2009 HSC Notes from the Marking Centre General Mathematics

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2009 HSC NOTES FROM THE MARKING CENTRE
GENERAL MATHEMATICS

Introduction

This document has been produced for the teachers and candidates of the Stage 6 General Mathematics course. It contains comments on candidate responses to the 2009 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2009 Higher School Certificate examination, the marking guidelines and other support documents which have been developed by the Board of Studies to assist in the teaching and learning of General Mathematics.

Teachers and students are advised that, in December 2008, the Board of Studies approved changes to the examination specifications and assessment requirements for a number of courses. These changes will be implemented for the 2010 HSC cohort. Information on a course-by-course basis is available on the Board's website at www.boardofstudies.nsw.edu.au/syllabus_hsc

General Comments

One of the main difficulties that markers continue to have is marking responses that involve an incorrect answer with little or no working shown. In these cases it is not possible to give part marks, since markers have no indication of the candidate's reasoning behind the solution. Candidates are advised to write their working down so that part marks can be awarded for some correct steps towards their answer. A simple example of this occurs when candidates have to round their answer to a certain degree of accuracy. Candidates should always write their calculator display before rounding their answer, and only round their answer in the last step of working, not in an earlier step. Markers can then see that candidates have rounded correctly, even if the answer is not correct.

Some questions required candidates to explain their answer and/or justify their result in words and/or by using calculations. This presented a problem for a significant number of candidates. They need to become familiar with appropriate terminology and read their answers after writing them to ensure that the answers make sense.

Candidates need to pay attention to the number of marks allocated to each part of a question so that they know how extensive their answers should be. Candidates should pay particular attention to the situation where a question asks them to justify with calculations or examples, and ensure that they respond appropriately.

Candidates should bring a ruler to the General Mathematics HSC examination for drawing graphs and diagrams accurately. Candidates should also take note of diagrams where 'Not to scale' is indicated, since in these cases measuring lines or angles to obtain a result will not be awarded any marks.

In the better responses, candidates:

- showed a clear, concise and appropriate method to solve each problem. Those who worked in a logical manner, clearly stated what they were doing and showed all necessary working were at an advantage compared to those who showed poor or no working, or who did not indicate where they were heading

- referred correctly to the formulae sheet, were familiar with it and used it carefully where necessary
- drew large, clear, well-labelled diagrams and included given information as well as information calculated while doing the question
- did not round off too early in their calculations
- articulated their explanations, either with the support of calculations or in clear, written form
- considered the reasonableness of their answers within the context of the question.

Section II

Question 23

(a)(i) Typical responses correctly used $\tan 38° = \frac{x}{25}$ to show the height was 19.5 m. Other responses correctly found the other angle (52°) and then used the sine rule to obtain the answer or used Pythagoras' Theorem to find the hypotenuse, and then used either the sine or cosine ratio. Both of these latter methods were correct but more time-consuming. Weaker incorrect responses used $\cos 38° = \frac{x}{25}$ and concluded incorrectly that an answer of 19.7 was close enough to 19.5. Several incorrect responses used a circular argument, finding the hypotenuse as 31.7 (assuming the 19.5 value) and then working back to find the short side as 19.5, assuming the hypotenuse was 31.7.

(ii) Better responses used $\tan \theta = \frac{19.5}{62}$. Many found the complement (73°) and subtracted it from 90° to obtain the correct answer. Many candidates found the hypotenuse using Pythagoras' Theorem and then used the cosine rule or other trigonometric relationships to find the angle. In weaker responses, candidates did not know which angle was the angle of depression and found the complement of the correct angle. Rounding was tested in this part, with many candidates losing a mark because they left their answers in degrees and minutes.

(b)(i) Better responses correctly stated that the number of possibilities was 10 000. A common error was to assume incorrectly that repetitions were not allowed. Careful consideration of the example given in the question would have avoided this mistake.

(ii) A large number of responses failed to show an understanding of probability. Many candidates found the number of possible combinations (2000) but did not express this as a probability. Other typical weaker responses used 2000/10 000, or simply wrote 'very unlikely' or similar. This was insufficient for the award of the mark. A correct fraction or percentage was required.

(c)(i) Better responses calculated the area correctly using rectangles. The use of a diagram indicating the areas being used was often a successful strategy. In weaker responses, candidates divided the shape incorrectly into smaller areas frequently using 1.35 (2.7 ÷ 2) as one of the lengths. Others either simply multiplied all the lengths together or tried to find the perimeter by adding all the lengths.

(ii) In better responses, candidates increased the area correctly by 10% and rounded-off their answer to the next whole box before calculating the cost of the tiles. In many typical weaker responses, candidates found the cost of the tiles first and increased this by 10%. Often these candidates failed to recognise the need to buy whole boxes. Others rounded-up to the number of boxes of tiles, before increasing by 10%. Some candidates were unable to increase correctly by 10%.

(d)(i) The correct total of the fees was \$12.50. A very common response was \$8.50 with candidates failing to recognise that the monthly account fee of \$4 needed to be included.

(ii) Some candidates failed to read the question correctly and offered combinations of services that the customer could obtain for the money saved. Others misinterpreted the question, offering answers such as \$2.99 or \$2.95.

Question 24

(a)(i) Most candidates identified the mode as 78. Common errors included omitting the stem which resulted in giving an answer of 8; calculating the range as 78 – 23 = 55; and misidentifying the score with the highest frequency, often by incorrectly selecting 44.

(ii) Most candidates calculated the median as $\frac{45+47}{2} = 46$. Common errors included providing the value for the mean, stating either 45 or 47, or listing both scores without calculating the average.

(b)(i) A significant number of incorrect responses came from assuming that the area chart did not provide cumulative totals, resulting in incorrectly calculated values for the total profit by adding values for India (2 million), Belgium (4 million) and USA (8 million), to give an incorrect answer of 14 million. Some candidates also chose to answer using a higher degree of accuracy, eg 7.9 million. A small number of candidates did not indicate knowledge that the vertical scale on the graph was given in millions.

(ii) Most candidates were successful in answering this part.

(c) Many candidates successfully provided examples of government decisions with clear descriptions of how the data might justify the decision, for example building a new primary school if the number of school-age children in the area was increasing or providing additional aged-care facilities if the number of people over a certain age was increasing. A number of candidates answered only one part of the question – providing a specific example or making reference to the data, but not both.

(d)(i) Very few candidates successfully answered this part of the question. Most of those who were successful used the statement provided in the stem of the question, stating the equation as $x + y = 200$, rather than using the graph. Those attempting to calculate the equation using $y = mx + b$ and subsequent calculations from the graph were generally not successful. Common errors included incorrect reading of the scale of the vertical axis, incorrect calculation of the gradient, or failure to recognise the negative slope.

(ii) Many candidates correctly identified the segment as the section of the line that satisfied all three production limits stated in the stem of the question.

(iii) This part was not well answered by many candidates. A significant number incorrectly identified the coordinates of *C* as (120,75), ignoring the detail that the factory makes 200 shoes each week. An equally significant number of candidates wrongly calculated $24 \times 120 = 2280$ and $15 \times 150 = 2450$, rather than using the given formula to calculate the two profit values. Other candidates calculated the profit at either *B* or *C*, but did not go on to compare the profits at both points.

(e)(i) The overwhelming majority of candidates identified the annual amount of depreciation as \$1200. While the use of the formula was common, a small number of candidates also successfully used trial and error to find this amount.

(ii) Candidates did not answer this part well, with many simply restating the question as an explanation for why the salvage value never reaches zero. Many candidates demonstrated their knowledge of the declining balance formula, typically finding the salvage value of the computer after 3 years. In the better responses, candidates typically justified explanations with a combination of calculations after longer periods of time (such $n = 20$ or $n = 100$) and descriptions of the nature of exponential functions and graphs. A small number of candidates also correctly used logarithms to justify an explanation. Other candidates correctly argued that the declining balance formula results in a value of zero only if either the rate of depreciation is 100% or the purchase price is \$0.

Question 25

(a) Incorrect responses included incorrect expansion followed by correct collection of like terms. Many candidates assumed the expression was an equation.

(b) Many candidates correctly divided 50 (mg) by 2.5 × 106. However, most candidates did the division 'upside-down'. Very few correctly converted from mg to g, and many candidates, if they correctly converted their answers, did not express their answer in correct scientific notation.

(c)(i) Some candidates correctly used Simpson's Rule. However, many candidates had trouble transposing the correct formula from the Formula Sheet. Better responses calculated the area of the lake directly while other responses subtracted from a rectangle or split the lake with a horizontal line.

(ii) Very few candidates successfully made the correct conversions from metres squared to centimetres squared.

(d)(i) Better responses included a diagram displaying *z*-scores on a distribution.

(ii) Many candidates failed to use the answer to (d)(i) above. A common error was omitting to subtract the 0.15% area pertaining to more than 3 standard deviations above the mean, leading to an answer of 84%.

Question 26

(a)(i) Many candidates displayed a good understanding of the meaning of interquartile range.

A common error was to give 6 as the answer (range rather than interquartile range). Some candidates gave the answer 2 to 6, not understanding that 'range' in this context means a single numerical value.

Teachers are advised to emphasise the different uses of the word 'range' in different contexts.

(ii) Many candidates obtained the correct answer of 75%.

A common error was 83%, based on 5/6 of the length of the graph, indicating a misunderstanding of 'quartile'.

(iii) There were few correct responses to this part. Many of the incorrect responses simply rephrased the question, changed the data in some way, spoke about the box and whisker plot or compared quartiles and missed the point of the question.

(b)(i) Many candidates obtained a mark in this part. However, candidates are reminded that 'to show' means to write a numerical expression with each line of working. It was clear that many candidates knew the solution to this question but neglected to set out each step and therefore failed to gain full marks.

(ii) There were many successful responses to this part. A common incorrect response of '5 am Monday' resulted from candidates subtracting, rather than adding, 16 hours. Candidates are reminded to read the question carefully as, in this case, they were required to state a time and day.

(iii) Candidates had difficulty recognising whether to add or subtract the 14-hour and 16-hour time differences.

(c)(i) Many candidates did not recognise the simplicity of this question and attempted to use the compound interest or present value formulae, etc. Others calculated simple interest and added it onto the \$300 000. Some responses which obtained the correct answer (\$528 000) then either added or subtracted \$300 000.

(ii) Although this part was reasonably well done, there was clearly a poor understanding of the terms *interest*, *repayment* and *balance*.

Some candidates calculated A correctly but did not understand the process of adding the monthly interest and deducting the monthly repayment to ascertain the end-of-month balance. Many subtracted both of these amounts.

A common incorrect value for A was \$1500, obtained by assuming that the monthly interest for month 2 would be the same as for month 1, and therefore simply copying \$1500 from the table.

(iii) Many candidates did not correctly substitute into the formula. Weaker responses struggled with the conversions of n and r, while others did not recognise what N and M represented in the equation.

A common error was to substitute $300 000 (the amount borrowed) for the amount of each repayment.

There were very few successful responses for this part. Most candidates who successfully answered (c)(iii)(1) above calculated the amount of each repayment correctly.

Question 27

(a)(i) Most candidates calculated the correct answer, or at least showed the numerical expression they intended to use to calculate the answer.

Common errors included substituting into a financial formula instead of using the table then using 0.3 instead of 0.03, or continuing beyond the correct answer by adding another $5000 to it.

(ii) Most candidates neglected to use the given table and used the Future Value formula or, in some cases, the Present Value formula.

A common error was to multiply instead of divide 407 100 by 8.1420 or, if using a formula, substituting 407 100 for M.

(iii) Most candidates failed to realise that the given table still applied to this part of the question and attempted to use a formula.

In using a formula, most had problems finding both the correct value for r and for n. Most candidates realised that a total of $8000 was invested.

A common error was to find the total value of the investment instead of subtracting $8000 to obtain the interest as the answer required.

(b) Many candidates did not attempt any of this section of the question despite its straightforward nature and the relatively familiar context. Some candidates used incorrect formulae such as the cosine rule but with sine instead of cosine, or the area rule using cosine instead of sine, without any apparent reason.

(b)(i) Many candidates could not calculate the correct bearing. Some showed little knowledge of the concept of a bearing, giving a distance instead. Some candidates assumed that the diagram was an isosceles triangle.

(ii) Many candidates using the cosine rule did not realise that they had to obtain the square root of their answer to get the final distance. A common error was to assume the angles in the diagram were right angles, then proceed to use Pythagoras' Theorem.

(iii) Many candidates did not interpret the question correctly, instead explaining the reason for a 'no-go zone' in a yacht race instead of finding its area.

A common error was to assume that the 'no-go' area was an isosceles triangle and attempt to find its area using $A = 1/2bh$.

(c) There was a variety of mixed responses for this question. Most candidates attempted the question, but very few gained full marks. Most responses stated the correct probability of Mary's winning a prize. Very few candidates successfully calculated the probability of Jane's winning. Most success was gained by those drawing some kind of probability tree. Most candidates realised that the closer the probability of an event approaches 1, the more likely it is to occur. However, some candidates did not see anything wrong with obtaining probabilities greater than 1.

Question 28

(a)(i) A sketch of the appropriate part of the given graph was required, using a set of axes. The main objectives were for a concave-up curve starting at the origin and remaining in the first quadrant.

Weaker responses had difficulties in maintaining correct concavity, or in not starting at the origin, or in reproducing the graph from the examination paper, or in not taking enough care with accuracy.

(ii) The question required candidates to accurately read the value for stopping distance at 40 km/hr from the graph and substitute this value into the given formula to obtain the value at 70 km/hr. Alternatively candidates could calculate the stopping distance at 40 km/hr by substitution into the formula.

Candidates needed to read the scale on the graph accurately. A common error in weaker responses gave 42 as the stopping distance at a speed of 40 km/hr, i.e. 98 – 42 = 56 m.

The mathematical term 'difference' was poorly comprehended. A variety of explanations described the impact of driving at different speeds rather than a mathematical calculation for finding the difference.

Many responses gave incorrect calculations from the formula for the stopping distance at 70 km/hr, such as $(0.01 \times 70)^2 + 0.7 \times 70 = 49.49$ or $0.01 \times 70^2 + 0.07 \times 70 = 53.9$.

(b)(i) This section was well answered, with typical responses correctly stating that the graph shows positive correlation.

(ii) Candidates were required to find an equation for the given line of best fit, providing a correct gradient and y-intercept for the general equation of a straight line. Some difficulties evidently arose due to the fact that the horizontal axis did not start at the origin, meaning that additional work was required to calculate the y-intercept. This was taken into consideration in the marking process.

There was some recognition of the requirements for $y = mx + b$, but limited success in finding the correct components of this equation. Common errors were incorrect values of $m = \frac{\text{rise}}{\text{run}}$; misreading the graph scale to find b; and the need to use values from the line of best fit (rather than from the table of observed values) in their calculations. Some candidates did not put the line into the form of an equation.

Note: Candidates should always write their calculator display before rounding their answer, and only round their answer in the last step of working, not in an earlier step. An instance of rounding off too soon became evident when candidates inserted the rounded-down value $m = 0.2$ in their equation instead of the correct gradient $m = 0.23\ldots$.

(c) This direct variation question required the use of $h = kd^2$ to solve for a specific height. Better responses recognised the relationship of height to the square of the distance and managed to successfully manipulate the proportionality to obtain $h = 17.8$ m. Weaker responses attempted linear proportionality using $h = kd$, obtaining $h = 5.333\ldots$. Other variants were attempted, such as the inverse form $h = \frac{k}{d^2}$, which earned some marks depending on the success of the calculation.

(d) This question was a variation on the probability of throwing two dice. Candidates were asked to explain the closeness of an observation to the theoretical probability, given two different experiments. A guiding statement was provided that the explanation should be based on finding the sample space, and on giving a comparison with theoretical probabilities.

In mathematics, the word ‘explain’ requires mathematical calculations to support an argument. Weaker responses to this question gave a lengthy explanation without any mathematical justification.

Some candidates set up a correct sample space but did not link it to the theoretical outcomes for 18 throws. A number of candidates interpreted the sample space as comprising only 21 elements, failing to recognise that 2–6 and 6–2 were different outcomes.