2016 Notes from the Marking Centre - Mathematics

Question 11

(a) This part was generally done well. Most candidates indicated either the radius or the centre. Common problems were:

- sketching a circle with the correct centre but an incorrect radius, or vice versa
- placing the centre $(3, -2)$ in the wrong quadrant
- drawing the circle incorrectly with a tangent being either the *y*–axis or both axes
- drawing a sketch without any labels.

(b) This part was generally done well with the majority of candidates attempting the quotient rule and substituting correctly. Common problems were:

- not using brackets or expanding incorrectly when dealing with the product $-u \times v'$
- mixing up the expressions for *u* and *v*.

(c) Many candidates only found one part of the solution. Common problems were:

- incorrectly solving the negative case, having the incorrect inequality sign or ignoring the negative value at some point
- writing a solution using an 'or' statement rather than an 'and' statement, leaving the answer as $x \leq 5$, $x \geq -1$ instead of $-1 \leq x \leq 5$.

(d) This part was attempted well by the majority of candidates. Common problems were:

- incorrectly integrating and using a denominator of 4 instead of 8
- assuming a substitution of 0 into the integral gives a result of 0
- differentiating instead of integrating.

(e) Most candidates realised that they needed to equate the two given equations and solve for x. Common problems were:

- not finding the correct quadratic or incorrectly solving $x^2 - 2x - 8 = 0$ to be $x = 2$ and $x = -4$
- correctly solving for *x* but not finding the corresponding *y*-values
- not showing the substitution of *x* to find *y* or making careless errors.

(f) This part was found to be challenging for many candidates. Most candidates found the derivative of $y = \tan x$. Common problems were:

- using degree mode and not radian mode on the calculator, or leaving the answer in terms of $\sec^2\left(\frac{\pi}{8}\right)$
- not recognising that $\sec^2\left(\frac{\pi}{8}\right) = \frac{1}{\cos^2 x}$.

(g) This part was found to be challenging for many candidates with the 2 in $\frac{x}{2}$ being problematic. Common problems were:

- finding only one solution
- giving the answer in degrees
- finding an angle but not showing whether it was for *x* or $\frac{x}{2}$
- using the incorrect domain or selecting the incorrect quadrants.

Question 12

(a)(i) This part was done well by the majority of candidates, most correctly using the point-gradient form or two-point form. Common problems were:

- incorrectly multiplying by the gradient
- not recognising which formula to use.

(a)(ii) This part was done well by the majority of candidates, although some may have spent a long time using alternative methods to find the coordinates of D. Common problems were:

- not recognising the need to use the perpendicular distance formula
- substituting incorrect values into the perpendicular distance formula
- incorrectly substituting the values for a and b taken from BC in general form
- making numerical errors in calculations.

(a)(iii) This part was done well by the majority of candidates. Common problems were:

- not using the perpendicular height of the triangle from part (a)(ii)
- using an incorrect area formula
- using the formula $A = \frac{1}{2}ab\sin C$ but not being able to correctly calculate the angle at C.

(b)(i) and (ii) These parts were done well by the majority of candidates. Common problems were:

- poor setting out with little or no reasoning
- not writing a conclusion
- interpreting the question as a circle geometry problem.

(c) Most candidates recognised the need to use trigonometry involving the use of the cosine rule and identifying complementary angles. Common problems were:

- assuming that angle $\measuredangle P$ was a right angle
- finding the obtuse $\measuredangle P$ and not knowing how to proceed
- re-arranging the cosine rule incorrectly.

(d)(i) Most candidates realised that they needed to use the product rule. Common problems were:

- using the product rule incorrectly
- differentiating e^{3x} incorrectly.

(d)(ii) The candidates who had successfully answered (d) (i) were more able to recognise the connection and make the appropriate substitution. Common problems were:

- not rewriting the integral in terms of the expression from part (d)(i) and integrating incorrectly
- dividing, instead of multiplying, by 3
- substituting into the integral incorrectly.

Question 13

(a)(i) Most candidates answered this part well, setting work out in clear, logical steps. Candidates used either the first derivative test or the second derivative test to determine the nature of the stationary point. Common problems were:

- not solving the first derivative equal to zero
- when using a table for either the first or second derivative test, not showing which derivative it was, y' or y''
- when testing the stationary point at $x = 0$ via the second derivative test, it was often assumed that the stationary point was a point of inflexion when $y'' = 0$ without completing any further test to verify
- many solved $y'' = 0$ and found possible points of inflexion which seemed to confirm their results from $y' = 0$. However, many did not test that y' changed sign at $x = 0$ or that there was a change in concavity via y''.

(a)(ii) The better responses often included a graph of at least one third of a page and the detail required was clearly indicated and labelled. Common problems were:

- not clearly indicating the coordinates of the maximum stationary point
- after finding a horizontal point of inflexion at $(0,0)$, often something different was sketched
- not showing the x and y-intercepts. Also, many graphs finished above the x-axis.

(b)(i) In better responses, candidates demonstrated that they knew how to complete the square and use the equation of their parabola to find the focal length. Common problems were:

- not correctly factorising the bracket involving y
- calculating that $a = 3$ and then stating the focal length as $2a$, the distance from the focus to the directrix
- a misunderstanding of the concavity of the parabola. Often $-4a$ or $\pm 4a$ was used instead of just$4a$.

(b)(ii) Most found the vertex of their parabola. In better responses, candidates drew the parabola, noting the vertex on their graph and showing the relationship between the vertex, focal length and focus. Common problems were:

- using the coordinates of the vertex for the focus
- using an incorrect orientation of the parabola
- not correctly using the given formula from the Reference Sheet. Often opposite signs were used for h and k.

(c)(i) The value of A was stated by the majority. The difference between the instructions find, evaluate and state is not understood by some.

(c)(ii) Most responses demonstrated a correct method to find the value of k. Common problems were:

- not understanding that $M(t)$ stood for the mass of Curium after t days but instead multiplied the given mass of Curium by the number of days
- making errors in subsequent lines of working. For example, a change from $-k$ to $+k$ without necessity.

(d) Most responses included finding two areas by integration then subtracting them to find the shaded area. Common problems were:

- using incorrect boundary values for either the area under the cosine curve or the area under the line $y = x$. This included using the y-values instead of the x-values.
- not using or reading the Reference Sheet correctly, eg, having an incorrect sign when finding the primitive for the cosine function.
- when integrating $\cos(\frac{\pi}{4}x)$, multiplying the primitive by $\frac{\pi}{4}$ instead of $\frac{4}{\pi}$
- changing $\sqrt{2}$ to $2^{\frac{1}{2}}$ and then integrating that expression
- using $A = \frac{1}{2}r^2\theta$ to calculate the area as though the shaded area was a sector
- not using the fact that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and that the question asked for an exact value.

Question 14

(a) Most candidates recognised the need to find the difference between the applications of Simpson's Rule to each cross-section. Some candidates identified that the heights were symmetrical and so they understood they could find half the area and then double their answer. Few candidates subtracted the heights first. Common problems were:

- using x-values for the heights instead of function values
- using the trapezoidal rule
- applying Simpson's rule incorrectly by swapping the weightings of 4 and 2
- using an incorrect value of h
- calculating the percentage increase in area rather than the increase in area in square metres
- when the difference in heights was found before applying Simpson's rule errors were made with the endpoints, leaving the heights as 2 rather than the correct value of 0.

(b) Most showed a good understanding of how to develop the series and identify it as a GP.

(b)(i) The majority showed an understanding of the nature of the question and the link between A1 and A2. Common problems were:

- using 1.65 or 0.35 instead of 0.65
- finding A_1 only
- deriving an incorrect expression for A_1 leading to an incorrect expression for A_2
- incorrect use or omission of brackets when writing the expressions for A_1 and A_2 or inconsistently omitting zeros in 100 000 or 5000.

(b)(ii) Most showed the progression from A2 to A3 and onto An, displaying the geometric series with three consecutive terms and the last term, and correctly substituting into the GP sum formula. Common problems were:

- attempting to identify a GP with only 2 terms
- attempting to work backwards from the required result
- omitting the first or last term of the GP.

(b)(iii) This question was completed well by most candidates. Common problems were:

- calculator errors
- misinterpreting the statement 'correct to the nearest 100' to mean $n = 100$, hence incorrectly substituting 100 into the formula
- using $n = 13$ or $n = 15$.

(c)(i) This question was well attempted. Common problems were:

- not showing that the length of the unknown longer side of the enclosure was $\frac{720}{x}$
- multiplying the '$5x$' by 'x' to give 720.

(c)(ii) The majority identified that calculus was necessary and used the first derivative to find the stationary point. Most used the second derivative test to show the value was a maximum. Common problems were:

- testing their value of x using the first derivative test and agreeing with the question that the result was a minimum without any calculations to support their statement
- finding the second derivative incorrectly
- not finding the length of fencing required.

(d) Many candidates recognised the GP and apply the sum of a GP formula to find the required limit. A small number of candidates used the method of testing the limit either side of 1. Common problems were:

- not seeing the connection between the limit and the required sum
- incorrectly attempting to factorise x^5-1
- using the sum to infinity formula for a GP.

(e) Many candidates rewrote the series as 1log2 + 2log2+ 3log 2 + …+9log2. Common problems were:

- incorrectly treating the series as being geometric rather that arithmetic
- not making the connection between 1log2 + 2log2+ 3log 2 + …+9log2 and the final answer by simply summing the coefficients
- not writing the final solution in the form $a$log$b$ where a, b are integers.

Question 15

(a) Common problems were:

- errors made when squaring after making y the subject of both equations
- using the same limits for both integrals
- subtracting the volumes rather than adding them
- halving or doubling the volume
- rotating about the y-axis rather than the x-axis
- not showing substitutions into the primitive function when finding a definite integral.

(b)(i) In the better responses, candidates correctly drew the tree diagram with the correct probabilities on each branch and ensuring the branch did not extend after a win. This was followed by an explanation using words or probability notation to show how the result was obtained. Common problems were:

- drawing an incorrect tree diagram, with multiple extra branches
- not writing the probabilities on each branch
- attempting to work backwards from the expression and expanding
- not providing an explanation for the given expression.

(b)(ii) A significant number of candidates recognised that the pattern formed a geometric series and knew to sum the series. Common problems were:

- not realising what the last term in the series is and using an incorrect number of terms in the GP sum formula
- using the T_n formula rather than the S_n formula for a geometric series
- not solving the inequality even if realising the solution involved logs
- not realising that $\log\left(\frac{7}{8}\right)$ was negative and that a change of the inequality sign was required when dividing by this value.

(c)(i) This part was done well by most candidates. In better responses, the diagram was copied or traced into the answer booklet and then used to establish appropriate relationships between angle pairs. In successful responses, reasons were provided for every geometric step. Common problems were:

- confusing alternate angles with vertically opposite angles
- omitting the diagram, incorrectly labelling the diagram or copying the diagram without adding the given information
- using the wrong pair of triangles
- using pairs of sides in the same ratio instead of equal angles
- incorrectly labelling angles in their proof
- not concluding with the specific reason or test for similarity
- abbreviating reasons to the extent that they lacked clarity.

(c)(ii) This part was challenging for most candidates. In successful responses, candidates started the similarity proof by introducing θ to identify an angle and worked from there to establish equality. Common problems were:

- identifying pairs of equal angles without correct reason
- assuming *SA* and *TB* were parallel.

(c)(iii) This part was challenging for many candidates. In better responses, candidates drew the similar triangles from (c)(i) and (ii) and found the pairs of corresponding sides of the similar triangles. They were then able to use ratios to find the relationships, $AT = \frac{1}{x}$ and $AT = \frac{h}{y}$, and then correctly find h in terms of x and y. Common problems were:

- not realising that all four sides of the square were of length 1 unit
- using the wrong pairs of triangles
- using the wrong ratios or not stating the ratios
- unsuccessfully using Pythagoras' theorem and/or Trigonometry.

Question 16

(a)(i) This part was attempted well. The correct substitution of $t = 0$ was made to find the initial velocity. Common errors were:

- substituting $v = 0$ instead of $t = 0$
- transcribing $v = 2 - \dfrac{4}{t-1}$ instead of $v = 2 - \dfrac{4}{t+1}$.

(a)(ii) This part was attempted well by most candidates. Many let $v = 0$ to find t. Common errors were:

- not identifying $t = 1$ when $v = 0$

- incorrectly interpreting the meaning of stationary, assuming the particle was stationary when t = 0 instead of when $v = 0$
- attempting to use the quotient rule and finding the incorrect result of $a = -\frac{4}{(t+1)^2}$
- integrating instead of differentiating.

(a)(iii) Most candidates attempted this question. However, many displayed poor graphing skills and did not link the answers from previous parts. Common errors were:
- not showing the asymptote at $v = 2$
- labelling inconsistently on their graph including axes
- graphing values for $t < 0$.

(a)(iv) This part was quite challenging for most candidates. In successful responses, candidates used the time when the particle changed direction, found the correct primitive and substituted correctly. Common errors were:
- not addressing the change of direction
- not realising $|2-4\ln 2|$ is equal to $4\ln 2-2$
- not recognising that the primitive of $2-\frac{4}{t+1}$ involved a logarithm
- substituting incorrectly
- not finding the 'exact distance' as required.

(b)(i) Most candidates successfully used either the function of a function rule or the quotient rule to differentiate. Common errors were:
- neglecting to show the required negative signs
- incorrect use of brackets
- not showing the substitution of the values of v' and u' into the quotient rule formula and merely stating the result given in the question
- making algebraic errors.

(b)(ii) This question was found to be challenging. However, many candidates found at least one of the y values needed to determine the range.
Common errors were:
- leaving answers in terms of t instead of y
- having incorrect inequality signs
- neglecting to examine both situations , ie, when $t = 0$ and the limit as t tends to infinity.

(b)(iii) The algebraic manipulation required in this part was found to be challenging. Common errors were:
- incorrectly substituting or rearranging the modelling function
- writing $\left(\frac{200}{y}\right)^2 = \frac{400}{y^2}$.

(b)(iv) Most did not recognise the need to find the maximum value of the expression given in (b)(iii) or (b)(i). Those who differentiated the expression in (b)(i) were less successful than those who chose to use the result from(b)(iii). Very few candidates used the fact that the parabola in (b)(iii) was a concave down parabola to prove the function had a maximum value. Common errors were:
- solving $\frac{y}{200}(200-y)=0$ to find y
- not showing the rate of growth was a maximum
- substituting the values of 0 and 200 into the derivative of the rate.