NSW Education Standards Authority

2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Standard 2

General Instructions
- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 15 marks (pages 2–8)
- Attempt Questions 1–15
- Allow about 25 minutes for this section

Section II – 85 marks (pages 9–36)
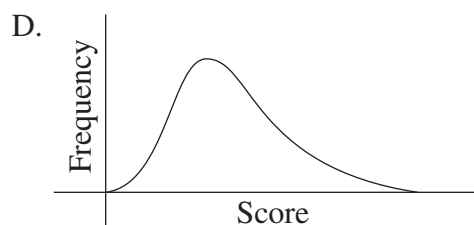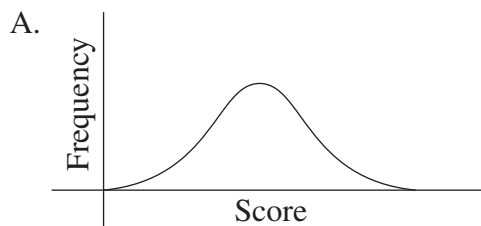- Attempt Questions 16–38
- Allow about 2 hours and 5 minutes for this section

Section I

15 marks
Attempt Questions 1–15
Allow about 25 minutes for this section

Use the multiple-choice answer sheet for Questions 1–15.

1 Which graph represents a negatively skewed distribution?

A.

B.

C.

D.

2 Which of the following could be the graph of $y = -2x + 2$?

A.

B.

C.

D.
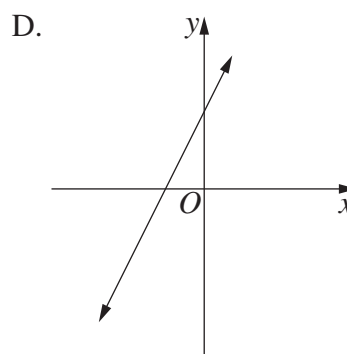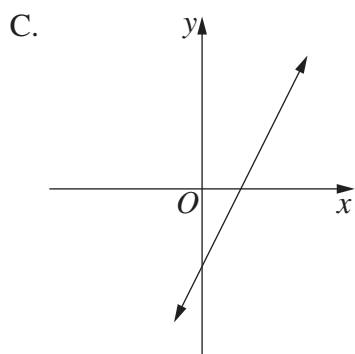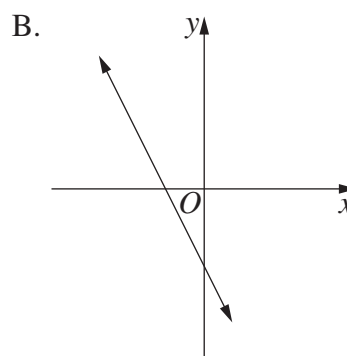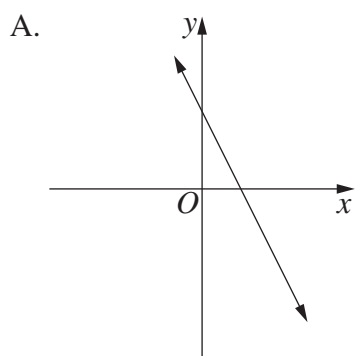
3   The network diagram shows the time needed for each step in order to complete a project.
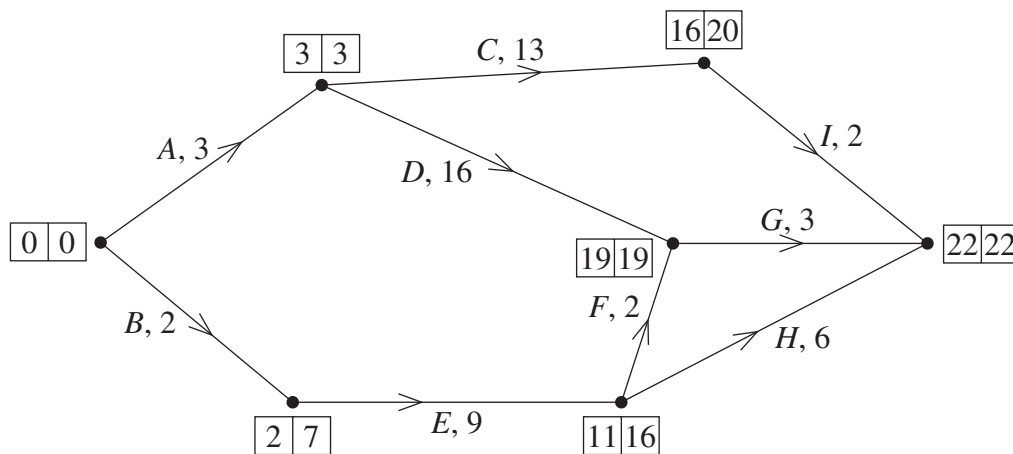
What is the critical path to complete the project?

A.   *ACI*

B.   *ADG*

C.   *BEH*

D.   *BEFG*

4   Lily wanted to estimate the number of fish in a lake.

She randomly captured 30 fish, then tagged and released them.

One week later she randomly captured 40 fish from the same lake. She found that 12 of these 40 fish were tagged.

What is the best estimate for the total number of fish in the lake?

A.   58

B.   70

C.   82

D.   100

5 Consider the following dataset.

$$13 \quad 16 \quad 17 \quad 17 \quad 21 \quad 24$$

Which row of the table shows how the median and mean are affected when a score of 5 is added to the dataset?

	Median	*Mean*
A.	Changes	Changes
B.	Stays the same	Stays the same
C.	Changes	Stays the same
D.	Stays the same	Changes

6 What is 20 minutes to one-third of a day, expressed as a ratio in simplest form?

A. $1:8$

B. $1:24$

C. $60:1$

D. $72:1$

7 Tian is paid $20.45 per hour, as well as a meal allowance of $16.20 per day.

What are Tian's total earnings if she works 9 hours per day for 5 days?

A. $329.85

B. $936.45

C. $1001.25

D. $1649.25

8 Which true bearing is the same as S48°W?

A. 132°

B. 222°

C. 228°

D. 312°

9 An object is projected vertically into the air. Its height, h metres, above the ground after t seconds is given by $h = -5t^2 + 80t$.
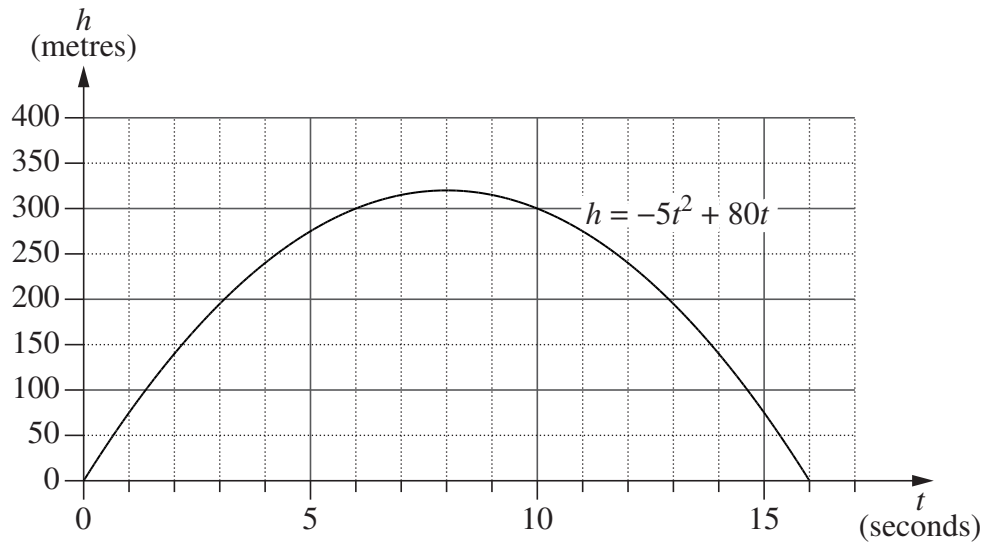
For how long is the object at a height of 300 metres or more above the ground?

A.   4 seconds

B.   6 seconds

C.   8 seconds

D.   10 seconds

10 Alex purchased 800 shares. The total cost was $2.60 per share. Alex sold the shares one year later for $3.40 each and paid a fee of $24.95 for selling the shares.

What profit did Alex make on these shares?

A. $590.10

B. $615.05

C. $640.00

D. $664.95

11 In ten years, the future value of an investment will be \$150 000. The interest rate is 4% per annum, compounded half-yearly.

Which equation will give the present value (PV) of the investment?

A. $PV = \dfrac{150\,000}{(1 + 0.04)^{10}}$

B. $PV = \dfrac{150\,000}{(1 + 0.04)^{20}}$

C. $PV = \dfrac{150\,000}{(1 + 0.02)^{10}}$

D. $PV = \dfrac{150\,000}{(1 + 0.02)^{20}}$

12 For a particular course, the recorded data show a relationship between the number of hours of study per week and the marks achieved out of 100.

A least-squares regression line is fitted to this dataset. The equation of this line is given by

$$M = 20 + 3H,$$

where M is the predicted mark and H is the number of hours of study per week.

Based on this regression equation, which of the following is correct regarding the predicted mark of a student?

A. It will be 3 for zero hours of study per week.

B. It will be 20 for zero hours of study per week.

C. It will increase by 20 for every additional hour of study per week.

D. It will increase by 1 for every 3 additional hours of study per week.

13 A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable lies below z for some positive values of z.

z	1.90	1.91	1.92	1.93	1.94	1.95	1.96	1.97	1.98	1.99
Probability	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

The probability values given in the table are represented by the shaded area in the following diagram.
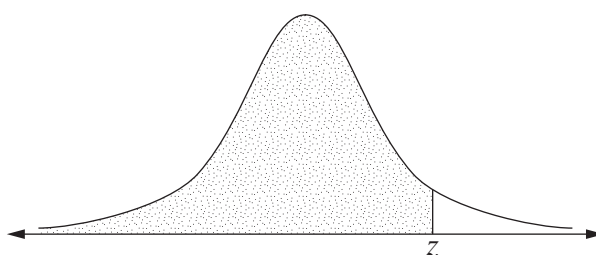
What is the probability that a normally distributed random variable with mean 0 and standard deviation 1 lies between 0 and 1.94?

A.   0.0262

B.   0.4738

C.   0.5262

D.   0.9738

14 Which of the following correctly expresses x as the subject of $y = \dfrac{ax - b}{2}$?

A. $x = \dfrac{2y + b}{a}$

B. $x = \dfrac{y + b}{2a}$

C. $x = \dfrac{2y}{a} + b$

D. $x = \dfrac{y}{2a} + b$

15 The cumulative frequency graph shows the distribution of the number of movie downloads made by 100 people in one month.
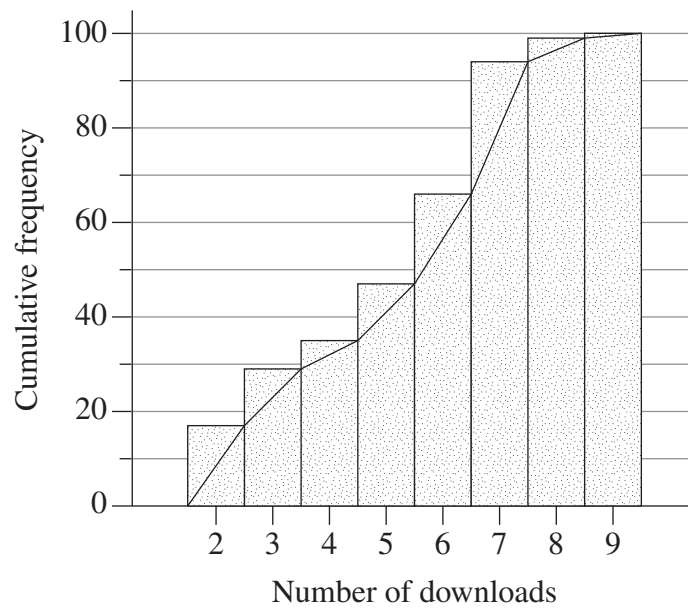
Which box-plot best represents the same data as displayed in the cumulative frequency graph?

A.

Number of downloads

B.

Number of downloads

C.

Number of downloads

D.

Number of downloads

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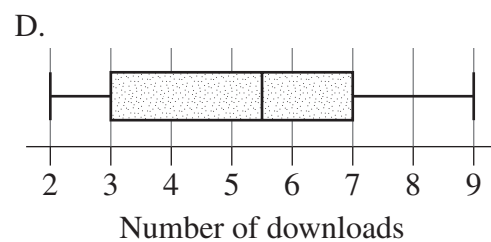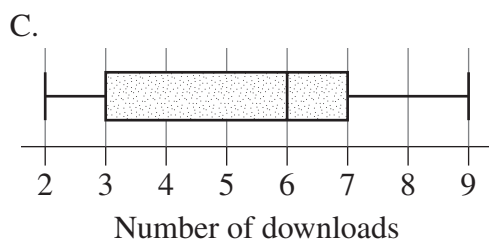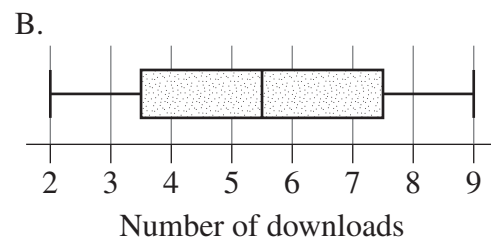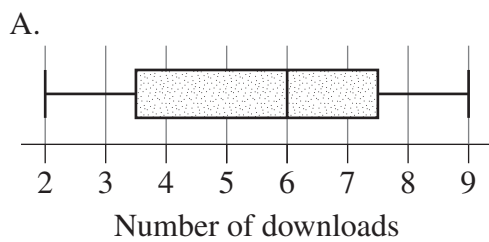
Centre Number

Student Number

Mathematics Standard 2

Section II Answer Booklet

85 marks
Attempt Questions 16–38
Allow about 2 hours and 5 minutes for this section

Instructions

• Write your Centre Number and Student Number at the top of this page.

• Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

• Your responses should include relevant mathematical reasoning and/or calculations.

• Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

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Question 16 (3 marks)

Tom is 25 years old, and likes to keep fit by exercising.

(a) Use this formula to find his maximum heart rate (bpm). **1**

$$\text{Maximum heart rate} = 220 - \text{age in years}$$

Tom's maximum heart rate is .... bpm.

(b) Tom will get the most benefit from this exercise if his heart rate is between 50% **2**
and 85% of his maximum heart rate.

Between what two heart rates should Tom be aiming for to get the most benefit
from his exercise?

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Question 17 (3 marks)

The numbers 1, 2, 3, 4, 5 and 6 are each written on separate cards.

Amy has cards 1, 3 and 5 and Bob has cards 2, 4 and 6.

They play a game in which each person randomly chooses one of their own cards and compares it with the other person's card. The person with the higher card wins.

(a)   A partially completed tree diagram is shown.   **2**

Amy's card   Bob's card

1 — 2, 4, __

3 — 2, __, __

5 — __, __, __
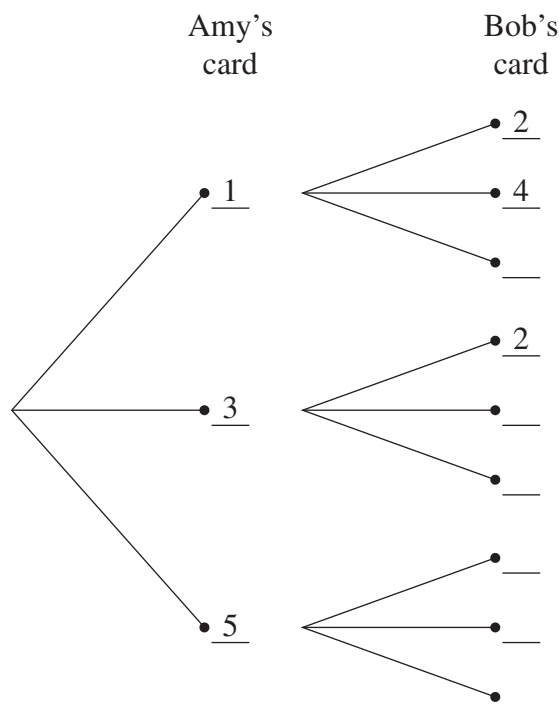
Complete the tree diagram and find the probability that Bob wins.

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(b)   Suppose Amy and Bob play this game 30 times.   **1**

How many times would Bob be expected to win?

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Question 18 (2 marks)

The marks in a test were normally distributed. The mean mark was 60 and the standard deviation was 15. **2**

What was the percentage of marks higher than 90?

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Question 19 (3 marks)

The table shows the types of customer complaints received by an online business in a month.

Type of complaint	Frequency	Cumulative frequency	Cumulative percentage
Stock shortage	98	98	49
Delivery fee	62	*A*	80
Delivery time	24	184	92
Damaged item	8	192	*B*
Returns policy	6	198	99
Product information	2	200	100
Total	200		

(a)   What are the values of *A* and *B*?   **2**

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(b)   The data from the table are shown in the following Pareto chart.   **1**

Customer complaints
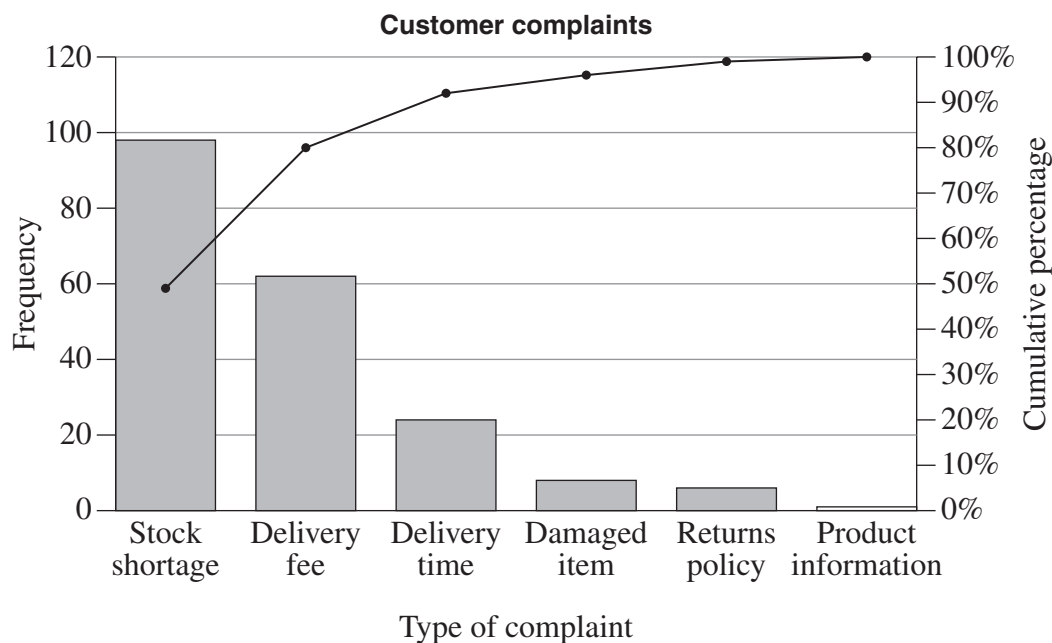
The manager will address 80% of the complaints.

Which types of complaints will the manager address?

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Question 20 (5 marks)

The table below shows the distances, in kilometres, between a number of towns.

Towns	Snowtown (S)	Clairville (C)	Yuma (Y)	Bosten (B)	Morrella (M)
(S)	–	–	280	275	–
(C)	–	–	60	150	–
(Y)	280	60	–	–	530
(B)	275	150	–	–	790
(M)	–	–	530	790	–

(a) Using the vertices given, draw a weighted network diagram to represent the information shown in the table.   **2**

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              Y
S             •                          M
•                                        •
                    C
                    •



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        B
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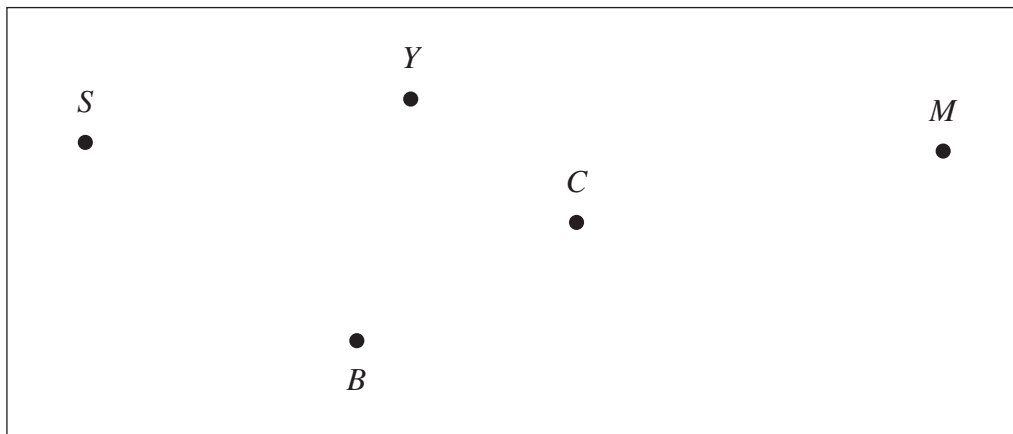
(b) A tourist wishes to visit each town.   **3**

Draw the minimum spanning tree which will allow for this AND determine its length.

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              Y
S             •                          M
•                                        •
                    C
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        B
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Length of minimum spanning tree = ...

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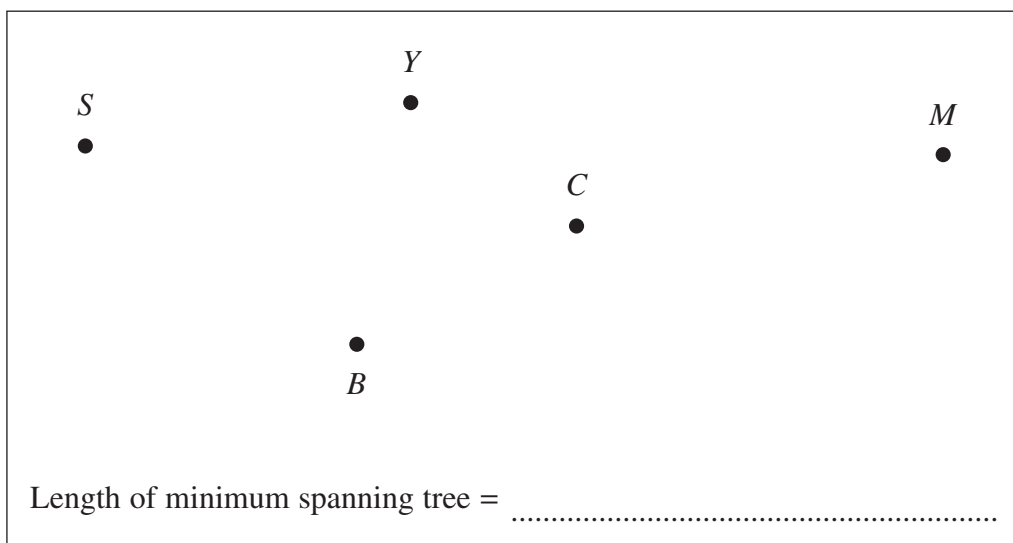
Question 21 (2 marks)

A real estate agent's commission for selling houses is 2% for the first $800 000 of the sale price and 1.5% for any amount over $800 000. **2**

Calculate the commission earned in selling a house for $1 500 000.

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Question 22 (3 marks)

The formula $C = 100n + b$ is used to calculate the cost of producing laptops, where C is the cost in dollars, n is the number of laptops produced and b is the fixed cost in dollars.

(a)   Find the cost when 1943 laptops are produced and the fixed cost is $20 180.   **1**

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(b)   Some laptops have some extra features added. The formula to calculate the   **2**
production cost for these is

$$C = 100n + an + 20\,180,$$

where a is the additional cost in dollars per laptop produced.

Find the number of laptops produced if the additional cost is $26 per laptop and the total production cost is $97 040.

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Question 23 (4 marks)

A teacher surveyed the students in her Year 8 class to investigate the relationship between the average number of hours of phone use per day and the average number of hours of sleep per day.

The results are shown on the scatterplot below.
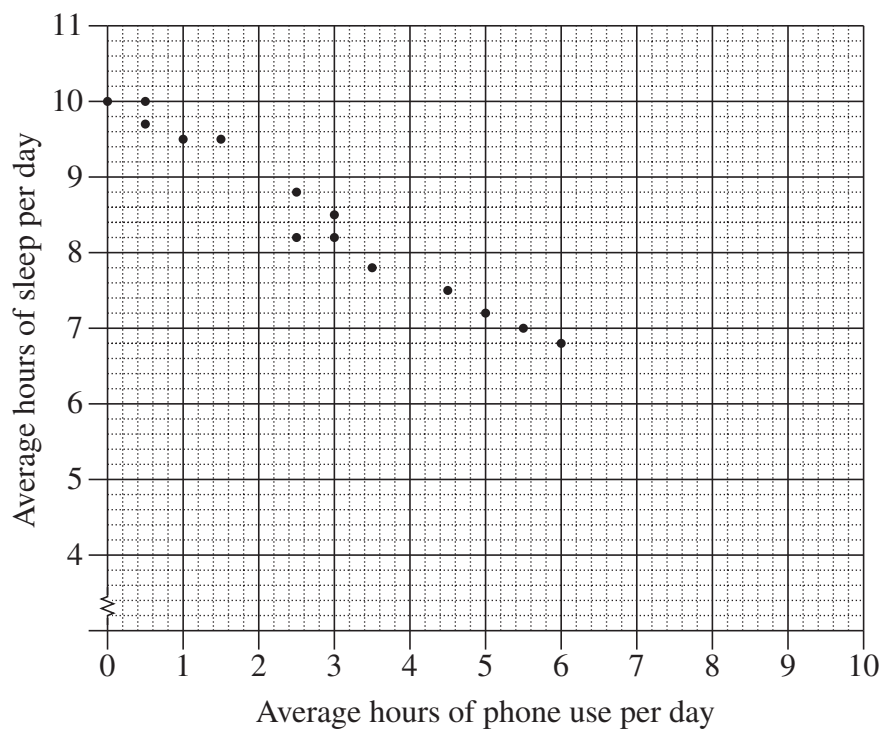
(a) The data for two new students, Alinta and Birrani, are shown in the table below. Plot their results on the scatterplot. **2**

	Average hours of phone use per day	Average hours of sleep per day
Alinta	4	8
Birrani	0	10.5

(b) By first fitting the line of best fit by eye on the scatterplot, estimate the average number of hours of sleep per day for a student who uses the phone for an average of 2 hours per day. **2**

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Question 24 (4 marks)

A student believes that the time it takes for an ice cube to melt (M minutes) varies inversely with the room temperature (T°C). The student observes that at a room temperature of 15°C it takes 12 minutes for an ice cube to melt.

(a) Find the equation relating M and T. **2**

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(b) By first completing this table of values, graph the relationship between temperature and time from $T = 5$°C to $T = 30$°C. **2**

T	5	15	30
M			

Temperature (T°C)

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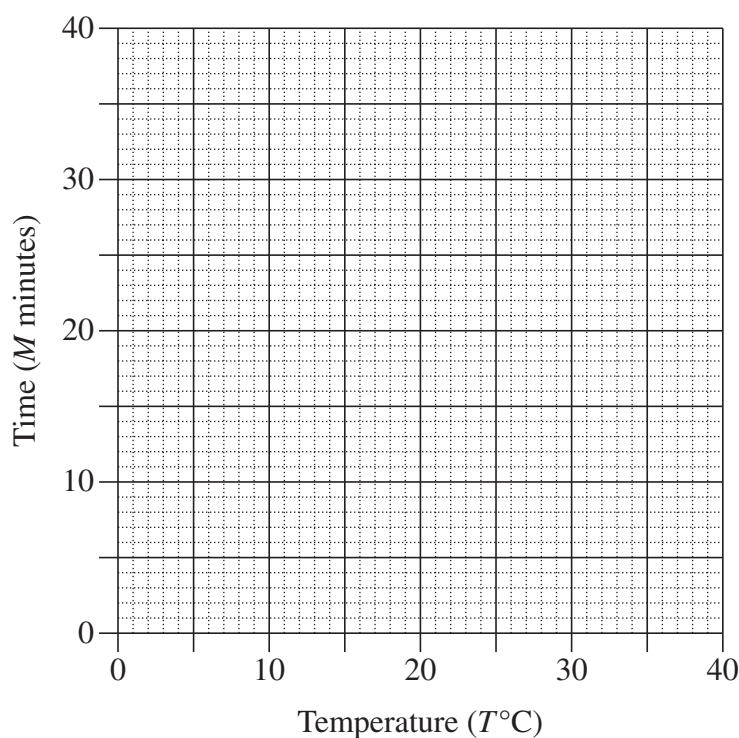
Question 25 (4 marks)

The table shows the future value of an annuity of $1.

Future values of an annuity of $1

YEARS	INTEREST RATE PER ANNUM			
	1%	*2%*	*3%*	*4%*
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

Zal is saving for a trip and estimates he will need $15 000. He opens an account earning 3% per annum, compounded annually.

(a) How much does Zal need to deposit every year if he wishes to have enough money for the trip in 4 years time? **2**

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(b) How much interest will Zal earn on his investment over the 4 years? Give your answer to the nearest dollar. **2**

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Question 26 (4 marks)

The diagram shows two right-angled triangles, *ABC* and *ABD*, **4**

where $AC = 35$ cm, $BD = 93$ cm, $\angle ACB = 41°$ and $\angle ADB = \theta$.
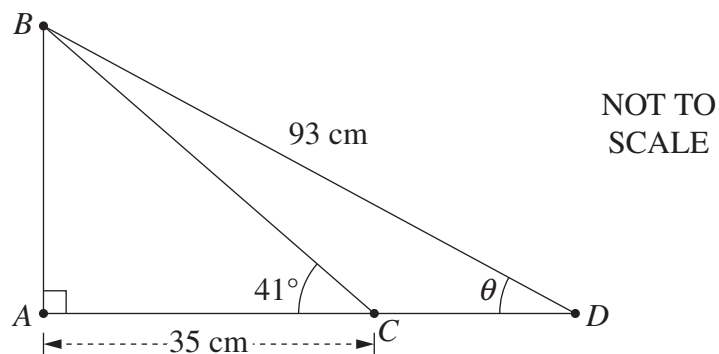
Calculate the size of angle θ, to the nearest minute.

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Question 27 (4 marks)

A company purchases a machine for \$50 000. The two methods of depreciation being considered are the declining-balance method and the straight-line method.

(a) For the declining-balance method, the salvage value of the machine after n years is given by the formula

$$S = V_0 \times (0.80)^n,$$

where S is the salvage value and V_0 is the initial value of the asset.

   (i) What is the annual rate of depreciation used in this formula? **1**

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   (ii) Calculate the salvage value of the machine after 3 years, based on the given formula. **1**

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(b) For the straight-line method, the value of the machine is depreciated at a rate of 12.2% of the purchase price each year. **2**

When will the value of the machine, using this method, be equal to the salvage value found in part (a) (ii)?

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Questions 16–27 are worth 41 marks in total

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Question 28 (4 marks)

A dam is in the shape of a triangular prism which is 50 m long, as shown.   **4**

Both ends of the dam, *ABC* and *DEF*, are isosceles triangles with equal sides of length 25 metres. The included angles *BAC* and *EDF* are each 150°.

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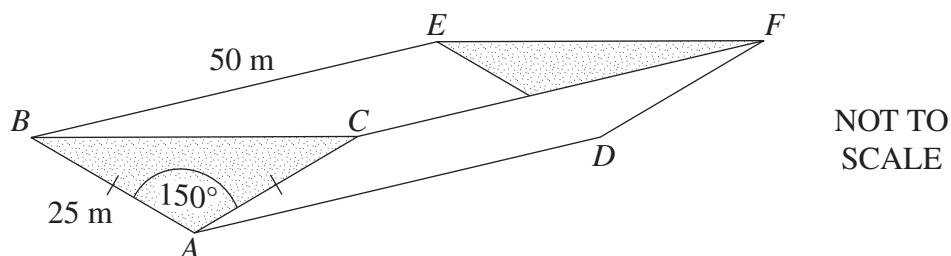
Calculate the number of litres of water the dam will hold when full.

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Question 29 (3 marks)

Sydney is 10 hours ahead of Coordinated Universal Time (UTC +10) and New York **3**
is 5 hours behind Coordinated Universal Time (UTC −5).

Tony travels from Sydney to New York. His plane leaves Sydney at 8:20 pm on
Wednesday local time and flies non-stop to New York.

The flight takes 20 hours and 24 minutes.

What time and day is it in New York when the plane lands?

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Question 30 (4 marks)

Eli is choosing between two investment options.

Option 1: Depositing a single amount of $40 000 today, earning interest of 1.2% per annum, compounded monthly.

Option 2: Depositing $1000 at the end of each quarter, earning interest of 2.4% per annum, compounded quarterly.

A table of future value interest factors for an annuity of $1 is shown.

N \ r	Interest rate per period as a decimal					
	0.002	0.006	0.020	0.024	0.060	0.240
10	10.09048	10.27437	10.94972	11.15211	13.18079	31.64344
20	20.38460	21.18211	24.29737	25.28909	36.78559	303.60062
30	30.88646	32.76227	40.56808	43.20983	79.05819	2640.91639
40	41.60026	45.05630	60.40198	65.92708	154.76197	22 728.80260

(a)   What is the value of Eli's investment after 10 years using Option 1?   **2**

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(b)   What is the difference between the future values after 10 years using Option 1 and Option 2?   **2**

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Question 31 (5 marks)

A wildlife park has 5 main attractions (A, B, C, D, E) connected by directional paths. A simple network is drawn to represent the flow through the park's paths. The number of visitors who can access each path at any one time is also shown.
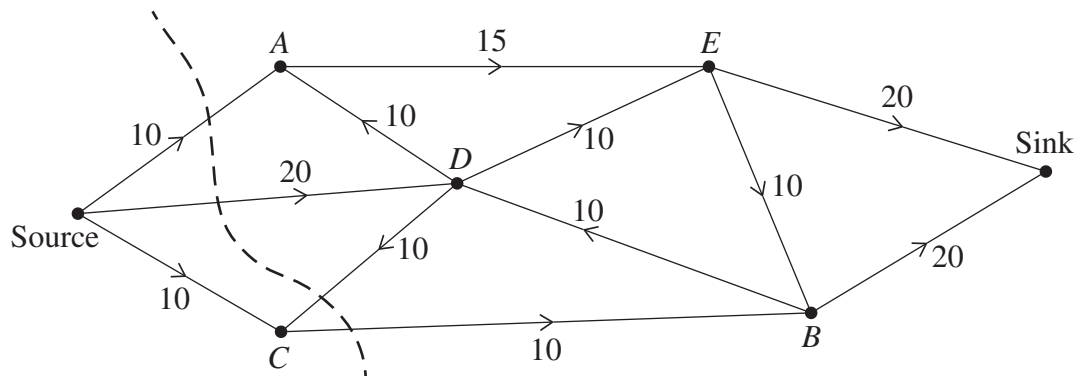
(a) What is the flow capacity of the cut shown? **1**

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(b) By showing a suitable cut on the diagram below, explain why the network's current maximum flow capacity is less than 40 visitors.

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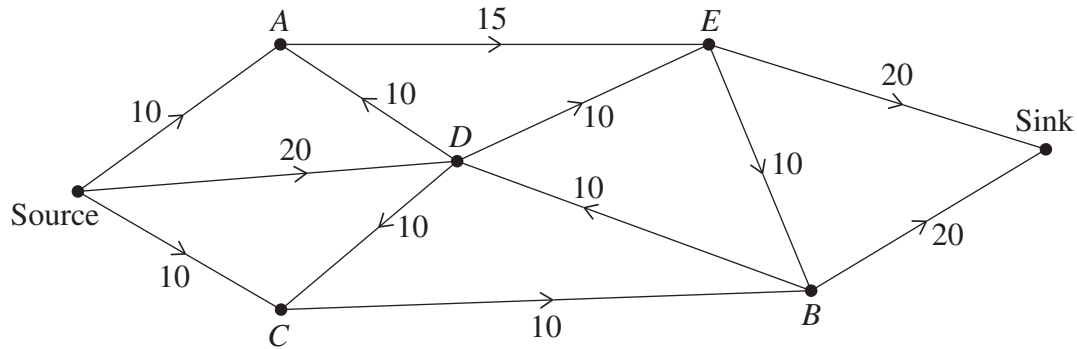
(c) One path is to be increased in capacity so that the overall maximum flow will be 40 visitors at any one time.

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Which path could be increased and by how much?

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End of Question 31

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Question 32 (4 marks)

The diagram shows a park consisting of a rectangle and a semicircle. The semicircle has a radius of 100 m. The dimensions of the rectangle are 200 m and 250 m.

A lake occupies a section of the park as shown. The rest of the park is a grassed section. Some measurements from the end of the grassed section to the edge of the lake are also shown.

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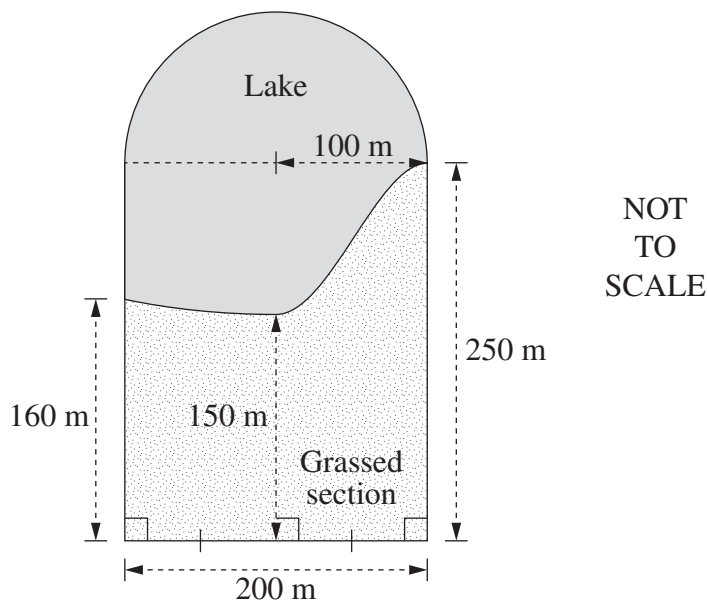
(a) Using two applications of the trapezoidal rule, calculate the approximate area of the grassed section.   2

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(b) Hence calculate the approximate area of the lake, to the nearest square metre.   2

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Question 33 (5 marks)

The diagram shows an aeroplane that was flying towards an airport at *A* on a bearing of 135°T. When it was at point *O*, 20 km away from the airport at *A*, the flight course was changed. The aeroplane landed at an airport at *B* directly south of *O*. The distance from *O* to *B* is 50 km.

N

O

20 km

•*A*

50 km

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•*B*
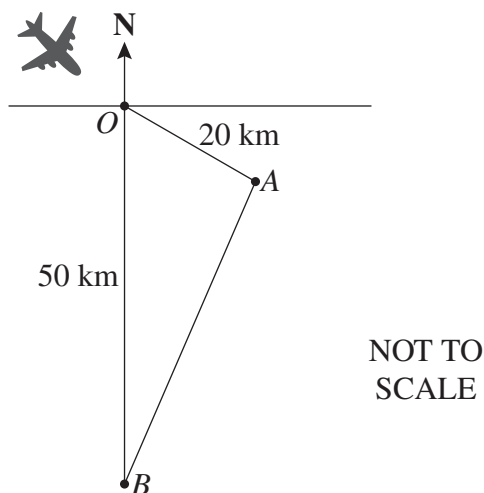
(a)   Show that the distance between the airport at *A* and the airport at *B* is 38.5 km, correct to 1 decimal place.   **2**

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(b)   Use the sine rule to find the angle *OBA* to the nearest degree.   **2**

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(c)   What is the bearing of the airport at *B* from the airport at *A*?   **1**

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Question 34 (4 marks)

A composite solid is shown. The top section is a cylinder with a height of 3 cm and **4**
a diameter of 4 cm. The bottom section is a hemisphere with a diameter of 6 cm. The
cylinder is centred on the flat surface of the hemisphere.
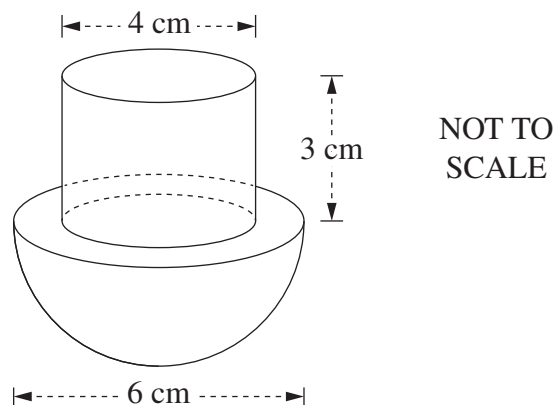
Find the total surface area of the composite solid in cm^2, correct to 1 decimal place.

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Question 35 (4 marks)

Jo is researching the relationship between the ages of teenage characters in television series and the ages of actors playing these characters.

4

After collecting the data, Jo finds that the correlation coefficient is 0.4564.

A scatterplot showing the data is drawn. The line of best fit with equation $y = -7.51 + 1.85x$, is also drawn.

Describe and interpret the data and other information provided, with reference to the context given.

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Question 36 (5 marks)

Frankie borrows $200 000 from a bank. The loan is to be repaid over 23 years at a rate of 7.2% per annum, compounded monthly. The repayments have been set at $1485 per month.

The interest charged and the balance owing for the first three months of the loan are shown in the spreadsheet below.

Month	Principal (at start of month)	Interest charged	Monthly repayment	Balance (at end of month)
1	$200 000	$1200	$1485	$199 715
2	$199 715	*A*	$1485	$199 428.29
3	$199 428.29	$1196.57	$1485	*B*

(a) What are the values of *A* and *B*? **2**

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(b) After 50 months of repaying the loan, Frankie decides to make a lump sum payment of $40 000 and to continue making the monthly repayments of $1485. The loan will then be fully repaid after a further 146 monthly repayments. **3**

How much less will Frankie pay overall by making the lump sum payment?

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Question 37 (3 marks)

The life span of batteries from a particular factory is normally distributed with a mean **3** of 840 hours and a standard deviation of 80 hours.

It is known from statistical tables that for this distribution approximately 60% of the batteries have a life span of less than 860 hours.

What is the approximate percentage of batteries with a life span between 820 and 920 hours?

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Please turn over

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Question 38 (3 marks)

A full container has 4.8 L of a mixture of cordial and water in the ratio 1 : 3.   **3**

After removing 1.2 L of the mixture, more water is added to refill the container.

What is the ratio of cordial to water in the final mixture?

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Section II extra writing space

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Mathematics Standard 1
Mathematics Standard 2

REFERENCE SHEET

Measurement

Limits of accuracy

Absolute error $= \dfrac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement − absolute error

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

$$A \approx \frac{h}{2}(d_f + d_l)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV(1 + r)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$
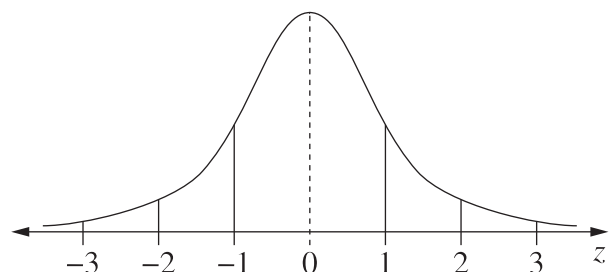
Statistical Analysis

An outlier is a score

   less than $Q_1 - 1.5 \times IQR$

   or

   more than $Q_3 + 1.5 \times IQR$

$$z = \frac{x - \mu}{\sigma}$$

Normal distribution

- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

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