BOSTES
BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

2015 HSC Mathematics Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	C
3	A
4	A
5	B
6	C
7	B
8	C
9	D
10	A

Section II

Question 11 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$4x-(8-6x)$$
$$=4x-8+6x$$
$$=10x-8$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Finds common factor of 3, or equivalent merit	1

Sample answer:

$$3x^2-27$$
$$=3(x^2-9)$$
$$=3(x-3)(x+3)$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Attempts to use $2-\sqrt{7}$, or equivalent merit	1

Sample answer:

$$\frac{8}{2+\sqrt{7}}=\frac{8}{2+\sqrt{7}}\times\frac{2-\sqrt{7}}{2-\sqrt{7}}$$
$$=\frac{8(2-\sqrt{7})}{4-7}$$
$$=\frac{8(2-\sqrt{7})}{-3}$$
$$=\frac{8\sqrt{7}-16}{3}$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Identifies the common ratio, or equivalent merit	1

Sample answer:

$a = 1 \qquad r = -\frac{1}{4}$

$$\begin{aligned} S &= \frac{a}{1-r} \\ &= \frac{1}{1-\left(-\frac{1}{4}\right)} \\ &= \frac{4}{5} \end{aligned}$$

Question 11 (e)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use chain rule, or equivalent merit	1

Sample answer:

Let $y = \left(e^x + x\right)^5$

$$\begin{aligned} \frac{dy}{dx} &= 5\left(e^x + x\right)^4 \times \left(e^x + 1\right) \\ &= 5\left(e^x + 1\right)\left(e^x + x\right)^4 \end{aligned}$$

Question 11 (f)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use product rule, or equivalent merit	1

Sample answer:

Let $u = x + 4 \qquad v = \ln x$

$u' = 1 \qquad v' = \frac{1}{x}$

$$\therefore\ y' = u'v + v'u$$

$$= 1.\ln x + \frac{1}{x}(x+4)$$

$$= \ln x + \frac{x+4}{x}$$

Question 11 (g)

Criteria	Marks
• Provides correct solution	2
• Provides correct primitive, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{4}} \cos 2x\, dx = \left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{2}\sin 0$$

$$= \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

Question 11 (h)

Criteria	Marks
• Provides correct primitive	2
• Recognises $\dfrac{f'(x)}{f(x)}$, or equivalent merit	1

Sample answer:

$$\int \frac{x}{x^2-3}\,dx$$

$$= \frac{1}{2}\int \frac{2x}{x^2-3}\,dx$$

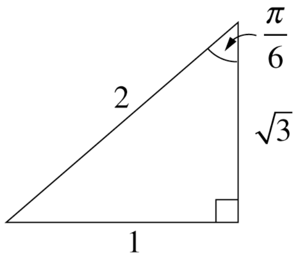
$$= \frac{1}{2}\ln\left(x^2-3\right)+C$$

Question 12 (a)

Criteria	Marks
• Provides correct solutions	2
• Provides one correct solution, or equivalent merit	1

Sample answer:

$$2\sin\theta = 1$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6},\ \frac{5\pi}{6}$$



Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the fact that the diagonals are perpendicular, or equivalent merit	1

Sample answer:

ℓ_1 is $y = m_1 x + b$

ℓ_2 is $y = -\frac{x}{3} \Rightarrow m_2 = -\frac{1}{3}$

$\ell_1 \perp \ell_2$ $\quad$ $OABC$ rhombus (diagonals ℓ_1, ℓ_2 are perpendicular)

$$m_1 m_2 = -1$$
$$m_1 \times \left(-\frac{1}{3}\right) = -1$$
$$\therefore\ m_1 = 3$$

ℓ_1 is $y = 3x + b$

$$11 = 3 \times 7 + b$$
$$b = -10$$
$$\therefore\ y = 3x - 10$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct coordinates	2
• Makes some progress towards correct answer	1

Sample answer:

Solving $y = -\dfrac{x}{3}$

and $y = 3x - 10$ simultaneously

$$
\begin{aligned}
-\frac{x}{3} &= 3x - 10 \\
x &= -9x + 30 \\
10x &= 30 \\
x &= 3
\end{aligned}
$$

Hence, $y = -\dfrac{3}{3}$

$\quad = -1$

$\therefore D(3,\ -1)$

Question 12 (c)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use the quotient rule, or equivalent merit	1

Sample answer:

Let $u = x^2 + 3 \qquad v = x - 1$

$u' = 2x \qquad v' = 1$

$$
\begin{aligned}
y' &= \frac{u'v - v'u}{v^2} \\
&= \frac{2x(x-1) - 1(x^2+3)}{(x-1)^2} \\
&= \frac{2x^2 - 2x - x^2 - 3}{(x-1)^2} \\
&= \frac{x^2 - 2x - 3}{(x-1)^2}
\end{aligned}
$$

Question 12 (d)

Criteria	Marks
• Provides correct solution	2
• Evaluates discriminant, or equivalent merit	1

Sample answer:

For real roots, $\triangle \geq 0$

$$\triangle = b^2 - 4ac$$
$$= (-8)^2 - 4(1)(k)$$
$$= 64 - 4k$$
$$\therefore 64 - 4k \geq 0$$
$$64 \geq 4k$$
$$k \leq 16$$

Question 12 (e) (i)

Criteria	Marks
• Provides a correct equation	2
• Finds correct gradient, or equivalent merit	1

Sample answer:

$$y = \frac{x^2}{2} \qquad P\left(1,\ \frac{1}{2}\right)$$
$$\frac{dy}{dx} = x$$

when $x = 1, \quad m = 1$

The equation of the tangent is

$$y - y_1 = m\left(x - x_1\right)$$
$$y - \frac{1}{2} = 1(x - 1)$$
$$\therefore\ y = x - \frac{1}{2}$$

Question 12 (e) (ii)

Criteria	Marks
• Provides a correct equation	1

Sample answer:

Focus $S(0,\ a) = \left(0,\ \dfrac{1}{2}\right) \therefore a = \dfrac{1}{2}$

Equation directrix is: $y = -a$

$$\therefore y = -\frac{1}{2}$$

Question 12 (e) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Tangent: $y = x - \dfrac{1}{2}$

Directrix: $y = -\dfrac{1}{2}$

Solution: $-\dfrac{1}{2} = x - \dfrac{1}{2}$

$x = 0$

$\therefore Q$ lies on y-axis

Question 12 (e) (iv)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned} SQ &= SO + OQ \\ &= \frac{1}{2} + \frac{1}{2} \\ &= 1 \\ PS &= 1 \end{aligned}$$

$\therefore SQ = PS$

$\therefore \triangle PQS$ is isosceles (two equal sides)

Question 13 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

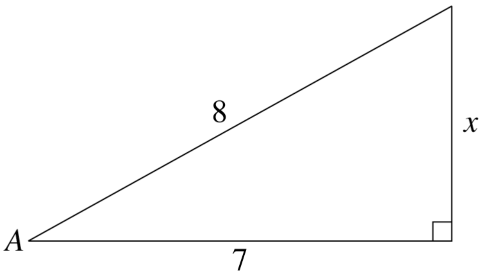
$$\begin{aligned}\cos A &= \frac{8^2 + 6^2 - 4^2}{2 \times 8 \times 6} \\ &= \frac{84}{96} \\ &= \frac{7}{8}\end{aligned}$$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds exact value of $\sin A$, or equivalent merit	1

Sample answer:

$\cos A = \dfrac{7}{8}$



$x^2 = \sqrt{8^2 - 7^2}$

$x = \sqrt{15}$

$\therefore \sin A = \dfrac{\sqrt{15}}{8}$

$\therefore \text{Area } \triangle ABC = \dfrac{1}{2} bc \sin A$

$= \dfrac{1}{2} \times 8 \times 6 \times \dfrac{\sqrt{15}}{8}$

$\therefore \text{Area} = 3\sqrt{15} \text{ cm}^2$

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Identifies –3 and 3 as important values, or equivalent merit	1

Sample answer:

Domain: $9 - x^2 \geq 0$

$x^2 \leq 9$

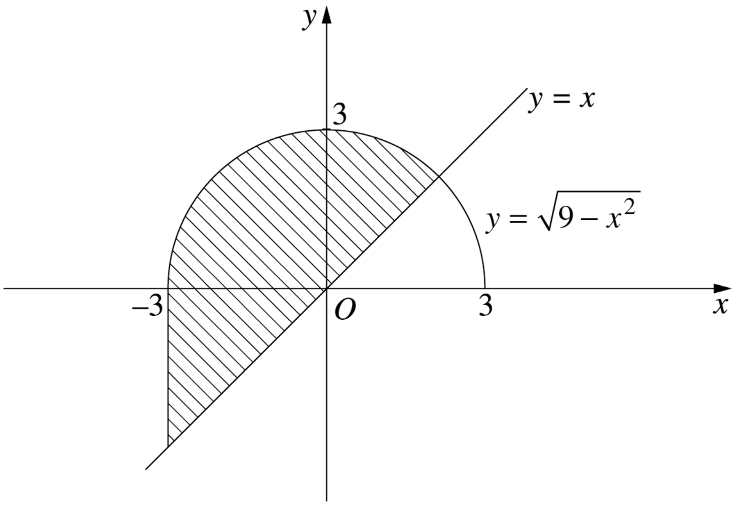
$-3 \leq x \leq 3$

Range: $0 \leq y \leq 3$

Question 13 (b) (ii)

Criteria	Marks
• Shades correct region	2
• Shades a region involving the correct semicircle, or equivalent merit	1

Sample answer:



Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	4
• Finds coordinates of both stationary points, or equivalent merit	3
• Factorises quadratic in $\frac{dy}{dx} = 0$ correctly, or equivalent merit	2
• Attempts to solve $\frac{dy}{dx} = 0$, or equivalent merit	1

Sample answer:

$$y = x^3 - x^2 - x + 3$$
$$y' = 3x^2 - 2x - 1$$
$$y'' = 6x - 2$$

Stationary points when $y' = 0$ ie $3x^2 - 2x - 1 = 0$

$$(3x + 1)(x - 1) = 0$$
$$\therefore x = -\frac{1}{3} \quad \text{or} \quad x = 1$$

when $x = -\frac{1}{3}$, $y = 3\frac{5}{27}$ and $y'' = -4 < 0$ (cd)

$\therefore \left(-\frac{1}{3}, 3\frac{5}{27}\right)$ is a local maximum

when $x = 1$, $y = 2$ and $y'' = 4 > 0$ (cu)

$\therefore (1, 2)$ is a local minimum

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows second derivative = 0 at $x = \frac{1}{3}$, or equivalent merit	1

Sample answer:

$P\left(\frac{1}{3}, \frac{70}{27}\right)$

Point of inflexion where $y'' = 0$ and concavity changes

ie $6x - 2 = 0$

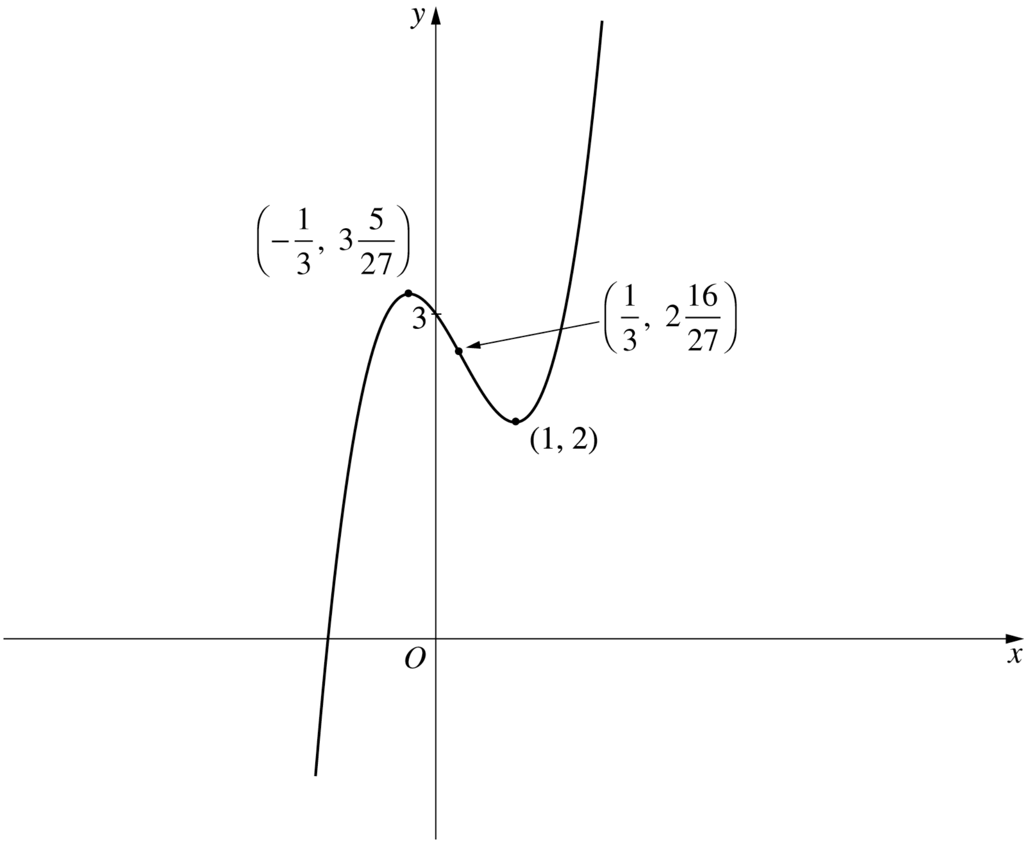
$$x = \frac{1}{3}$$

There is a point of inflexion at $x = \frac{1}{3}$ since from (i) concavity changes (maximum at $x = -\frac{1}{3}$ and minimum at $x = 1$).

Question 13 (c) (iii)

Criteria	Marks
• Provides correct sketch	2
• Correctly plots and labels stationary points and point of inflexion, or equivalent merit	1

Sample answer:



Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Provides correct expression for $\dot{x}$ including determination of the constant, or equivalent merit	1

Sample answer:

$\ddot{x} = -10, \qquad t = 0,\ x = 110,\ \dot{x} = 0$

$$\dot{x} = \int -10\,dt$$
$$= -10t + C_1$$

When $t = 0,\ \dot{x} = 0$

$$\therefore 0 = -10(0) + C_1$$
$$\therefore C_1 = 0$$
$$\therefore \dot{x} = -10t$$

Now, $x = \int -10t\,dt$

$$= -5t^2 + C_2$$

When $t = 0, x = 110$

$$\therefore 110 = -5(0)^2 + C_2$$
$$\therefore C_2 = 110$$
$$\therefore\ x = -5t^2 + 110$$

Question 14 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds correct time when brakes are applied, or equivalent merit	1

Sample answer:

When $\dot{x} = -37$

$$-37 = -10t$$

$$t = \frac{37}{10}$$

$$\therefore\ x = -5\left(\frac{37}{10}\right)^2 + 110$$

$$= 41.55$$

$\therefore$ It has fallen $(110 - 41.55)$ m

$$= 68.45 \text{ m}$$

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$P(\text{Saturday dry}) = P(WD) + P(DD)$$

$$= \frac{1}{2} \times \frac{5}{6} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{2}{3}$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Provides correct expression for probability of one of Saturday or Sunday being wet, or equivalent merit	1

Sample answer:

$P(\text{both Saturday and Sunday wet})$

$= P(WWW) + P(DWW)$

$= \frac{1}{2} \times \frac{1}{6} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{72} + \frac{1}{24}$

$= \frac{1}{18}$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$P(\text{at least one of Saturday and Sunday dry})$

$= 1 - P(\text{both are wet})$

$= 1 - \frac{1}{18}$

$= \frac{17}{18}$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$r = 0.6\% \text{ per month}$$
$$= 0.006 \text{ per month}$$

$$A_1 = 100\,000(1.006) - M$$
$$A_2 = A_1(1.006) - M$$
$$= \big(100\,000(1.006) - M\big)(1.006) - M$$
$$A_2 = 100\,000(1.006)^2 - M(1.006) - M$$
$$= 100\,000(1.006)^2 - M(1 + 1.006)$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Recognises the pattern in the expression for A_n as a sum, or equivalent merit	1

Sample answer:

$$A_3 = A_2(1.006) - M$$
$$= 100\,000(1.006)^3 - M(1.006)^2 - M(1.006) - M$$
$$\vdots$$
$$A_n = 100\,000(1.006)^n - M\left[1 + (1.006) + (1.006)^2 + \ldots + (1.006)^{n-1}\right]$$

Now $\left[1 + (1.006) + (1.006)^2 + \ldots + (1.006)^{n-1}\right]$ is a geometric series, with n terms where $a = 1$ and $r = 1.006$

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$
$$= \frac{1\big((1.006)^n - 1\big)}{(1.006) - 1}$$

$$\therefore A_n = 100\,000(1.006)^n - M\left[\frac{1\big((1.006)^n - 1\big)}{0.006}\right]$$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$M = 780, n = 120$

$$A_{120} = 100\,000(1.006)^{120} - 780\left[\frac{(1.006)^{120} - 1}{0.006}\right]$$
$$= 68\,499.46$$

$\therefore$ amount owing $= \$68500$ (nearest \$100)

Question 14 (c) (iv)

Criteria	Marks
• Provides correct solution	3
• Finds correct expression for $(1.006)^n$, or equivalent merit	2
• Attempts to use $A_n = 0$, or equivalent merit	1

Sample answer:

Since the amount owing is \$48 500,

$$A_n = 48\,500(1.006)^n - 780\left[\frac{(1.006)^n - 1}{0.006}\right]$$

Let $A_n = 0$ when the amount owing is completely repaid

$$\therefore 48\,500(1.006)^n = 780\left[\frac{(1.006)^n - 1}{0.006}\right]$$
$$\therefore 291(1.006)^n = 780(1.006)^n - 780$$
$$\therefore 489(1.006)^n = 780$$
$$\therefore (1.006)^n = 1.59509\ldots$$
$$n\log(1.006) = \log(1.59509\ldots)$$
$$n \approx \frac{\log(1.59509)}{\log 1.006}$$

time required ≈ 78.055 months

Question 15 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$C = Ae^{-0.14t}$$
$$\frac{dC}{dt} = -0.14Ae^{-0.14t}$$
$$= -0.14C$$

Question 15 (a) (ii)

Criteria	Marks
• Provides correct value of A	1

Sample answer:

When $t = 0,\ C = 130$

$130 = Ae^0$

$\therefore A = 130$

Question 15 (a) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$C = 130e^{-0.14t}$

When $t = 7$,

$C = 130e^{-0.14\times 7}$

$\simeq 48.79044285$

amount of caffeine ≈ 49 mg

Question 15 (a) (iv)

Criteria	Marks
• Provides correct solution	2
• Attempts to use logarithms to solve $65 = 130e^{-0.14t}$, or equivalent merit	1

Sample answer:

Taking $C = 65$

$$65 = 130e^{-0.14t}$$

$$\frac{1}{2} = e^{-0.14t}$$

$$\ln\frac{1}{2} = -0.14t$$

$$t = \frac{\ln\frac{1}{2}}{-0.14}$$

$$= 4.95105129$$

time taken ≈ 4.95 hours

Question 15 (b) (i)

Criteria	Marks
• Provides correct proof	2
• Shows $\angle FDC$ equal to $\angle ADE$, or $\angle ADE$ equal to $\angle DAE$, or equivalent merit	1

Sample answer:

$\angle FDC = \angle ADE$ (vertically opposite angles equal)

$\angle ADE = \angle DAE$ (base angles, isosceles triangle)

$\therefore \angle FDC = \angle DAE$

$\angle FCD = \angle ACB$

$= 90°$ (given)

$\therefore \triangle ACB \,|||\, \triangle DCF$ (equal angles)

Question 15 (b) (ii)

Criteria	Marks
• Provides correct explanation	1

Sample answer:

$\angle DFC = \angle ABC$ (corresponding angles in similar triangles)

$\therefore \triangle EFB$ is isosceles (base angles equal)

Question 15 (b) (iii)

Criteria	Marks
• Provides correct solution	2
• Provides correct relationship between lengths of AB and FD, or drops perpendicular from E onto AD, or equivalent merit	1

Sample answer:

$AB = 2FD$ (using (i), corresponding sides in proportion $AC = 2DC$)

ie $AE + EB = 2FD$

Also, $FD = EF - ED$

so $FD = EB - AE$ (by (ii) since $EF = EB$)

Hence $AE + EB = 2(EB - AE)$

$\therefore EB = 3AE$

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to solve $\frac{dV}{dt} = 0$, or equivalent merit	1

Sample answer:

Set $\frac{dV}{dt} = 0$

$80\sin(0.5t) = 0$

$\sin 0.5t = 0$

$0.5t = 0, \pi, 2\pi, \ldots$

first starts to decrease when $0.5t = \pi$

so $t = 2\pi$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Provides correct primitive, or equivalent merit	1

Sample answer:

$\frac{dV}{dt} = 80\sin(0.5t)$

Integrating,

$V = -160\cos(0.5t) + C$

When $t = 0$, $1200 = -160\cos(0.5 \times 0) + C$

$C = 1360$

Hence, $V = -160\cos(0.5t) + 1360$

When $t = 3$, $V = -160\cos(0.5 \times 3) + 1360$

Volume ≈ 1349 L

Question 15 (c) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Greatest volume when $\dfrac{dV}{dt} = 0$

ie when $t = 2\pi$ and volume starts to decrease

$$
\begin{aligned}
V &= -160\cos(0.5 \times 2\pi) + 1360 \\
&= 1520
\end{aligned}
$$

volume $= 1520$ L

Question 16 (a) (i)

Criteria	Marks
• Provides correct x-coordinates	1

Sample answer:

When $y = 0,\ x^2 - 7x + 10 = 0$

$\Rightarrow (x-2)(x-5) = 0$

$\therefore\ x = 2,\ x = 5$

So A is $(2,0)$ and B is $(5,0)$

Question 16 (a) (ii)

Criteria	Marks
• Provides correct coordinates	1

Sample answer:

When $x = 0,\ y = 10$

$\Rightarrow C$ is $(7,10)$ (by symmetry)

Question 16 (a) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\int_0^2 \left(x^2 - 7x + 10\right)dx = \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x\right]_0^2$$

$$= \left(\frac{2^3}{3} - 7\times\frac{2^2}{2} + 10\times 2\right) - \left(\frac{0^3}{3} - 7\times\frac{0^2}{2} + 10\times 0\right)$$

$$= \frac{26}{3}$$

Question 16 (a) (iv)

Criteria	Marks
• Provides correct solution	2
• Attempts to find the difference of two areas or integrals, or equivalent merit	1

Sample answer:

By symmetry, $\displaystyle\int_0^2 \left(x^2 - 7x + 10\right)dx = \int_5^7 \left(x^2 - 7x + 10\right)dx$

$$\therefore \text{Shaded area} = \text{area of } \triangle - \int_5^7 \left(x^2 - 7x + 10\right)dx$$

$$= \frac{1}{2} \times (7 - 2) \times 10 - \frac{26}{3}$$

$$= \frac{49}{3} \text{ units}^2$$

Question 16 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains $\pi \int_0^6 e^{\frac{y}{4}} + 2e^{\frac{y}{8}} + 1 \quad dy$, or equivalent merit	2
• Writes down formula for the volume including limits OR • Writes x as a function of y, or equivalent merit	1

Sample answer:

$$V = \pi \int_0^6 x^2 \, dy$$

Now $y = 8\log_e(x-1)$

$$\Rightarrow \quad e^{\frac{y}{8}} = x - 1$$

ie $\quad x = e^{\frac{y}{8}} + 1$

$$\therefore V = \pi \int_0^6 \left(e^{\frac{y}{8}} + 1\right)^2 dy$$

$$= \pi \int_0^6 e^{\frac{y}{4}} + 2e^{\frac{y}{8}} + 1 dy$$

$$= \pi \left[4e^{\frac{y}{4}} + 16e^{\frac{y}{8}} + y\right]_0^6$$

$$= \pi \left[\left(4e^{\frac{3}{2}} + 16e^{\frac{3}{4}} + 6\right) - (4 + 16 + 0)\right]$$

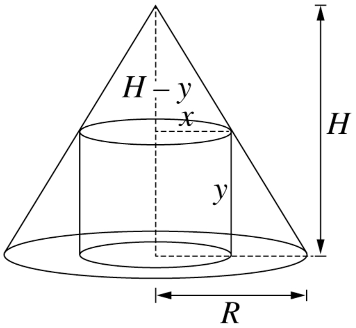
$$= \pi \left[4e^{\frac{3}{2}} + 16e^{\frac{3}{4}} - 14\right]$$

volume ≈ 118.7

Question 16 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Uses similar triangles to obtain correct equation relating x and y, or equivalent merit	2
• Attempts to use similar triangles, or equivalent merit	1

Sample answer:



Using similar triangles

$$\frac{H-y}{H} = \frac{x}{R}$$

$$RH - Ry = xH$$

$$RH - xH = Ry$$

$$y = \frac{H(R-x)}{R}$$

$$\left[\text{OR,} \quad \frac{H}{R} = \frac{y}{R-x} \qquad y = \frac{H}{R}(R-x)\right]$$

$$\therefore V = \pi r^2 h = \pi x^2 y$$

$$= \pi x^2 \frac{H}{R}(R-x)$$

$$= \frac{H}{R}\pi x^2 (R-x)$$

Question 16 (c) (ii)

Criteria	Marks
• Provides correct solution	4
• Finds the value at which the stationary point occurs and verifies it is a maximum, or equivalent merit	3
• Finds the value of x at which $\frac{dV}{dx} = 0$, or equivalent merit	2
• Finds correct expression for $\frac{dV}{dx}$, or equivalent merit	1

Sample answer:

Maximum V occurs when $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} < 0$

$$V = \frac{H}{R}\pi x^2 (R - x) = H\pi x^2 - \frac{H}{R}\pi x^3$$

$$\frac{dV}{dx} = 2H\pi x - \frac{3H}{R}\pi x^2 = 0 \text{ for maximum}$$

$$\therefore\ 0 = \pi H x\left(2 - \frac{3}{R}x\right)$$

$$x = 0 \text{ (discount) or } x = \frac{2R}{3}$$

$$\frac{d^2V}{dx^2} = 2H\pi - \frac{6H}{R}\pi x$$

$$= 2H\pi - \frac{6H}{\cancel{R}}\pi\frac{2\cancel{R}}{3} \quad \text{at } x = \frac{2R}{3}$$

$$= 2H\pi - 4H\pi$$

$$= -2H\pi \quad < \ 0 \text{ since } H > 0$$

$$\therefore\ \text{Maximum volume when } x = \frac{2R}{3}$$

$$\therefore V = \frac{H}{R}\pi\left(\frac{2R}{3}\right)^2\left(R - \frac{2R}{3}\right)$$

$$= \frac{H}{\cancel{R}}\pi\frac{4R^2}{9}\left(\frac{\cancel{R}}{3}\right)$$

$$V = \frac{4H\pi R^2}{27}$$

2015 HSC Mathematics
Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	1.1	P3, P4
2	1	6.2	P4
3	1	7.1	H5
4	1	3.3	H5
5	1	11.3	H4
6	1	13.4, 13.5, 13.7	H5
7	1	11.4	H8
8	1	9.1, 12.1	H3
9	1	14.3	H4, H5
10	1	12.5	H3, H8

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	1	1.3	P3
11 (b)	2	1.3	P4
11 (c)	2	1.1	P3
11 (d)	2	7.3	H5
11 (e)	2	8.9, 12.5	P7, H3
11 (f)	2	8.9, 12.5	P7, H3
11 (g)	2	11.1, 13.7	H5
11(h)	2	11.2, 12.5	H5
12 (a)	2	5.3, 13.1	P4, H5
12 (b) (i)	2	2.3, 6.8	H5
12 (b) (ii)	2	1.4, 6.3, 6.8	P4, H5
12 (c)	2	8.8	P7
12 (d)	2	1.4, 9.2, 9.3	P4
12 (e) (i)	2	6.2, 8.5	P4
12 (e) (ii)	1	9.5	P4, P5
12 (e) (iii)	1	1.4, 6.3	P4, P5
12 (e) (iv)	1	2.2, 6.8	P4, H2, H5
13 (a) (i)	1	5.5	P3, P4
13 (a) (ii)	2	5.5	P3, P4

Question	Marks	Content	Syllabus outcomes
13 (b) (i)	2	4.1	P5
13 (b) (ii)	2	4.1, 4.4	P5
13 (c) (i)	4	10.2, 10.4	H6
13 (c) (ii)	2	10.4	H6
13 (c) (iii)	2	10.5	H6
14 (a) (i)	2	14.3	H4, H5
14 (a) (ii)	2	14.3	H4, H5
14 (b) (i)	1	3.2, 3.3	H5
14 (b) (ii)	2	3.2, 3.3	H5
14 (b) (iii)	1	3.2, 3.3	H5
14 (c) (i)	1	7.5	H4, H5
14 (c) (ii)	2	7.2, 7.5	H4, H5
14 (c) (iii)	1	7.5	H4, H5
14 (c) (iv)	3	7.5, 12.2	H3, H5
15 (a) (i)	1	14.2	H3, H5
15 (a) (ii)	1	14.2	H3
15 (a) (iii)	1	14.2	H3
15 (a) (iv)	2	12.2, 14.2	H3
15 (b) (i)	2	2.3, 2.5	H2, H5
15 (b) (ii)	1	2.2, 2.5	H2, H5
15 (b) (iii)	2	2.3, 2.5	H2, H5
15 (c) (i)	2	14.1	H4, H5
15 (c) (ii)	2	14.1	H4, H5
15 (c) (iii)	1	14.1	H4, H5
16 (a) (i)	1	9.1	P4
16 (a) (ii)	1	1.4, 4.2	P4
16 (a) (iii)	1	11.1	H8
16 (a) (iv)	2	11.4	H8, H9
16 (b)	3	11.4, 12.3, 12.5	H3, H4, H8
16 (c) (i)	3	2.3, 2.5	P4, P5
16 (c) (ii)	4	10.6	H5