BOARD OF STUDIES
NEW SOUTH WALES

2009 HSC Mathematics
Sample Answers

This document contains 'sample answers'. These are developed by the examination committee for two purposes. The committee does this:

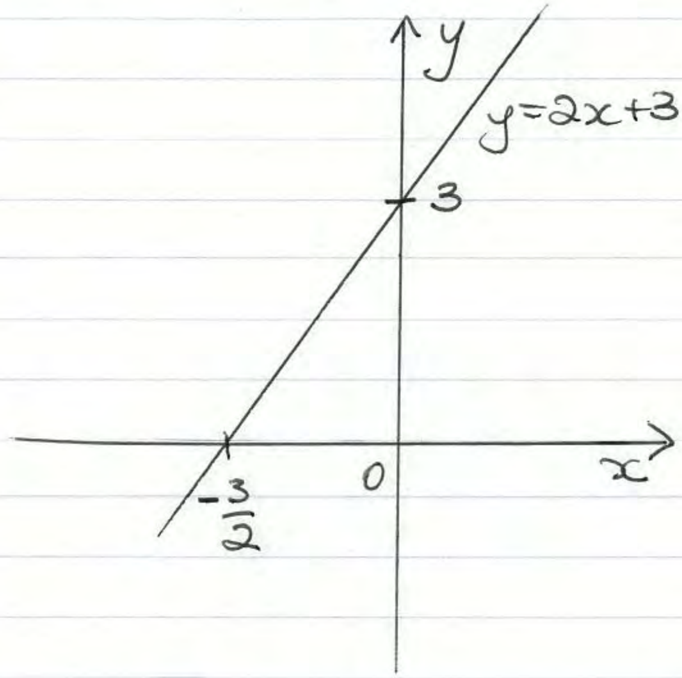
(a) as part of the development of the examination paper to ensure the questions will effectively assess students' knowledge and skills, and

(b) in order to provide some advice to the Supervisor of Marking about the nature and scope of the responses expected of students.

The 'sample answers' or similar advice, are not intended to be exemplary or even complete responses. They have been reproduced in their original form as part of the examination committee's 'working document'.

Question 1

1(a) $y - 2x = 3$
$y = 2x + 3$



(b) $\frac{5x-4}{x} = 2$
$5x - 4 = 2x$
$3x = 4$
$x = \frac{4}{3}$

(c) $x + 1 = \pm 5$
$x = 4, -6$

(d) $y = x^4 - 3x$
$\frac{dy}{dx} = 4x^3 - 3$
When $x = 1$, $\frac{dy}{dx} = 4 - 3 = 1$

(e) $2\cos\theta = 1$
$\cos\theta = \frac{1}{2}$
$\theta = \frac{\pi}{3}$

(f) $\ln x = 2$
$x = 7.3891$

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Question 2.

(a) (i) $y = x \sin x$

$y' = x \times \cos x + \sin x$

(ii) $y = (e^x + 1)^2$

$y' = 2(e^x + 1) \cdot e^x$

(b) (i) $\int 5\,dx = 5x + c$

(ii) $\int \frac{3}{(x-6)^2}\,dx = \frac{-3}{x-6} + c$

(iii) $\int_1^4 x^2 + \sqrt{x}\,dx = \left[\frac{x^3}{3} + \frac{2}{3}x\sqrt{x}\right]_1^4$

$= \frac{63}{3} + \frac{2}{3} \times 7 = \frac{77}{3}$

(c) $\sum_{k=1}^{4} (-1)^k k^2 = -1 + 4 - 9 + 16 = 10$

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Question 3

(a) $S = \dfrac{3+53}{2} \times 21 = 28 \times 21 = 588$

(b) (i) $\dfrac{y-1}{x-2} = \dfrac{5-1}{5-2} = \dfrac{4}{3}$

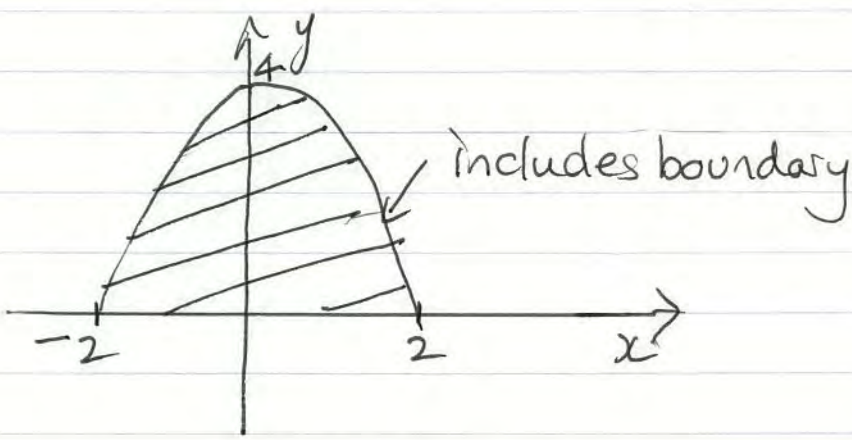
$4(x-2) = 3(y-1)$

$4x - 3y - 5 = 0$

(ii) $NP = \left| \dfrac{4\times1 - 3\times3 - 5}{\sqrt{4^2+3^2}} \right| = 2$

(iii) $(x-1)^2 + (y-3)^2 = 2^2$

(c)



(d) Area $\simeq \dfrac{210 + 4\times220 + 2\times200 + 4\times190 + 2\times210 + 4\times240 + 2[illegible]}{1+4+2+4+2+4+1} \times 300$

$= 64500\ m^2$

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Question 4.

(a) eventual height $= \dfrac{1\cdot 2}{1-\frac{9}{10}}$

$= 12\text{m}$

(b) $x^2 - (k+4)x + (k+7) = 0$

$\Delta = (k+4)^2 - 4(7+k) = 0$

$k^2 + 4k - 12 = 0$

$k = -6 \text{ or } 2$

(c) (i) $PM \perp AC$, $BC \perp AC$ so $PM \parallel CB$

so $\angle PMA = \angle CBA$ (corresponding angles)

$\angle APM = \angle ACB$

and $\angle PAM = \angle CAB$

so $\triangle AMP \,|||\, \triangle ABC$ (first two lines are unnecessary)

(ii) $AP : AC = AM : AB = 1:2$

(iii) From (ii) $AP = CP$

$MP = MP$ (Common)

$\angle CPM = \angle APM$ $(= 90°)$

$\therefore \triangle CPM \equiv \triangle APM$ (SAS)

$\therefore CM = AM$

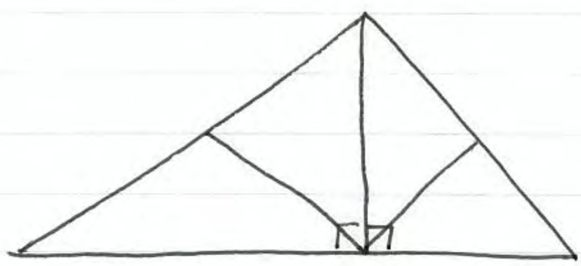
$\therefore \triangle AMC$ is isosceles

(iv) $MC = MA = MB$, so $\triangle BMC$ is isosceles

so $\triangle ABC$ can be subdivided into two isosceles triangles.

Question 4 (continued).

(v)



Two right-angled triangles, each of which is subdivided into 2 isosceles triangles.

End of Question 4

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Question 5.

(a) (i) $B(\sqrt{3},0)$ slope of $BC = -\frac{1}{\sqrt{3}}$

Equation of BC: $y = -\frac{1}{\sqrt{3}}(x+\sqrt{3})$
or $\sqrt{3}y + x = \sqrt{3}$

(ii) $C(0,1)$
$BC = 2,\quad AB = 2\sqrt{3}$
Area = $2\sqrt{3}$ sq. units

(b) (i) $\frac{1}{3}$

(ii) $\frac{2}{3} \times \frac{1}{2} \times 1 = \frac{1}{3}$

(iii) $\left(\frac{2}{3}\right)^5 = \frac{32}{243}$

(c) (i) $\frac{1}{2} \times 2^2 \sin\theta = \sqrt{3}$
$\sin\theta = \frac{\sqrt{3}}{2}$
$\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

(ii) (1) Area of sector $= \frac{1}{2} \times 2^2 \times \frac{\pi}{3} = \frac{2\pi}{3}$

(2) $AB = 2$ (equilateral Δ)
arc $AB = \frac{1}{6} \times 2 \times \pi \times 2 = \frac{2\pi}{3}$
perimeter $= \frac{2\pi}{3} + 2$.

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Question 6.

(a) $V = \int \pi y^2 \, dx$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \pi \sec^2 x \, dx$

$= \pi \left[\tan x \right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$

$= 2\pi\sqrt{3}$

(b) (i) $Q = Ae^{-kt}$

$\frac{1}{2}A = Ae^{-1600k}$

$e^{1600k} = 2$

$1600k = \ln 2$

$k = \frac{\ln 2}{1600}$

(ii) $A = 3S e^{-\frac{\ln 2}{1600}t} = S$ (S = safe level)

$e^{\frac{\ln 2}{1600}t} = 3$

$\frac{\ln 2}{1600}t = \ln 3$

$t = \frac{1600 \ln 3}{\ln 2} \simeq 2536$ years

(c)(i) $y = ax^2 + bx$
$y' = 2ax + b$

At O, $x = 0$ $y' = b = 1.2$
so $y = ax^2 + 1.2x$
$y' = 2ax + 1.2$

At $x = 30$ $y' = 60a + 1.2 = -1.8$
$60a = -3.0$
$a = -0.05$
$y = -0.05x^2 + 1.2x$

(ii) at max.
$y' = -0.1x + 1.2 = 0$
$x = 12$
$y = -0.05 \times 12^2 + 1.2 \times 12$
$= -7.2 + 14.4$
$= 7.2$

$x = 30$ $y = -0.05 \times 900 + 1.2 \times 30$
$= -45 + 36$
$= -9$
$d = 16.2$ (metres)

End of Question 6

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Question 7

(a) (i) $\ddot{x} = 8e^{-2t} + 3e^{-t}$

$\dot{x} = -4e^{-2t} - 3e^{-t} + c$

when $t = 0$, $\dot{x} = -6$ so $c = 1$

$\dot{x} = -4e^{-2t} - 3e^{-t} + 1$

$x = 2e^{-2t} + 3e^{-t} + t + c$

when $t = 0$ $x = 5$ so $c = 0$

$x = 2e^{-2t} + 3e^{-t} + t$

(ii) $\dot{x} = -4e^{-2t} - 3e^{-t} + 1$
$\dot{x} = 0$

$4(e^{-t})^2 + 3(e^{-t}) - 1 = 0$

$e^{-t} = \frac{-3 \pm \sqrt{9+16}}{8}$

$= \frac{1}{4}$ (since $e^{-t} > 0$)

$e^{t} = 4$

$t = \ln 4$

Alternately: $(4e^{-t} - 1)(e^{-t} + 1) = 0$

$\therefore 4e^{-t} = 1$

$e^{-t} = \frac{1}{4}$

$t = \ln 4$

(iii) $x = 2 \times \frac{1}{16} + 3 \times \frac{1}{4} + \ln 4$

$= \frac{7}{8} + 2\ln 2$

Question 7 (continued)

(b) $h = 1 + 0.7 \sin \frac{\pi}{6} t$ for $0 \leq t \leq 12$

(i) 12 hours

(ii) $1 - 0.7 = 0.3$

$\sin \frac{\pi}{6} t = -1 \qquad \frac{\pi}{6} t = \frac{3\pi}{2} \Rightarrow t = 9$

Low tide - at 2pm.

(iii) $1 + 0.7 \sin \frac{\pi}{6} t \geq 1.35$

$\sin \frac{\pi}{6} t \geq 0.5$

$\frac{\pi}{6} \leq \frac{\pi}{6} t \leq \frac{5\pi}{6}$

$1 \leq t \leq 5$

between 6am and 10 am

End of Question 7

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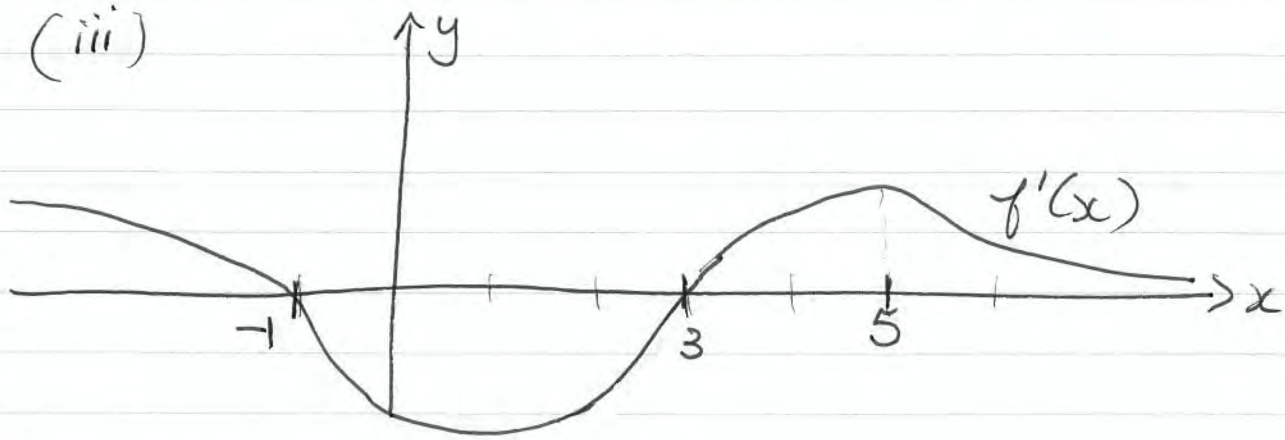
Question 8

(a) (i) $f'(x) < 0$ for $-1 < x < 3$

(ii) $f'(x) \to 0$ as $x \to \infty$

(iii)



(b) (i) $\$350000 \times 1.0075 - 2937$

$= \$349688$

(ii) $346095 \times 1.605^{288}$
$- M(1 + 1.005 + \dots + 1.005^{287})$
$= 0$

$$346095 \times 1.005^{288} - M\left(\frac{1.005^{288} - 1}{0.005}\right) = 0$$

$$M = \frac{346095 \times 0.005 \times 1.005^{288}}{1.005^{288} - 1}$$

$$= \frac{346095 \times 0.005}{1 - 1.005^{-288}}$$

$$= \frac{1730.475}{0.762220607}$$

$= 2270.31$ So suppose $M = 2270$

Question 8 (continued)

(b) (iii)

$$346095 \times 1{\cdot}005^n - 2937\left(\frac{1{\cdot}005^n - 1}{0{\cdot}005}\right) = 0$$

$$346095 \times 1{\cdot}005^n - 587400 \times 1{\cdot}005^n + 587400 = 0$$

$$1{\cdot}005^n = \frac{587400}{241305} = 2{\cdot}434263691$$

$$n \ln 1{\cdot}005 = \ln 2{\cdot}434263691$$

$$n = \frac{0{\cdot}889644325}{0{\cdot}004987541511}$$

$$= 178{\cdot}37$$

So, about 178 payments or 14 years, 10 months.

(iv) $288 \times \$2270 = \$653760$

$178 \times \$2937 = \$522786$

$178{\cdot}37 \times \$2937 = \$523872$

Saving $\simeq \$130000$

End of Question 8

Question 9.

(a) $\text{probability} = 1 - \left(\frac{8}{9}\right)^3\left(\frac{15}{16}\right)^3$

$= 1 - \left(\frac{5}{6}\right)^3$

$= 1 - \frac{125}{216}$

$= \frac{91}{216}$

(b) (i) $5 \times \$1000 + 3 \times \$2600 = \$12800$

(ii) $\sqrt{5^2+3^2} \times \$2600 = \$15160$

(iii) $C = 1000 \times (5-x) + 2600\sqrt{x^2+9}$

$= 1000\left(5 - x + 2.6\sqrt{x^2+9}\right)$

(iv) $\frac{dC}{dx} = 1000\left(-1 + \frac{2.6x}{\sqrt{x^2+9}}\right)$

$= 0$

when $2.6x = \sqrt{x^2+9}$

$6.76x^2 = x^2 + 9$

$5.76x^2 = 9$

$x^2 = \frac{9}{5.76}$

$x = \frac{3}{2.4} = 1.25$

$C = 12200$ is a minimum (below the other values)

Question 9 (continued)

9 (b) (v) Now $C = 1000\left(5 - x + 1.1\sqrt{x^2+9}\right)$

$$C' = 1000\left(-1 + \frac{1.1x}{\sqrt{x^2+9}}\right)$$

$$= 0$$

when $1.1x = \sqrt{x^2+9}$

$$1.21x^2 = x^2 + 9$$

$$0.21x^2 = 9$$

$$x^2 = \frac{9}{0.21} > 25$$

ie when $x > 5$

Indeed $C' < 0$ for $0 < x < 5$

so min occurs at $x = 5$.

The cable should be laid straight from P to S.

End of Question 9

Question 10

(a) $f'(x) = 1 - x + x^2$
$= (x-\frac{1}{2})^2 + \frac{3}{4}$
$\geqslant \frac{3}{4}$

So $f'(x) \neq 0$, $f(x)$ has no turning points.

(b) $f''(x) = -1 + 2x$ changes sign at $x = \frac{1}{2}$

$x = \frac{1}{2}$ $f(x) = \frac{1}{2} - \frac{1}{8} + \frac{1}{24} = \frac{5}{12}$

inflection is $(\frac{1}{2}, \frac{5}{12})$

(c) (i) $1 - x + x^2 - \frac{1}{1+x}$

$= \frac{(1-x+x^2)(1+x) - 1}{1+x}$

$= \frac{1 + x^3 - 1}{1+x}$

$= \frac{x^3}{1+x}$

(ii) $f(x) - g(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ln(1+x)$

$f'(x) - g'(x) = 1 - x + x^2 - \frac{1}{1+x}$

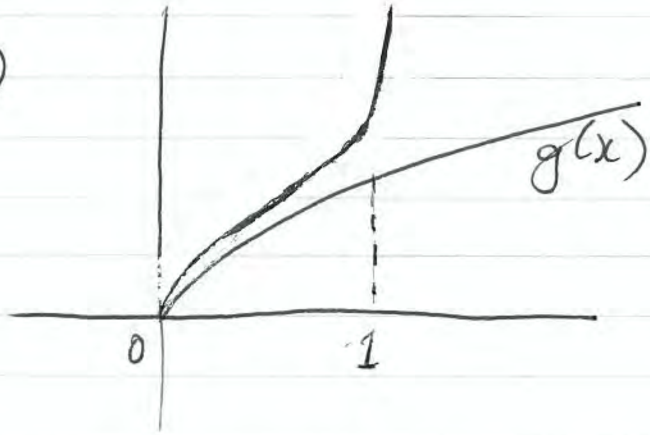
$= \frac{x^3}{1+x}$

$\geqslant 0$ for $x \geqslant 0$

$f'(x) \geqslant g'(x)$ for $x \geqslant 0$

Question 10 (continued)

(d)



(e) $\frac{d}{dx}\left((x+1)\ln(1+n) - (1+x)\right)$

$= (1+x) \times \frac{1}{1+x} + \ln(1+x) \times 1 - 1$

$= \ln(1+x)$

(f) $\text{Area} = \int_0^1 f(x) - g(x)\,dx$

$= \int_0^1 x - \frac{x^2}{2} + \frac{x^3}{3} - \ln(1+x)\,dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - (1+x)\ln(1+x) + (1+x)\right]_0^1$

$= \frac{1}{2} - \frac{1}{6} + \frac{1}{12} - 2\ln 2 + 2 - 1$

$= \frac{17}{12} - 2\ln 2$

End of Question 10