BOARD OF STUDIES
NEW SOUTH WALES

2013 HSC Mathematics Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	B
3	A
4	A
5	C
6	D
7	B
8	B
9	C
10	D

Section II

Question 11 (a)

Criteria	Marks
• Correct answer	1

Sample answer:

$$\begin{aligned}\ln 3 &= 1.09861\ldots \\ &= 1.10 \text{ (to 3 significant figures)}\end{aligned}$$

Question 11 (b)

Criteria	Marks
• Correct solution	2
• Attempts to factorise numerator and denominator, or equivalent merit	1

Sample answer:

$$\begin{aligned}\lim_{x\to 2}\frac{x^3-8}{x^2-4} &= \lim_{x\to 2}\frac{(x-2)\left(x^2+2x+4\right)}{(x-2)(x+2)} \\ &= \lim_{x\to 2}\frac{x^2+2x+4}{x+2} \\ &= \frac{2^2+2\times 2+4}{2+2} = 3\end{aligned}$$

Question 11 (c)

Criteria	Marks
• Correct derivative	2
• Attempts to use chain rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}(\sin x - 1)^8 = 8(\sin x - 1)^7 \cos x$$

Question 11 (d)

Criteria	Marks
• Correct derivative	2
• Attempts to use the product rule	1

Sample answer:

$$\frac{d}{dx}\left(x^2e^x\right) = 2xe^x + x^2e^x$$

Question 11 (e)

Criteria	Marks
• Correct primitive	2
• Obtains primitive of the form ae^{4x+1}	1

Sample answer:

$$\int e^{4x+1}\,dx = \frac{1}{4}e^{4x+1} + C$$

Question 11 (f)

Criteria	Marks
• Correct solution	3
• Obtains correct primitive OR • Substitutes limits into primitive of the form $a\ln\left(x^3+1\right)$, or equivalent merit	2
• Obtains primitive of the form $a\ln\left(x^3+1\right)$, or equivalent merit	1

Sample answer:

$$\int_0^1 \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\int_0^1 \frac{3x^2}{x^3+1}\,dx$$

$$= \frac{1}{3}\left[\ln\left(x^3+1\right)\right]_0^1$$

$$= \frac{1}{3}\left[\ln\left(1^3+1\right)-\ln\left(0^3+1\right)\right]$$

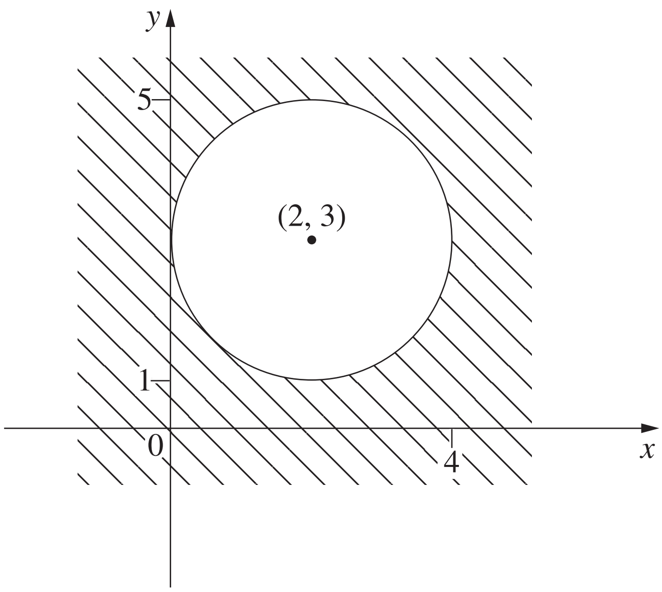
$$= \frac{1}{3}\ln 2$$

Question 11 (g)

Criteria	Marks
• Correct sketch	3
• Shows circle with TWO of the following correct: radius, centre, shading	2
• Shows circle with correct radius OR correct centre OR correct shading	1

Sample answer:

Exterior of a circle centre at (2, 3) and a radius 2.



Question 12 (a)

Criteria	Marks
• Correct solution	2
• Finds correct second derivative, or equivalent merit	1

Sample answer:

For a point of inflexion

$y'' = 0$ at $x = p$

$y' = 3ax^2 + 2bx + c$

$y'' = 6ax + 2b$

Hence, $6ax + 2b = 0$

$\therefore \quad x = \dfrac{-2b}{6a}$

at $x = p$, $\quad p = \dfrac{-b}{3a}$

Question 12 (b) (i)

Criteria	Marks
• Correct solution	2
• Finds correct slope, or equivalent merit	1

Sample answer:

Slope of the line is $\dfrac{17-(-1)}{22-(-2)} = \dfrac{18}{24}$

$$= \frac{3}{4}$$

Since $(-2,-1)$ is on the line, the equation of the line is

$$y-(-1) = \frac{3}{4}(x-(-2))$$

$$4y+4 = 3x+6$$

$$0 = 3x-4y+2$$

Question 12 (b) (ii)

Criteria	Marks
• Correct solution	1

Sample answer:

Using the perpendicular distance formula,

$$d = \frac{|3(-2)-4\times 24+2|}{\sqrt{3^2+4^2}}$$

$$= \frac{100}{5}$$

$$= 20$$

$\therefore$ the distance is 20 units

Question 12 (b) (iii)

Criteria	Marks
• Correct solution	1

Sample answer:

$$\begin{aligned} EC &= \sqrt{(42-39)^2+(22-18)^2} \\ &= \sqrt{3^2+4^2} \\ &= 5 \end{aligned}$$

$\therefore$ the length of EC is 5 units

Question 12 (b) (iv)

Criteria	Marks
• Correct solution	2
• Finds the length of *FD*, or equivalent merit	1

Sample answer:

$$\begin{aligned} AD &= \sqrt{(22-(-21))^2+(17-(-11))^2} \\ &= \sqrt{24^2+18^2} \\ &= 30 \end{aligned}$$

Since F is the midpoint of *AD*, *FD* = 15

The height of the trapezium is the perpendicular distance from part (ii).

$$\begin{aligned} \therefore \text{ Area of trapezium } &= \frac{1}{2}\times 20\times(5+15) \text{ units}^2 \\ &= 200 \text{ units}^2 \end{aligned}$$

Question 12 (c) (i)

Criteria	Marks
• Correct answer	2
• Obtains Kim's or Alex's salary after 10 years, or equivalent merit	1

Sample answer:

Kim's salary is a geometric progression where $a = 30\,000$ and $r = 1.05$

$$\begin{aligned} T_{10} &= 30\,000 + (1.05)^{10-1} \\ &= 46\,539.85 \end{aligned}$$

Alex's salary is an arithmetic progression where $a = 33\,000$ and $d = 1500$

$$\begin{aligned} T_{10} &= 33\,000 + (10-1) \times 1\,500 \\ &= 46\,500 \end{aligned}$$

Hence Kim's salary is higher than Alex's salary in the 10$^{\text{th}}$ year.

Question 12 (c) (ii)

Criteria	Marks
• Correct solution	2
• Attempts to find the sum of a relevant geometric series, or equivalent merit	1

Sample answer:

Sum of a geometric progression

$$\begin{aligned} S_{10} &= \frac{30\,000(1.05^{10} - 1)}{1.05 - 1} \\ &= 377\,336.78 \end{aligned}$$

Kim earned \$377 336.78 in the first 10 years.

Question 12 (c) (iii)

Criteria	Marks
• Correct solution	3
• Attempts to solve resulting quadratic equation	2
• Attempts to use the sum of an arithmetic series to get a quadratic equation or equivalent merit	1

Sample answer:

Alex saves

$11\,000$ in the first year
$11\,000 + 500$ in the second year
$11\,000 + 500 + 500$ in the third year
$\vdots$
$11\,000 + \underbrace{500 + \ldots + 500}_{n-1}$ in the n^{th} year
$\left(\text{ie} = 11\,000 + (n-1)500 \quad \text{in the n}^{th} \text{ year}\right)$

Total savings in n years:

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_n = \frac{n}{2}\left(2(11\,000) + (n-1)500\right)$$
$$= \frac{n}{2}\left(22\,000 + 500n - 500\right)$$
$$= \frac{n}{2}\left(21\,500 + 500n\right)$$

Now $S_n = 87\,500$

$21\,500n + 500n^2 = 2 \times (87\,500) = 175\,000$

$500n^2 + 21\,500n - 175\,000 = 0$

$n^2 + 43n - 350 = 0$

$(n+50)(n-7) = 0$

so $n = 7$ or $n = -50$

But years must be postive, so Alex will save \$87 500 after seven years.

Question 13 (a) (i)

Criteria	Marks
• Correct solution	2
• Obtains one of the times, or equivalent merit	1

Sample answer:

$$P(t) = 400 + 50\cos\frac{\pi}{6}t = 375$$

$$50\cos\frac{\pi}{6}t = -25$$

$$\cos\frac{\pi}{6}t = -\frac{1}{2}$$

$$\text{Hence} \quad \frac{\pi}{6}t = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$$

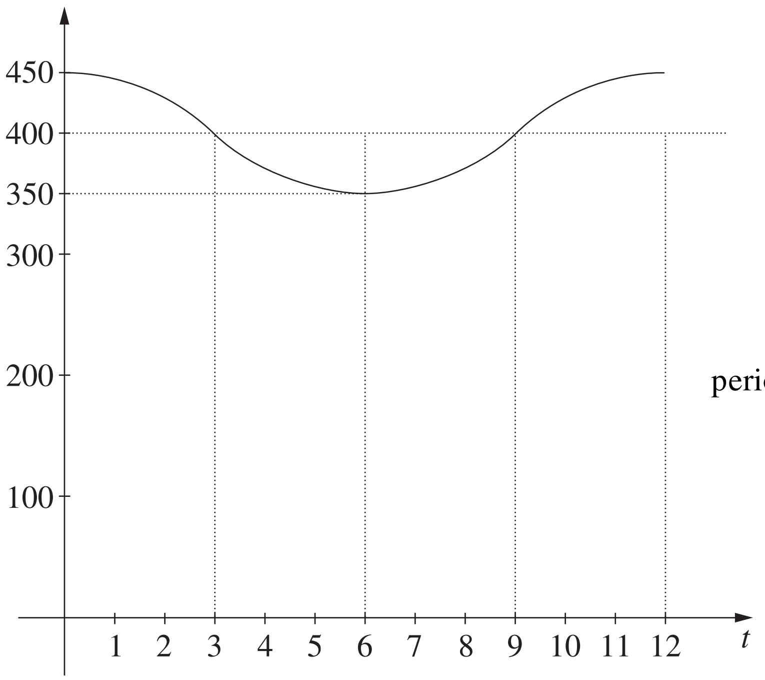
$$t = 4, 8$$

The population is 375 at 4 and at 8 months.

Question 13 (a) (ii)

Criteria	Marks
• Correct graph	2
• Shows a periodic function with two of the following features correct: period, amplitude, phase, centre of motion	1

Sample answer:



$$\text{period} = \frac{2\pi}{\frac{\pi}{6}} = 12$$

Question 13 (b) (i)

Criteria	Marks
• Correct solution	1

Sample answer:

x - coordinate of T satisfies $f(x) = g(x)$

$$4x^3 - 4x^2 + 3x = 2x$$
$$4x^3 - 4x^2 + x = 0$$
$$x\left(4x^2 - 4x + 1\right) = 0$$
$$x(2x-1)^2 = 0$$

Hence $x = 0$ or $x = \frac{1}{2}$

As T is not the origin, $x = \frac{1}{2}$.

Question 13 (b) (ii)

Criteria	Marks
• Correct solution	3
• Obtains correct primitive, or equivalent merit	2
• Obtains a definite integral for the area, or equivalent merit	1

Sample answer:

The area is given by

$$\int_0^{\frac{1}{2}} \left(4x^3 - 4x^2 + 3x - 2x\right) dx$$

$$= \int_0^{\frac{1}{2}} \left(4x^3 - 4x^2 + x\right) dx$$

$$= \left[\frac{4x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2}\right]_0^{\frac{1}{2}}$$

$$= \frac{1}{48}$$

$\therefore$ Area $= \frac{1}{48}$ units^2

Question 13 (c)

Criteria	Marks
• Correct solution	2
• Finds the area of the sector *ABC* in terms of θ, or equivalent merit	1

Sample answer:

Area of sector ABC is $\frac{1}{2} \times 30^2\theta$

Area of sector DEC is half of that:

$$\frac{1}{4} \times 30^2\theta = 225\theta$$

Hence $\frac{1}{2}(CD)^2\theta = 225\theta$

$$\begin{aligned} (CD)^2 &= 450 \\ CD &= \sqrt{450} \\ &= 15\sqrt{2} \end{aligned}$$

Length of CD is $15\sqrt{2}$ cm.

Question 13 (d) (i)

Criteria	Marks
• Correct solution	2
• Substitutes given information into $A_{360} = 0$, or equivalent merit	1

Sample answer:

M needs to be such that

$$\begin{aligned} 0 &= 500\,000(1.005)^{360} - M\left(1 + 1.005 + \ldots + (1.005)^{359}\right) \\ &= 500\,000(1.005)^{360} - M\frac{(1.005)^{360} - 1}{1.005 - 1} \\ &= 500\,000(1.005)^{360} - 1004.515\,M \end{aligned}$$

Hence

$$\begin{aligned} M &= \frac{500\,000(1.005)^{360}}{1004.515} \\ &= 2997.75 \end{aligned}$$

$\therefore$ monthly repayment is = \$2998 to the nearest dollar.

Question 13 (d) (ii)

Criteria	Marks
• Correct solution	1

Sample answer:

$$A_{240} = 500\,000(1.005)^{240} - 2998\left(1 + 1.005 + \ldots + (1.005)^{239}\right)$$

$$= 500\,000(1.005)^{240} - 2998\frac{(1.005)^{240} - 1}{1.005 - 1}$$

$$= 269\,903.63$$

$\therefore$ Balance owing is \$270 000 to the nearest thousand.

Question 13 (d) (iii)

Criteria	Marks
• Correct solution	2
• Finds equation in terms of $(1.005)^n$, or equivalent merit	1

Sample answer:

Need to find n so that

$$370\,000(1.005)^n - 2998\left(1 + 1.005 + \ldots + (1.005)^{n-1}\right) = 0$$

$$370\,000(1.005)^n = 2998\frac{(1.005)^n - 1}{1.005 - 1}$$

$$370\,000(1.005)^n = 599\,600\left((1.005)^n - 1\right)$$

$$599\,600 = (599\,600 - 370\,000)(1.005)^n$$

$$599\,600 = 229\,600(1.005)^n$$

$$2.6115 = (1.005)^n$$

$$\ln(2.6115) = n\ln(1.005)$$

$$n = \frac{\ln(2.6115)}{\ln(1.005)}$$

$$= 192.46$$

Hence 193 months (that is, 16 years and 1 month) are required to repay the \$370 000.

Question 14 (a) (i)

Criteria	Marks
• Correct solution	1

Sample answer:

Acceleration is $\ddot{x}$

$\dot{x} = 10 - 2t$

$\ddot{x} = -2$

$\therefore$ acceleration is constant.

Question 14 (a) (ii)

Criteria	Marks
• Correct solution	1

Sample answer:

At rest when $\dot{x} = 0$

$\dot{x} = 10 - 2t = 0$

$t = 5$

The particle is at rest after 5 seconds.

Question 14 (a) (iii)

Criteria	Marks
• Correct solution	2
• Finds $x = 10t - t^2 + C$, or equivalent merit	1

Sample answer:

Displacement of particle is

$$x = \int (10 - 2t)\,dt$$

$$= 10t - t^2 + C$$

When $t = 0$, $x = 5$

$$x = C = 5$$

ie $x = 10t - t^2 + 5$.

After 7 seconds,

$$x = 10 \times 7 - 7^2 + 5$$

$$= 26$$

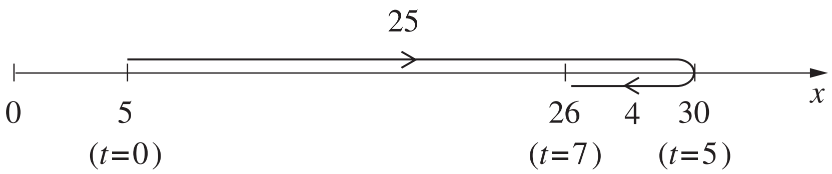
$\therefore$ particle is 26 m to the right of the origin.

Question 14 (a) (iv)

Criteria	Marks
• Correct solution	2
• Finds displacement when particle is at rest, or equivalent merit	1

Sample answer:

The particle is at rest when $t = 5$, that is, $x = 30$. Hence particle travels from $x = 5$ to $x = 30$, then to $x = 26$. Total distance : 25 m + 4 m = 29 m



Question 14 (b) (i)

Criteria	Marks
• Correct solution	2
• Finds *BR* in terms of *t*, or equivalent merit	1

Sample answer:

After time t, distance of A to R is $80\,t$

After time t, distance of B to R is $100 - 50\,t$

Using the cosine rule

$$D^2 = (100 - 50t)^2 + (80t)^2 - 2(100 - 50t)80t \cos 60^\circ$$

$$= 10\,000 - 10\,000t + 2\,500t^2 + 6\,400t^2 - 160t(100 - 50t)\left(\frac{1}{2}\right)$$

$$= 10\,000 - 10\,000t + 8\,900t^2 - 8\,000t + 4\,000t^2$$

$$= 12\,900t^2 - 18\,000t + 10\,000$$

Question 14 (b) (ii)

Criteria	Marks
• Correct solution	3
• Justifies that the value of t gives a minimum distance	2
• Finds the time when the distance is minimal, or equivalent merit	1

Sample answer:

Minimum of D occurs at the same time as the minimum of D^2. For maximum/minimum

$$\begin{aligned}
\frac{d\left(D^2\right)}{dt} &= 25\,800t - 18\,000 \\
&= 0 \\
t &= 0.69767\ldots \\
&= 0.7 \text{ (to 1 dp)} \\
D^2 &= 12\,900(0.7)^2 - 18\,000(0.7) + 10000 \\
&= 3721 \\
\therefore D &= 61
\end{aligned}$$

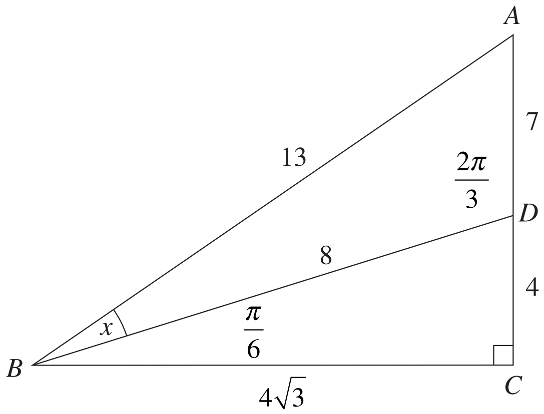
Test: $\dfrac{d^2\left(D^2\right)}{dt^2} = 25\,800 > 0$, so minimum

$\therefore$ Minimum distance is 61km.

Question 14 (c)

Criteria	Marks
• Correct solution	3
• States $\dfrac{\sin x}{AD} = \dfrac{\sqrt{3}}{26}$ and attempts to find AD, or equivalent merit	2
• States $\dfrac{\sin x}{AD} = \dfrac{\sin\frac{2\pi}{3}}{13}$, or equivalent merit	1

Sample answer:



$$\sin\frac{\pi}{6} = \frac{CD}{BD}$$
$$\frac{1}{2} = \frac{4}{BD}$$
$$\therefore BD = 8$$
$$\tan\frac{\pi}{6} = \frac{CD}{BC}$$
$$\frac{1}{\sqrt{3}} = \frac{4}{BC}$$
$$\therefore BC = 4\sqrt{3}$$

Now $AC = \sqrt{AB^2 - BC^2}$

$$= \sqrt{13^2 - \left(4\sqrt{3}\right)^2}$$
$$= \sqrt{121} = 11$$

Hence $AD = AC - DC$

$$= 11 - 4 = 7$$

Also $\angle ADB = \dfrac{\pi}{2} + \dfrac{\pi}{6} = \dfrac{2\pi}{3}$ (ext. angle of ΔBCD)

Using the sine rule in ΔABD

$$\frac{\sin x}{AD} = \frac{\sin\frac{2\pi}{3}}{AB}$$

$$\frac{\sin x}{7} = \frac{\sin\frac{2\pi}{3}}{13}$$
$$= \frac{\frac{\sqrt{3}}{2}}{13}$$
$$= \frac{\sqrt{3}}{26}$$

Hence $\sin x = \dfrac{7\sqrt{3}}{26}$

Question 14 (d)

Criteria	Marks
• Correct solution	1

Sample answer:

$a = 4.5$

Reason:

$$\int_{-3}^{-1} f(x)\,dx = 0 \text{ and } \int_{2}^{3} f(x) = 0$$

$$\text{and so } \int_{-3}^{3} f(x)\,dx = \int_{-1}^{2} f(x)\,dx$$

$$= 3 \times 1$$

$$= 3$$

So for

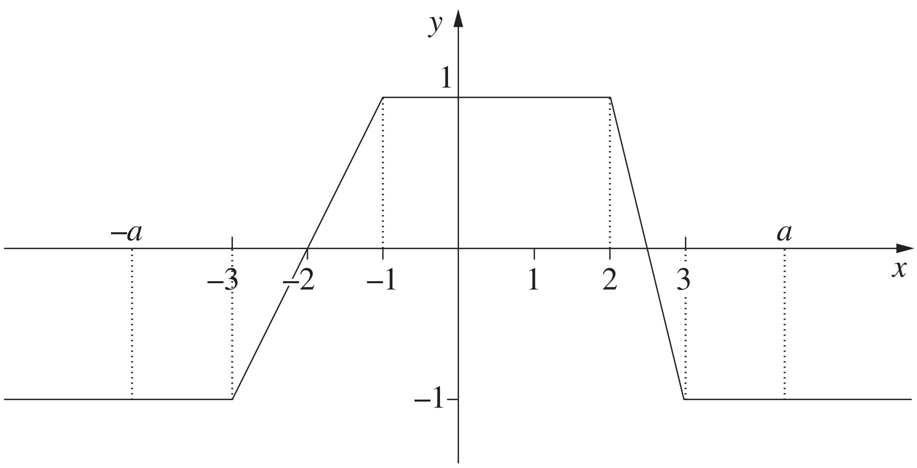
$$\int_{-a}^{a} f(x)\,dx = 0$$

$$\int_{-a}^{-3} f(x)\,dx + \int_{3}^{a} f(x)\,dx = 3$$

$$\text{ie } 1 \times 1.5 + 1 \times 1.5 = 3$$

$$\therefore a = 3 + 1.5$$

$$= 4.5$$



Question 15 (a) (i)

Criteria	Marks
• Correct solution	1

Sample answer:

Estimated area from trapezoidal rule

$$\frac{1.2}{2}\left(1.5+1.5+2(1.8)\right)=3.96\ \text{m}^2$$

Question 15 (a) (ii)

Criteria	Marks
• Correct solution	1

Sample answer:

Estimated area from Simpson's rule

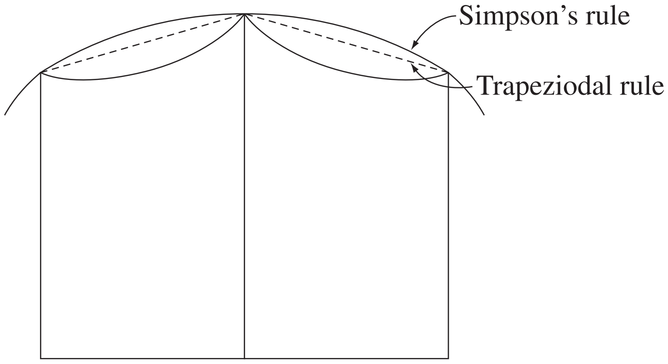
$$\frac{1.2}{3}\left(1.5+4\times 1.8+1.5\right)=4.08\ \text{m}^2$$

Question 15 (a) (iii)

Criteria	Marks
• Correct explanation	1

Sample answer:

Simpson's rule uses a parabola through the 3 points. For the trapezoidal rule, the points are connected by straight lines. The roof hangs below the straight lines, and the parabola is above the straight lines. Hence, the area A is closer to the estimate given by the trapezoidal rule than that given by Simpson's rule.

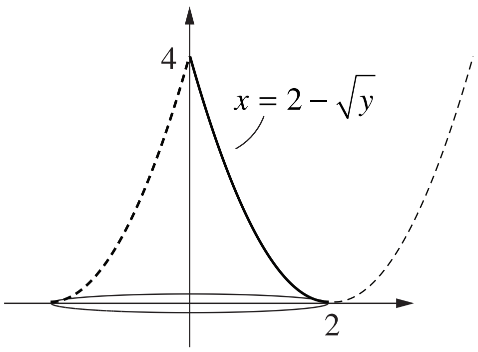


Alternative: For the trapezoidal rule, the top of the poles are connected by straight lines to form trapeziums. The exact area is less than the area of the trapeziums because the roof hangs between the poles. Since Simpson's rule gives a greater value than the trapezoidal rule, then the trapezoidal rule gives a better estimate.

Question 15 (b)

Criteria	Marks
• Correct solution	4
• Makes significant progress to get primitive of $\left(2 \pm \sqrt{y}\right)^2$	3
• Obtains $V = \pi \int_0^a \left(2 \pm \sqrt{y}\right)^2 dy$, or equivalent merit	2
• States $V = \pi \int_0^a x^2\, dy$, for some value of a OR • Finds the y-intercept, or equivalent merit	1

Sample answer:



$$\left(x-2\right)^2 = y$$
$$x - 2 = \pm\sqrt{y}$$
$$x = 2 \pm \sqrt{y}$$

But $x = 0$ when $y = 4$

$\therefore$ use the left branch

ie $x = 2 - \sqrt{y}$

The volume of revolution is

$$V = \pi \int_0^4 x^2\, dy = \pi \int_0^4 \left(2 - \sqrt{y}\right)^2 dy$$

$$= \pi \int_0^4 \left(4 - 4\sqrt{y} + y\right) dy$$

$$= \pi \left[4y - 4 \times \frac{2}{3} y^{\frac{3}{2}} + \frac{1}{2} y^2\right]_0^4$$

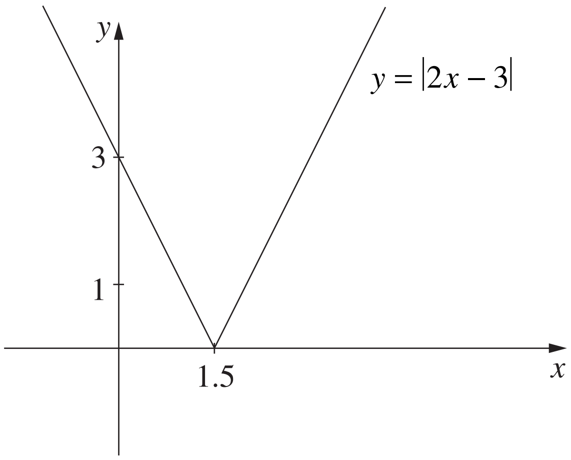
$$= \pi \left(16 - \frac{64}{3} + 8 - 0\right)$$

$$\therefore \text{volume} = \frac{8\pi}{3} \text{ units}^3$$

Question 15 (c) (i)

Criteria	Marks
• Correct sketch	1

Sample answer:



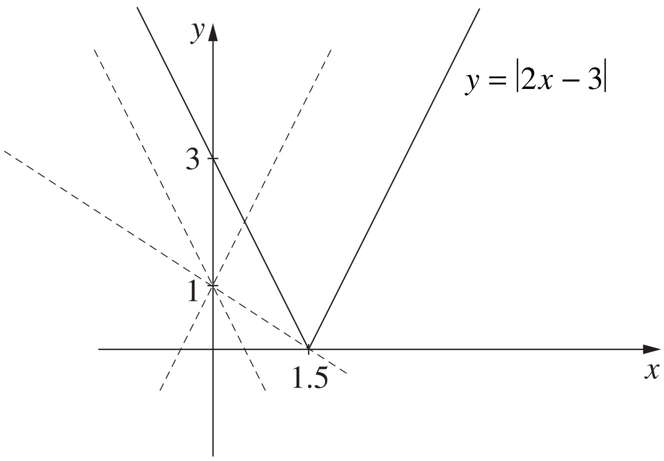
Question 15 (c) (ii)

Criteria	Marks
• Correct solution	2
• Finds at least one possible value for *m*, or equivalent merit	1

Sample answer:

From the graph:

$m < -2$ or $m \geq 2$ or $m = -\dfrac{2}{3}$



Question 15 (d) (i)

Criteria	Marks
• Correct answer	1

Sample answer:

As Pat starts, the probability for Pat to win the first throw is $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

Question 15 (d) (ii)

Criteria	Marks
• Correct solution	2
• Finds probability that Pat wins in second game OR • Attempts to find probability Pat wins in the second game, and adds to answer from part (i)	1

Sample answer:

Pat winning on the second throw means both lose in the first throw and Pat wins on the next.

That probability is $\frac{35}{36}\times\frac{35}{36}\times\frac{1}{36}$.

The probability that Pat wins on the first or second throw:

$$\frac{1}{36}+\frac{35}{36}\times\frac{35}{36}\times\frac{1}{36}=\frac{2521}{46656}=0.054$$

Question 15 (d) (iii)

Criteria	Marks
• Correct solution	2
• Finds probability that Pat wins in the third game, or equivalent merit	1

Sample answer:

For Pat to win on the third throw, probability is

$$\left(\frac{35}{36}\times\frac{35}{36}\times\frac{35}{36}\times\frac{35}{36}\times\frac{1}{36}\right)$$

$\therefore$ The probability that Pat eventually wins is

$$\frac{1}{36}+\left(\frac{35}{36}\times\frac{35}{36}\times\frac{1}{36}\right)+\left(\frac{35}{36}\times\frac{35}{36}\times\frac{35}{36}\times\frac{35}{36}\times\frac{1}{36}\right)+\dots$$

$$=\frac{1}{36}\left(1+\left(\frac{35}{36}\right)^{2}+\left(\frac{35}{36}\right)^{2\times 2}\times\left(\frac{35}{36}\right)^{2\times 3}+\dots\right)$$

(This forms an infinite GP with $a = 1$, $r=\left(\frac{35}{36}\right)^{2}$ and $S_{\infty}=\frac{a}{1-r}$)

$$=\frac{1}{36}\left(\frac{1}{1-\left(\frac{35}{36}\right)^{2}}\right)$$

$$=\frac{1}{36}\left(\frac{1}{1-\frac{1225}{1256}}\right)$$

$$=\frac{36}{71}$$

Question 16 (a)

Criteria	Marks
• Correct solution	3
• Finds primitive of $f'(x)$ and solves $f'(x) = 5$, or equivalent merit	2
• Finds a primitive of $f'(x)$ OR • Solves $f'(x) = 5$, or equivalent merit	1

Sample answer:

Slope of tangent is given by $f'(x) = 4x - 3$.
Slope of line $y = 5x - 7$ is 5.
For the line to be tangent to $y = f(x)$

$$4x - 3 = 5$$

$$\therefore \quad x = 2$$

When $x = 2$

$$y = 5x - 7$$

$= 3$ So the point (2,3) lies on the graph $y = f(x)$ and on the tangent.

Now $$f(x) = \int (4x - 3)\,dx$$

$$= \frac{4x^2}{2} - 3x + C$$

Since $(2,3)$ lies on the graph

$$\begin{aligned} 3 &= f(2) \\ &= 2 \times (2)^2 - 3(2) + C \\ &= 2 + C \end{aligned}$$

so $C = 1$

Hence $f(x) = 2x^2 - 3x + 1$

Question 16 (b) (i)

Criteria	Marks
• Correct answer	1

Sample answer:

When $t = 0$, N = $375 - e^0$

There were 374 trout.

Question 16 (b) (ii)

Criteria	Marks
• Correct solution	1

Sample answer:

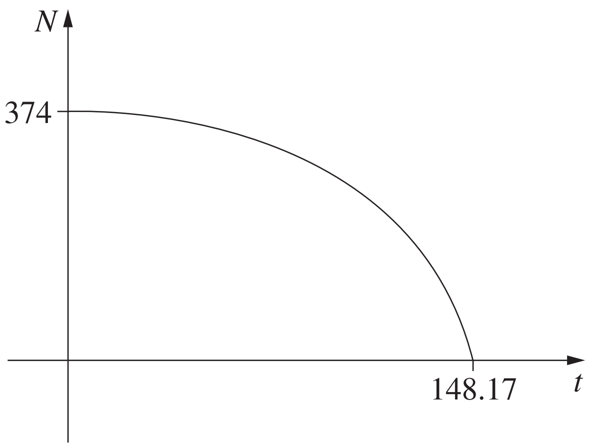
Solve

$$N = 375 - e^{0.04t} = 0$$
$$e^{0.04t} = 375$$
$$0.04t = \ln 375$$
$$t = \frac{\ln 375}{0.04} = 148.17$$

Question 16 (b) (iii)

Criteria	Marks
• Correct sketch	1

Sample answer:



Question 16 (b) (iv)

Criteria	Marks
• Correct solution	3
• Finds $0.2e^{0.02t} = 0.04e^{0.04t}$, or equivalent merit	2
• Finds $P = 10e^{0.02t}$ OR • States $\left\lvert \frac{dP}{dt} \right\rvert = \left\lvert \frac{dN}{dt} \right\rvert$, or equivalent merit	1

Sample answer:

P solves the equation $\frac{dP}{dt} = 0.02P$

So $P(t) = 10e^{0.02t}$, as $P(0) = 10$

For the rates to be equal,

$$\left\lvert \frac{dN}{dt} \right\rvert = \left\lvert \frac{dP}{dt} \right\rvert$$

ie

$$\begin{aligned} 0.04e^{0.04t} &= 0.02P(t) \\ &= 0.2e^{0.02t} \\ e^{0.02t} &= \frac{0.2}{0.04} \\ &= 5 \\ t &= \frac{\ln 5}{0.02} \\ &= 80.47 \end{aligned}$$

Question 16 (b) (v)

Criteria	Marks
• Correct solution	2
• Finds quadratic equation for $e^{0.02t}$, or equivalent merit	1

Sample answer:

$$P(t) = N(t)$$

$$10e^{0.02t} = 375 - e^{0.04t}$$

$$e^{0.04t} + 10e^{0.02t} - 375 = 0$$

$$\left(e^{0.02t}\right)^2 + 10e^{0.02t} - 375 = 0$$

$$\left(e^{0.02t} + 25\right)\left(e^{0.02t} - 15\right) = 0$$

$$\text{Hence } e^{0.02t} = -25 \text{ or } 15$$

Exponentials are positive, so

$$e^{0.02t} = 15$$

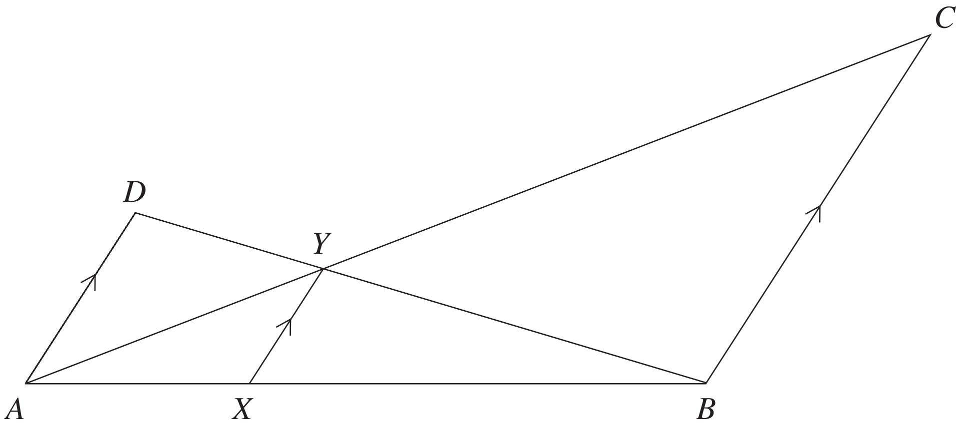
$$t = \frac{\ln 15}{0.02} = 135.4$$

The number of carp and trout are equal in the 136th month.

Question 16 (c) (i)

Criteria	Marks
• Correct proof	2
• Uses properties relating to parallel lines	1

Sample answer:



In ΔABC and ΔAXY
$<XAY$ is common
$<BCA = <XYA$ (corresponding angles $XY \| BC$)
Hence ΔABC is similar to ΔAXY (equiangular)

Question 16 (c) (ii)

Criteria	Marks
• Correct proof	2
• Obtains a ratio involving XY and AX or BX, or equivalent merit	1

Sample answer:

By similarity of ΔABC and ΔAXY:

$\frac{AX}{AB} = \frac{XY}{BC}$ (corresponding sides in similar triangles are in proportion).

Now, ΔABD is similar to ΔXBY (using similar argument used in part (i)). Hence

$$\frac{BX}{BA} = \frac{XY}{AD}$$

Add the two identities:

$$\frac{AX}{AB} + \frac{BX}{BA} = \frac{XY}{BC} + \frac{XY}{AD}$$

$$\frac{AB}{AB} = XY\left(\frac{1}{BC} + \frac{1}{AD}\right) \text{ (since } AX + BX = AB\text{)}$$

$$\frac{1}{XY} = \frac{1}{AD} + \frac{1}{BC}$$

Mathematics

2013 HSC Examination Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	1.4	P3, P4
2	1	5.3, 6.1	P5
3	1	1.4, 4.1	P5
4	1	8.9, 13.5	H5
5	1	3.3	H5
6	1	13.3	H5
7	1	9.5	P5
8	1	10.1, 10.4	H6
9	1	12.2, 12.3	H3
10	1	14.1	H5

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	1	1.1	P3
11 (b)	2	8.2	P3, P4
11 (c)	2	8.8, 13.5	P7, H5
11 (d)	2	8.8, 12.5	P7, H5
11 (e)	2	12.5	H5, H8
11 (f)	3	12.5	H5, H8
11 (g)	3	4.4	P4
12 (a)	2	10.4	H5
12 (b) (i)	2	6.2	P4
12 (b) (ii)	1	6.5	P4
12 (b) (iii)	1	6.5	P4
12 (b) (iv)	2	2.3, 6.7	P4
12 (c) (i)	2	7.1, 7.2	H5
12 (c) (ii)	2	7.2, 7.5	H5
12 (c) (iii)	3	7.1	H5
13 (a) (i)	2	5.2, 13.1	P4, H5
13 (a) (ii)	2	13.3	H5
13 (b) (i)	1	1.4, 9.2	P4

Question	Marks	Content	Syllabus outcomes
13 (b) (ii)	3	11.2, 11.4	H8
13 (c)	2	13.1	H5
13 (d) (i)	2	7.5	H4, H5
13 (d) (ii)	1	7.5	H4, H5
13 (d) (iii)	2	7.5	H4, H5
14 (a) (i)	1	14.3	H4, H5
14 (a) (ii)	1	14.3	H4, H5
14 (a) (iii)	2	14.3	H4, H5
14 (a) (iv)	2	14.3	H4, H5
14 (b) (i)	2	10.6	H4
14 (b) (ii)	3	10.6	H4, H5
14 (c)	3	5.3, 5.5, 13.2	P4, H5
14 (d)	1	11.4	H8
15 (a) (i)	1	11.3	H8
15 (a) (ii)	1	11.3	H8
15 (a) (iii)	1	11.3	H8
15 (b)	4	11.4	H8
15 (c) (i)	1	4.2	P5
15 (c) (ii)	2	1.2, 1.4, 4.2	P4, P5
15 (d) (i)	1	3.1	H5
15 (d) (ii)	2	3.2, 3.3	H5
15 (d) (iii)	2	3.2, 7.3, 7.5	H5
16 (a)	3	8.6, 10.8	P6, H8
16 (b) (i)	1	14.2	H3
16 (b) (ii)	1	14.2	H3
16 (b) (iii)	1	14.2	H3
16 (b) (iv)	3	14.2	H3
16 (b) (v)	2	14.2	H3
16 (c) (i)	2	2.5	H5
16 (c) (ii)	2	2.5	H5