
• Writes an expression for a perimeter in terms of x and y which includes the curved length of a quadrant, or equivalent merit	1

Sample answer:

$$xy + \frac{1}{4}\pi x^2 = 36$$

$$xy = 36 - \frac{1}{4}\pi x^2$$

$$y = \frac{36}{x} - \frac{1}{4}\pi x$$

$$P = 2y + 2x + \frac{1}{4} \times 2\pi x$$

$$= 2\left(\frac{36}{x} - \frac{1}{4}\pi x\right) + 2x + \frac{1}{2}\pi x$$

$$= \frac{72}{x} - \frac{1}{2}\pi x + 2x + \frac{1}{2}\pi x$$

$$= 2x + \frac{72}{x}$$

Question 25 (b)

Criteria	Marks
• Provides the correct solution	4
• Verifies that at $x = 6$ there is a minimum turning point, or equivalent merit	3
• Solves $\frac{dP}{dx} = 0$, or equivalent merit	2
• Finds $\frac{dP}{dx}$, or equivalent merit	1

Sample answer:

$$\frac{dP}{dx} = 2 - \frac{72}{x^2}$$

Setting $\frac{dP}{dx} = 0$, we have

$$2 = \frac{72}{x^2}$$

$$x^2 = 36$$

$$x = 6 \qquad (\text{since } x > 0)$$

$$\frac{d^2P}{dx^2} = \frac{144}{x^3}$$

When $x = 6$,

$\frac{d^2P}{dx^2} = \frac{144}{6^3}$ which is greater than zero.

$\therefore P$ is a minimum when $x = 6$.

The minimum perimeter $= 2 \times 6 + \frac{72}{6}$

$$= 12 + 12$$

$$= 24 \text{ m}$$

Question 26 (a)

Criteria	Marks
• Provides the correct solution	2
• Finds the value of A_1, or equivalent merit	1

Sample answer:

$$A_1 = 60\,000(1.005) - 800$$
$$= \$59\,500$$
$$A_2 = 59\,500(1.005) - 800$$
$$= \$58\,997.50$$
$$A_3 = 58\,997.50(1.005) - 800$$
$$\doteqdot \$58\,492.49$$

Question 26 (b)

Criteria	Marks
• Provides the correct solution	2
• Finds by how much the balance is reduced, or equivalent merit	1

Sample answer:

Total withdrawals after 3 months
$= 800 \times 3$
$= \$2400$

Balance is reduced by
$= 60\,000 - 58\,492.49$
$= \$1507.51$

Interest
$= \$2400 - \$1507.51$
$= \$892.49$

Question 26 (c)

Criteria	Marks
• Provides the correct solution	3
• Finds an expression for A_{94}, or equivalent merit	2
• Attempts to develop a formula for A_{94}, or equivalent merit	1

Sample answer:

$$A_1 = 60\,000(1.005) - 800$$

$$A_2 = 60\,000(1.005)^2 - 800(1.005) - 800$$

$$\vdots$$

$$\begin{aligned} A_{94} &= 60\,000(1.005)^{94} - 800\left(1 + 1.005 + \cdots + 1.005^{93}\right) \\ &= 60\,000(1.005)^{94} - 800\,\frac{1.005^{94} - 1}{0.005} \\ &= \$187.85 \end{aligned}$$

Question 27

Criteria	Marks
• Provides the correct solution	5
• Finds value of b, or equivalent merit	4
• Substitutes $\overline{x}$ and $\overline{y}$ into the regression equation, or equivalent merit	3
• Finds the value of $\overline{x}$ and $\overline{y}$, or equivalent merit	2
• Finds the value of $\overline{x}$ or $\overline{y}$ or the median of the temperature, or equivalent merit	1

Sample answer:

$$\overline{x} = 22 - 0.525$$
$$= 21.475$$
$$\overline{y} = \frac{684}{20}$$
$$= 34.2$$

Substituting $(\overline{x}, \overline{y})$ into the regression equation, we have

$$34.2 = -10.6063 + b(21.475)$$
$$\frac{44.8063}{21.475} = b$$
$$b = 2.08644\ldots$$

When $x = 19$, $y = -10.6063 + 2.08644\ldots \times 19$

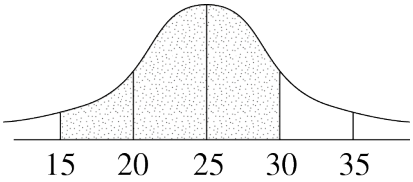
$$= 29.03606\ldots$$

$\therefore$ 29 chirps are expected.

Question 28 (a)

Criteria	Marks
• Provides the correct solution	3
• Finds the percentage of adults earning between \$15 and \$30 and finds a probability using $1 - P(\text{neither earns ...})$, or equivalent merit	2
• Finds a probability using $1 - P(\text{neither earns ...})$, or equivalent merit OR • Finds the percentage of adults earning between \$15 and \$30, or equivalent merit	1

Sample answer:



$P(\$15 < \text{hourly rate of pay} < \$30)$

$$= 68\% + \frac{95\% - 68\%}{2}$$
$$= 68\% + 13.5\%$$
$$= 81.5\%$$

$P(\text{at least one earns between \$15 and \$30 per hour})$

$= 1 - P(\text{neither earns between \$15 and \$30 per hour})$

$= 1 - (1 - 0.815)^2$

$= 0.965775$

Question 28 (b)

Criteria	Marks
• Provides the correct solution	2
• Finds the probability of randomly choosing an adult who works or the probability of an adult earning more than \$25 per hour	1

Sample answer:

$P(\text{chosen adult works}) = \dfrac{3}{4}$

$\therefore P(\text{chosen adult works and earns more than \$25 per hour}) = \dfrac{3}{4} \times \dfrac{1}{2}$

$= \dfrac{3}{8}$

Question 29 (a)

Criteria	Marks
• Provides the correct solution	2
• Finds the gradient of the tangent at $x = p$, or equivalent merit	1

Sample answer:

$$y = c\ln x$$

$$\frac{dy}{dx} = \frac{c}{x}$$

The gradient of the tangent at $x = p$ is $\dfrac{c}{p}$.

At $x = p \quad y = c\ln p$

$\therefore\ y - c\ln p = \dfrac{c}{p}(x - p)$

$\therefore\ y - c\ln p = \dfrac{c}{p}x - c$

$\therefore\ \quad y = \dfrac{c}{p}x - c + c\ln p$

Question 29 (b)

Criteria	Marks
• Provides the correct solution	2
• States that $\frac{c}{p}=1$ or $-c+c\ln p=0$, or equivalent merit	1

Sample answer:

$\frac{c}{p}=1$ (since the tangent has a gradient of 1)

$\therefore\ c=p$

$-c+c\ln p=0$ (since the tangent passes through the origin)

$$-c+c\ln c=0$$
$$c(-1+\ln c)=0$$
$$c=0,\quad \ln c=1$$
$$c=e \quad (\text{as } c>0)$$

Question 30 (a)

Criteria	Marks
• Provides the correct solution	2
• Attempts to solve a quadratic equation to find the x-coordinate of A, or equivalent merit	1

Sample answer:

$$ax^2=4x-x^2$$
$$(a+1)x^2-4x=0$$
$$x\left[(a+1)x-4\right]=0$$
$$x=0,\quad (a+1)x-4=0$$
$$(a+1)x=4$$
$$x=\frac{4}{a+1}$$

Question 30 (b)

Criteria	Marks
• Provides the correct solution	4
• Correctly substitutes the limits into the anti-derivative, or equivalent merit	3
• Finds the anti-derivative, or equivalent merit	2
• Writes an expression for the area involving an integral of the difference between two functions, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{4}{a+1}} \left(4x - x^2 - ax^2\right) dx = \frac{16}{3}$$

$$\therefore \quad \int_0^{\frac{4}{a+1}} \left(4x - (a+1)x^2\right) dx = \frac{16}{3}$$

$$\left[2x^2 - \frac{(a+1)x^3}{3}\right]_0^{\frac{4}{a+1}} = \frac{16}{3}$$

$$\frac{2 \times 16}{(a+1)^2} - \frac{(a+1)}{3} \times \frac{64}{(a+1)^3} - 0 = \frac{16}{3}$$

$$\frac{32}{(a+1)^2} - \frac{64}{3(a+1)^2} = \frac{16}{3}$$

$$\frac{96 - 64}{3(a+1)^2} = \frac{16}{3}$$

$$\frac{32}{3(a+1)^2} = \frac{16}{3}$$

$$\therefore \quad (a+1)^2 = 2$$

$$a = \sqrt{2} - 1 \qquad (\text{since } a > 0)$$

Question 31 (a)

Criteria	Marks
• Provides the correct values for a and b	2
• Provides the correct value of a or the correct value of b, or equivalent merit	1

Sample answer:

$$a = \frac{35\,000 - 5000}{2}$$

$$= 15\,000$$

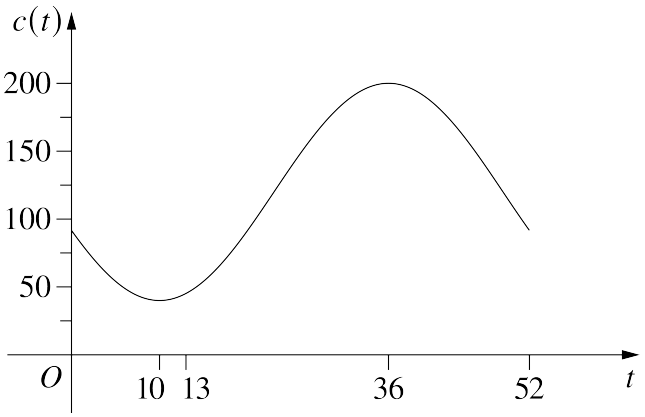
$$b = \frac{35\,000 + 5000}{2}$$

$$= 20\,000$$

Question 31 (b)

Criteria	Marks
• Provides the correct solution	3
• Finds the values of t when the population of mice increases and the values of t when the population of cats increases	2
• Finds the values of t when the population of mice increases, or equivalent merit	1

Sample answer:



The population of mice increases for $0 < t < 13$ and $39 < t < 52$.

The population of cats increases for $10 < t < 36$.

Therefore both populations are increasing for $10 < t < 13$.

Question 31 (c)

Criteria	Marks
• Provides the correct solution	2
• Correctly differentiates $m(t)$, or equivalent merit	1

Sample answer:

The cat population reaches a maximum at $t = 36$.

$$m'(t) = \frac{\pi}{26} \times 15\,000 \cos\left(\frac{\pi}{26} t\right)$$

$$m'(36) = \frac{\pi}{26} \times 15\,000 \cos\left(\frac{\pi}{26} \times 36\right)$$

$$= -642.7\ldots$$

Therefore the mice population is decreasing at 643 mice per week.

2020 HSC Mathematics Advanced Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-F1 Working with Functions	MA11-1
2	1	MA-F2 Graphing Techniques	MA12-1
3	1	MA-S1 Probability and Discrete Probability Distributions	MA12-8
4	1	MA-C4 Integral Calculus	MA12-7
5	1	MA-F1 Working with Functions	MA11-2
6	1	MA-T3 Trigonometric Functions	MA12-5
7	1	MA-C4 Integral Calculus	MA12-7
8	1	MA-C3 Applications of Differentiation	MA12-3
9	1	MA-S3 Random Variables	MA12-8
10	1	MA-C2 Differential Calculus	MA12-6

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	1	MA-F1 Working with Functions	MA11-2
11 (b)	2	MA-F1 Working with Functions	MA11-1
11 (c)	1	MA-F1 Working with Functions	MA11-2
12	3	MA-M1 Modelling Financial Situations	MA12–4
13	2	MA-C4 Integral Calculus	MA12-7
14 (a)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-9
14 (b)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
14 (c)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
15 (a)	1	MA-T1 Trigonometry and Measuring Angles	MA11-1
15 (b)	2	MA-T1 Trigonometry and Measuring Angles	MA11-3
15 (c)	2	MA-T1 Trigonometry and Measuring Angles	MA11-3
16	4	MA-C3 Applications of Differentiation	MA12-3, 6,10
17	2	MA-C4 Integral Calculus	MA12-7
18 (a)	2	MA-C2 Differential Calculus	MA12-6
18 (b)	1	MA-C4 Integral Calculus	MA12-3
19	2	MA-T2 Trigonometric Functions and Identities	MA11-4

Question	Marks	Content	Syllabus outcomes
20	2	MA-C4 Integral Calculus	MA12-7
21 (a)	1	MA-E1 Exponentials and Logarithms	MA11-6
21 (b)	2	MA-C3 Applications of Differentiation	MA12-6
21 (c)	3	MA-E1 Exponentials and Logarithms	MA11-6
22	4	MA-T1 Trigonometry and Measuring Angles	MA11-9
23 (a)	2	MA-S3 Random Variables	MA12-8
23 (b)	2	MA-S3 Random Variables	MA12-8
24	3	MA-F2 Graphing Techniques	MA12-1
25 (a)	3	MA-F1 Working with Functions	MA11-1
25 (b)	4	MA-C3 Applications of Differentiation	MA12-10
26 (a)	2	MA-M1 Modelling Financial Situations	MA12-2
26 (b)	2	MA-M1 Modelling Financial Situations	MA12-2
26 (c)	3	MA-M1 Modelling Financial Situations	MA12–4
27	5	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
28 (a)	3	MA-S3 Random Variables	MA12-8
28 (b)	2	MA-S3 Random Variables	MA12-8
29 (a)	2	MA-C2 Differential Calculus	MA12-6
29 (b)	2	MA-C2 Differential Calculus	MA12-6
30 (a)	2	MA-F1 Working with Functions	MA11-1
30 (b)	4	MA-C4 Integral Calculus	MA12-7
31 (a)	2	MA-T3 Trigonometric Functions	MA12-5
31 (b)	3	MA-T3 Trigonometric Functions	MA12-5
31 (c)	2	MA-C3 Applications of Differentiation	MA12-6