NSW Education Standards Authority

Mathematics Standard 2

General Instructions
- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 15 marks (pages 2–9)
- Attempt Questions 1–15
- Allow about 25 minutes for this section

Section II – 85 marks (pages 13–44)
- Attempt Questions 16–41
- Allow about 2 hours and 5 minutes for this section
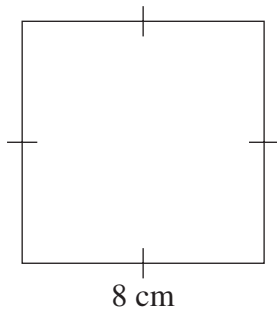
Section I

15 marks
Attempt Questions 1–15
Allow about 25 minutes for this section

Use the multiple-choice answer sheet for Questions 1–15.

1   Which of the following shapes has the largest perimeter?

A.

8 cm

B.

3 cm

11 cm

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C.

10 cm

D.

3 cm

4 cm

9 cm
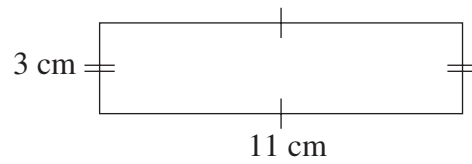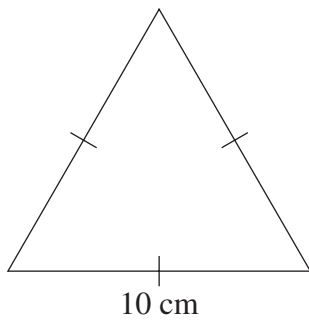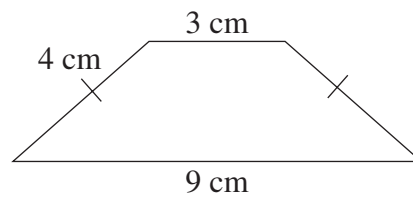
2 Consider the network diagram.
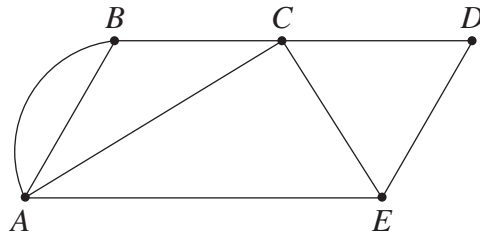
What is the sum of the degrees of all the vertices in this network?

A.   5

B.   8

C.   14

D.   16

3 The stem-and-leaf plot shows the number of goals scored by a team in each of ten netball games.

0	6 8
1	2 4 5
2	1 5 5 9
3	5

What is the mode of this dataset?

A.   5

B.   18

C.   25

D.   29

4 Three years ago an appliance was valued at $2467. Its value has depreciated by 15% each year, based on the declining-balance method.

What is its salvage value today, to the nearest dollar?

A. $952

B. $1110

C. $1357

D. $1515

5 Peter currently earns $21.50 per hour. His hourly wage will increase by 2.1% compounded each year for the next four years.

What will his hourly wage be after four years?

A. $21.50(1.21)^4$

B. $21.50(1.021)^4$

C. $21.50 + 21.50 \times 0.21 \times 4$

D. $21.50 + 21.50 \times 0.021 \times 4$

6 Suppose $a = \dfrac{b}{7}$, where $b = 22$.

What is the value of a, correct to three significant figures?

A. 3.14

B. 3.15

C. 3.142

D. 3.143

7   The number of downloads of a song on each of twenty consecutive days is shown in the following graph.
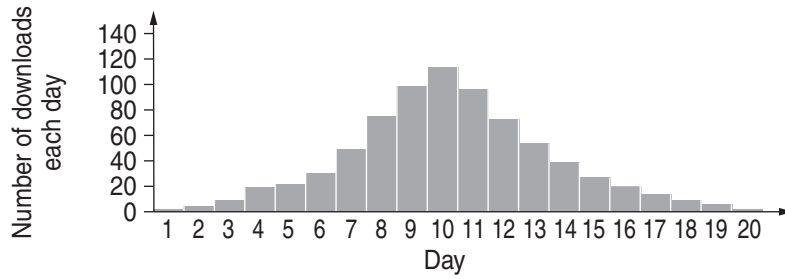
Which of the following graphs best shows the cumulative number of downloads up to and including each day?

A.
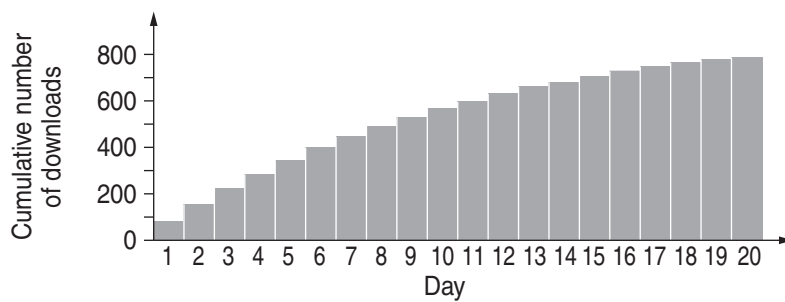
B.
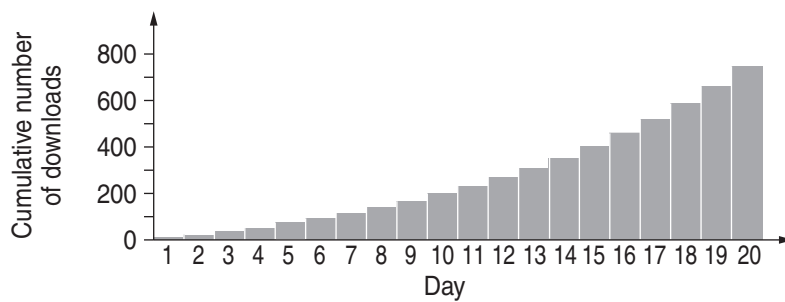
C.
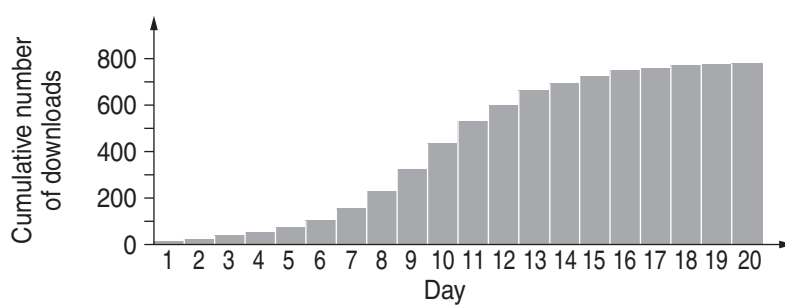
D.
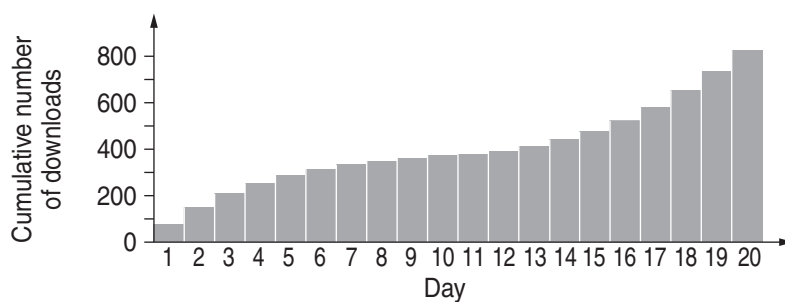
8   On a test, Zac's mark corresponded to a z-score of 2. The test scores had a mean of 63 and a standard deviation of 8.

What was Zac's actual mark on the test?

A.   65

B.   67

C.   73

D.   79

9   A student is thinking of a number. Let the number be x.

When the student subtracts 8 from this number and multiplies the result by 3, the answer is 2 more than x.

Which equation can be used to find x?

A. $3(x - 8) = 2x$

B. $3x - 8 = 2x$

C. $3(x - 8) = x + 2$

D. $3x - 8 = x + 2$

10   Which of the following best represents the graph of $y = 10(0.8)^x$?

A.

B.

C.

D.
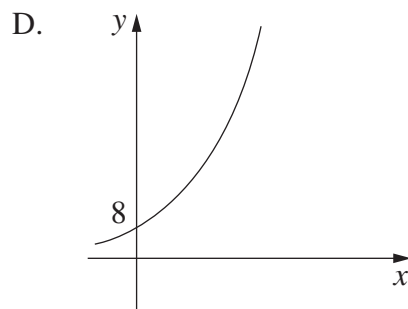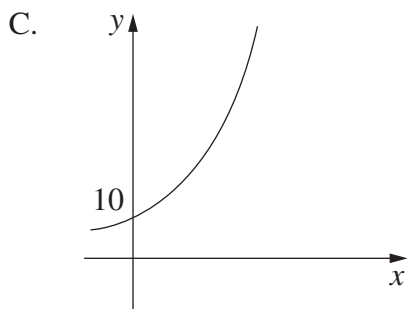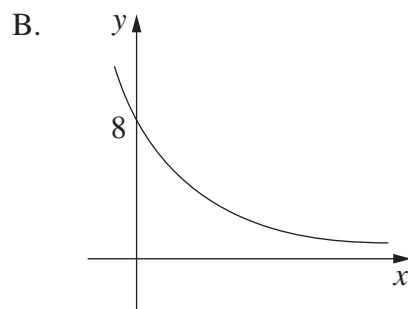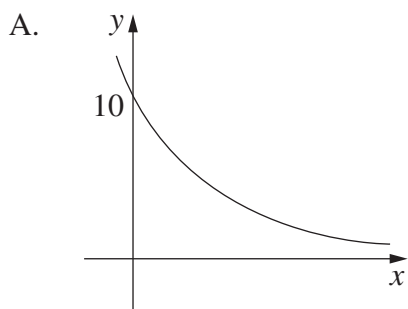
11 There are 8 chocolates in a box. Three have peppermint centres (P) and five have caramel centres (C).

Kim randomly chooses a chocolate from the box and eats it. Sam then randomly chooses and eats one of the remaining chocolates.

A partially completed probability tree is shown.
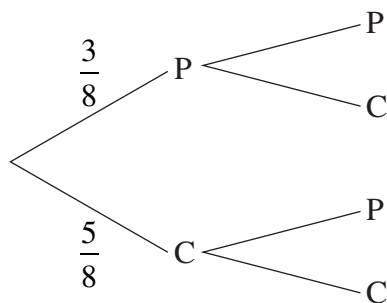
What is the probability that Kim and Sam choose chocolates with different centres?

A. $\dfrac{15}{64}$

B. $\dfrac{15}{56}$

C. $\dfrac{15}{32}$

D. $\dfrac{15}{28}$

12 A block of land is represented by the shaded region on the number plane. All measurements are in kilometres.
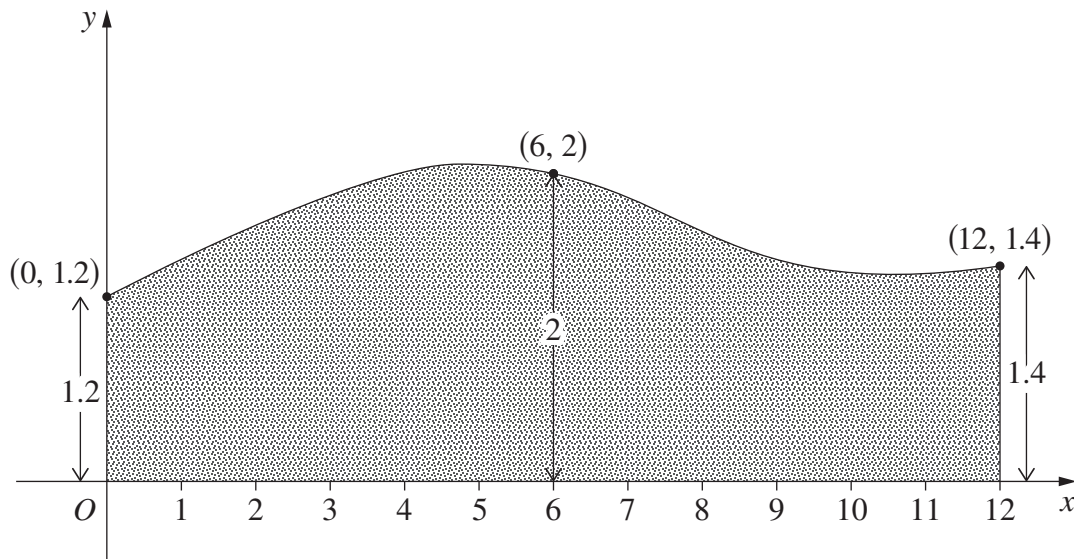
Which of the following is the approximation for the area of this block of land in square kilometres, using two applications of the trapezoidal rule?

A. 9.9

B. 19.8

C. 39.6

D. 72

13 The time taken to clean a warehouse varies inversely with the number of cleaners employed.

It takes 8 cleaners 60 hours to clean a warehouse.

Working at the same rate, how many hours would it take 10 cleaners to clean the same warehouse?

A. 45

B. 48

C. 62

D. 75

14 Consider the diagram below.

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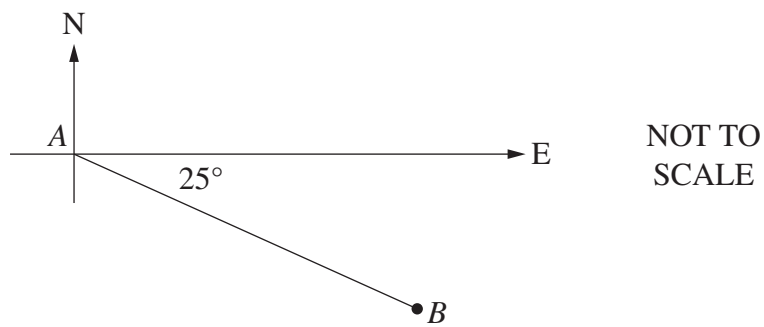
What is the true bearing of *A* from *B*?

A. 025°

B. 065°

C. 115°

D. 295°

15 A total of 11 400 people entered a running race. The ratio of professional runners to amateurs was $3:16$. All the professional runners completed the race while 600 of the amateurs did not complete the race.

For those who completed the race, what is the ratio, in simplest form, of professional runners to amateurs?

A. $1:2$

B. $1:5$

C. $1:8$

D. $1:19$

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Centre Number

Student Number

Mathematics Standard 2

Section II Answer Booklet 1

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Section II

85 marks
Attempt Questions 16–41
Allow about 2 hours and 5 minutes for this section

Booklet 1 — Attempt Questions 16–33 (55 marks)
Booklet 2 — Attempt Questions 34–41 (30 marks)

Instructions

• Write your Centre Number and Student Number at the top of this page.

• Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

• Your responses should include relevant mathematical reasoning and/or calculations.

• Extra writing space is provided on pages 30–32. If you use this space, clearly indicate which question you are answering.

Please turn over

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Question 16 (2 marks)

The volume, V, of a sphere is given by the formula

$$V = \frac{4}{3}\pi r^3,$$

where r is the radius of the sphere.

A tank consists of the bottom half of a sphere of radius 2 metres, as shown.

2
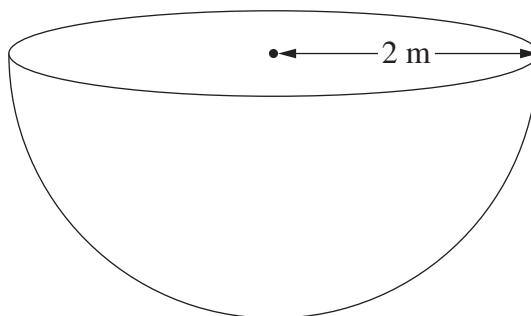
Find the volume of the tank in cubic metres, correct to one decimal place.

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Question 17 (2 marks)

The five-number summary of a dataset is given.

2

Lowest score = 1

Lower quartile $(Q_1) = 4$

Median $(Q_2) = 7$

Upper quartile $(Q_3) = 10$

Highest score = 20

Is 20 an outlier? Justify your answer with calculations.

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Question 18 (2 marks)

The fuel consumption for a car is 6.7 litres/100 km. On a road trip, the car travels a distance of 1560 km and the fuel cost is $1.45 per litre.

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What is the total fuel cost for this trip?

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– 15 –

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Question 19 (2 marks)

Adam purchased some office furniture five years ago. It depreciated by $2300 each year based on the straight-line method of depreciation. The salvage value of the furniture is now $7500.

2

Find the initial value of the office furniture.

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Question 20 (3 marks)

City *A* is in Sweden and is located at (58°N, 16°E). Sydney, in Australia, is located at (33°S, 151°E).

3

Robert lives in Sydney and needs to give an online presentation to his colleagues in City *A* starting at 5:00 pm Thursday, local time in Sweden.

What time and day, in Sydney, should Robert start his presentation?

It is given that 15° = 1 hour time difference. Ignore daylight saving.

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Question 21 (3 marks)

Julie invests $12 500 in a savings account. Interest is paid at a fixed monthly rate. At the end of each month, after the monthly interest is added, Julie makes a deposit of $500.

3

Julie has created a spreadsheet to show the activity in her savings account. The details for the first 6 months are shown.

Month	Amount in account at beginning of month	Monthly interest	Deposit	Amount in account at end of month
1	12 500.00	18.75	500	13 018.75
2	13 018.75	19.53	500	13 538.28
3	13 538.28	20.31	500	14 058.59
4	14 058.59	21.09	500	14 579.68
5	14 579.68	21.87	500	15 101.55
6	15 101.55	22.65	500	15 624.20
7			500	

By finding the monthly rate of interest, complete the final row above for the 7th month.

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Question 22 (3 marks)

The table shows the income tax rates for the 2020–2021 financial year.   **3**

Taxable income	Tax payable on this income
0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

William has a gross annual salary of $84 000. He has allowable tax deductions of $900 for home-office equipment and $474 for union fees. William must also pay a Medicare Levy of 2% of his taxable income.

Calculate the total tax payable by William including the Medicare Levy.

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Question 23 (4 marks)

The network diagram shows the travel times in minutes along roads connecting a number of different towns.
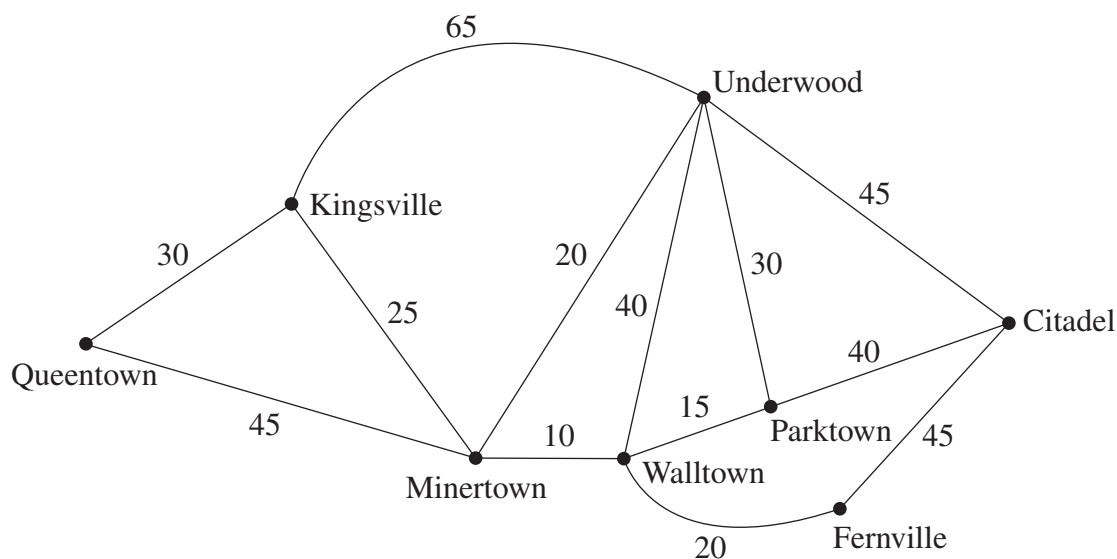
(a)   Draw a minimum spanning tree for this network and determine its length.   **3**

Length of minimum spanning tree = ...

(b)   How long does it take to travel from Queentown to Underwood using the fastest route?   **1**

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Question 24 (4 marks)

A population, P, is to be modelled using the function $P = 2000(1.2)^t$, where t is the time in years.

(a) What is the initial population? **1**

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(b) Find the population after 5 years. **1**

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(c) On the axes below, draw the graph of the population against time, showing the points at $t = 0$ and at $t = 5$. **2**

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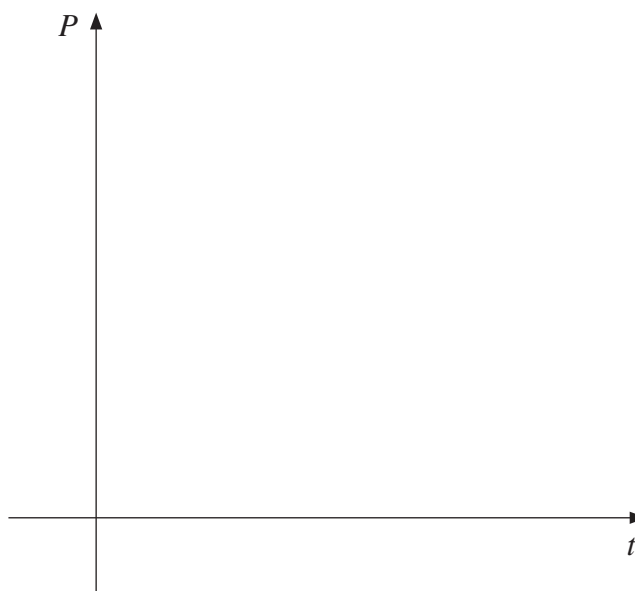
Question 25 (4 marks)

A rectangular sportsground has been drawn to scale on a 1-cm grid as shown. The scale used is $1 : 3000$.

4
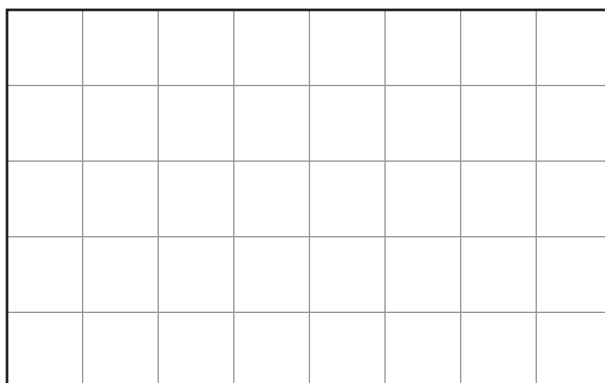
Kerry took 12 minutes to walk around the perimeter of this sportsground.

What was Kerry's average speed in kilometres per hour?

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Question 26 (4 marks)

Nina plans to invest $35 000 for 1 year. She is offered two different investment options.

Option *A*: Interest is paid at 6% per annum compounded monthly.

Option *B*: Interest is paid at *r*% per annum simple interest.

(a) Calculate the future value of Nina's investment after 1 year if she chooses Option *A*. **2**

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(b) Find the value of *r* in Option *B* that would give Nina the same future value after 1 year as for Option *A*. Give your answer correct to two decimal places. **2**

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Question 27 (4 marks)

The price and the power consumption of two different brands of television are shown.

Television A	*Television B*
Price: $900	Price: $921.90
Power: 176 W	Power: 160 W

The average cost for electricity is 25c/kWh. A particular family watches an average of 3 hours of television per day.

(a) The annual cost of electricity for Television *A* for this family is $48.18.   **2**

For this family, what is the difference in the annual cost of electricity between Television *A* and Television *B*?

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(b) For this family, how many years will it take for the total cost of buying and using Television *A* to be equal to the total cost of buying and using Television *B*?   **2**

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Question 28 (3 marks)

A salesperson is interested in the relationship between the number of bottles of lemonade sold per day and the number of hours of sunshine in the day.

The diagram shows the dataset used in the investigation and the least-squares regression line.
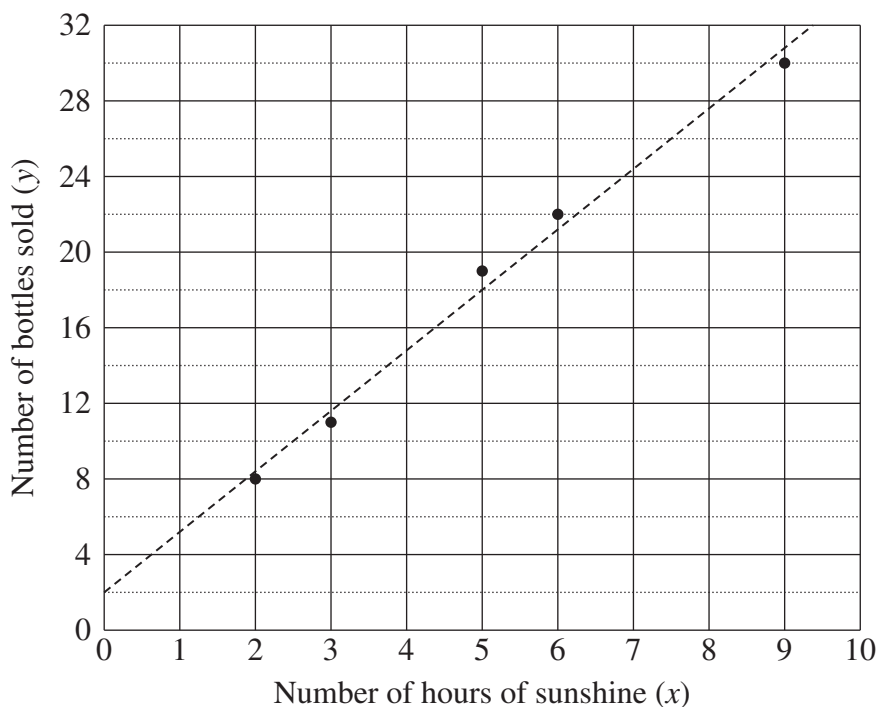
(a) Find the equation of the least-squares regression line relating to the dataset. **2**

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(b) Suppose a sixth data point was collected on a day which had 10 hours of sunshine. On that day 45 bottles of lemonade were sold. **1**

What would happen to the gradient found in part (a)?

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Question 29 (2 marks)

Solve $x + \dfrac{x-1}{2} = 9$.　　　　　　　　　　　　　　　　　　　　　　　**2**

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Question 30 (2 marks)

Ariana owns 1500 shares in a company. The market price for each share is $27.　**2**
Ariana's total dividend from these shares is $810.

Calculate the dividend yield for her shares.

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Questions 16–30 are worth 44 marks in total

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Present value interest factors for an annuity of $1 for various interest rates (r) and numbers of periods (N) are given in the table.

2

Table of present value interest factors

r / N	*Interest rate per period as a decimal*			
	0.001	0.00125	0.0015	0.00175
300	259.07072	250.03980	241.43789	233.24180
330	280.95771	270.26900	260.13532	250.52386
360	302.19816	289.75411	278.01062	266.92278

A bank lends Martina $500 000 to purchase a home, with interest charged at 1.5% per annum compounding monthly. She agrees to repay the loan by making equal monthly repayments over a 30-year period.

How much should the monthly payment be in order to pay off the loan in 30 years? Give your answer correct to the nearest cent.

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Question 32 (5 marks)

A right-angled triangle *XYZ* is cut out from a semicircle with centre *O*. The length of the diameter *XZ* is 16 cm and $\angle YXZ = 30°$, as shown on the diagram.

Y

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30°

X

O

Z

—16 cm—
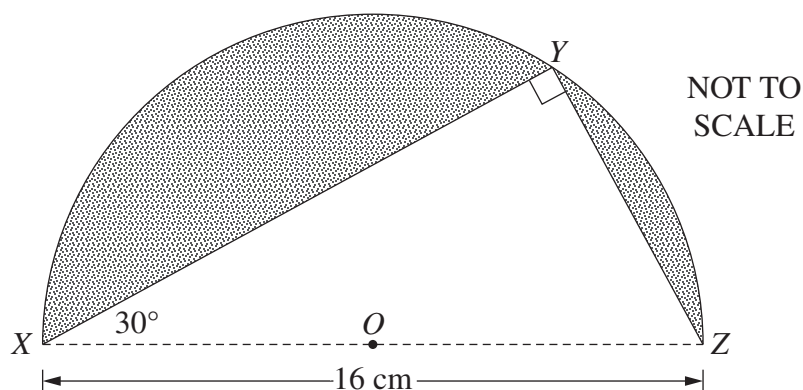
(a) Find the length of *XY* in centimetres, correct to two decimal places. **2**

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(b) Hence, find the area of the shaded region in square centimetres, correct to one decimal place. **3**

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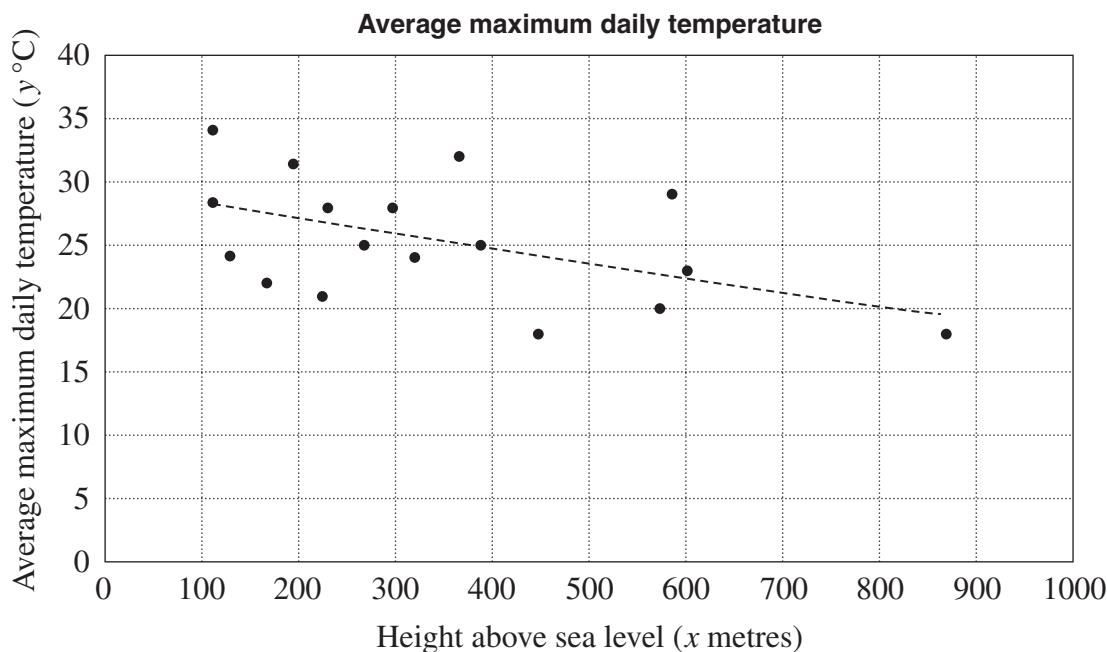..

Question 33 (4 marks)

For a sample of 17 inland towns in Australia, the height above sea level, x (metres), and the average maximum daily temperature, y (°C), were recorded.

The graph shows the data as well as a regression line.

Average maximum daily temperature

The equation of the regression line is $y = 29.2 - 0.011x$.

The correlation coefficient is $r = -0.494$.

(a)　(i)　By using the equation of the regression line, predict the average maximum daily temperature, in degrees Celsius, for a town that is 540 m above sea level. Give your answer correct to one decimal place.　**1**

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　　(ii)　The gradient of the regression line is −0.011. Interpret the value of this gradient in the given context.　**2**

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Question 33 continues on page 29

– 28 –

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(b)   The graph below shows the relationship between the latitude, x (degrees south),   **1**
       and the average maximum daily temperature, y (°C), for the same 17 towns, as
       well as a regression line.

Average maximum daily temperature
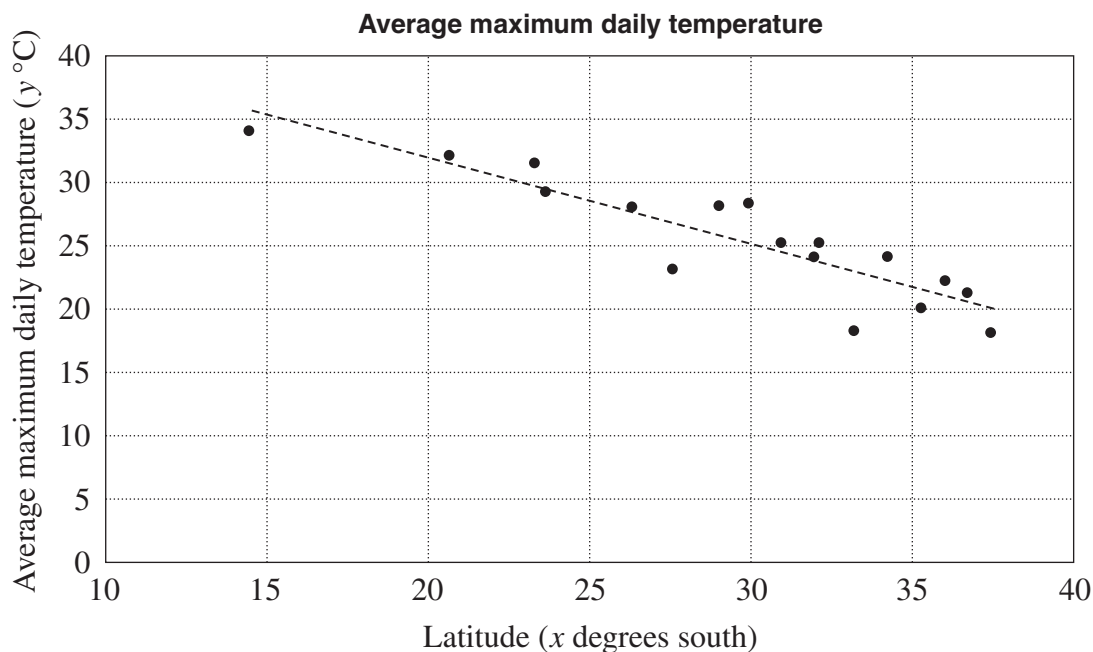
The equation of the regression line is $y = 45.6 - 0.683x$.

The correlation coefficient is $r = -0.897$.

Another inland town in Australia is 540 m above sea level. Its latitude is
28 degrees south.

Which measurement, height above sea level or latitude, would be better to use
to predict this town's average maximum daily temperature? Give a reason for
your answer.

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End of Question 33

Proceed to Booklet 2 for Questions 34–41

Section II extra writing space

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Section II extra writing space

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Centre Number

Student Number

Mathematics Standard 2

Section II Answer Booklet 2

Booklet 2 — Attempt Questions 34–41 (30 marks)

Instructions
- Write your Centre Number and Student Number at the top of this page.

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

- Your responses should include relevant mathematical reasoning and/or calculations.

- Extra writing space is provided on page 44. If you use this space, clearly indicate which question you are answering.

Please turn over

Do NOT write in this area.

Question 34 (4 marks)

In a park the only animals are goannas and emus. Let x be the number of goannas and let y be the number of emus.

4

The number of goannas plus the number of emus in the park is 31. Hence $x + y = 31$.

Each goanna has four legs and each emu has two legs. In total the emus and goannas have 76 legs.

By writing another relevant equation and graphing both equations on the grid on the following page, find the number of goannas and the number of emus in the park.

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Question 34 (continued)
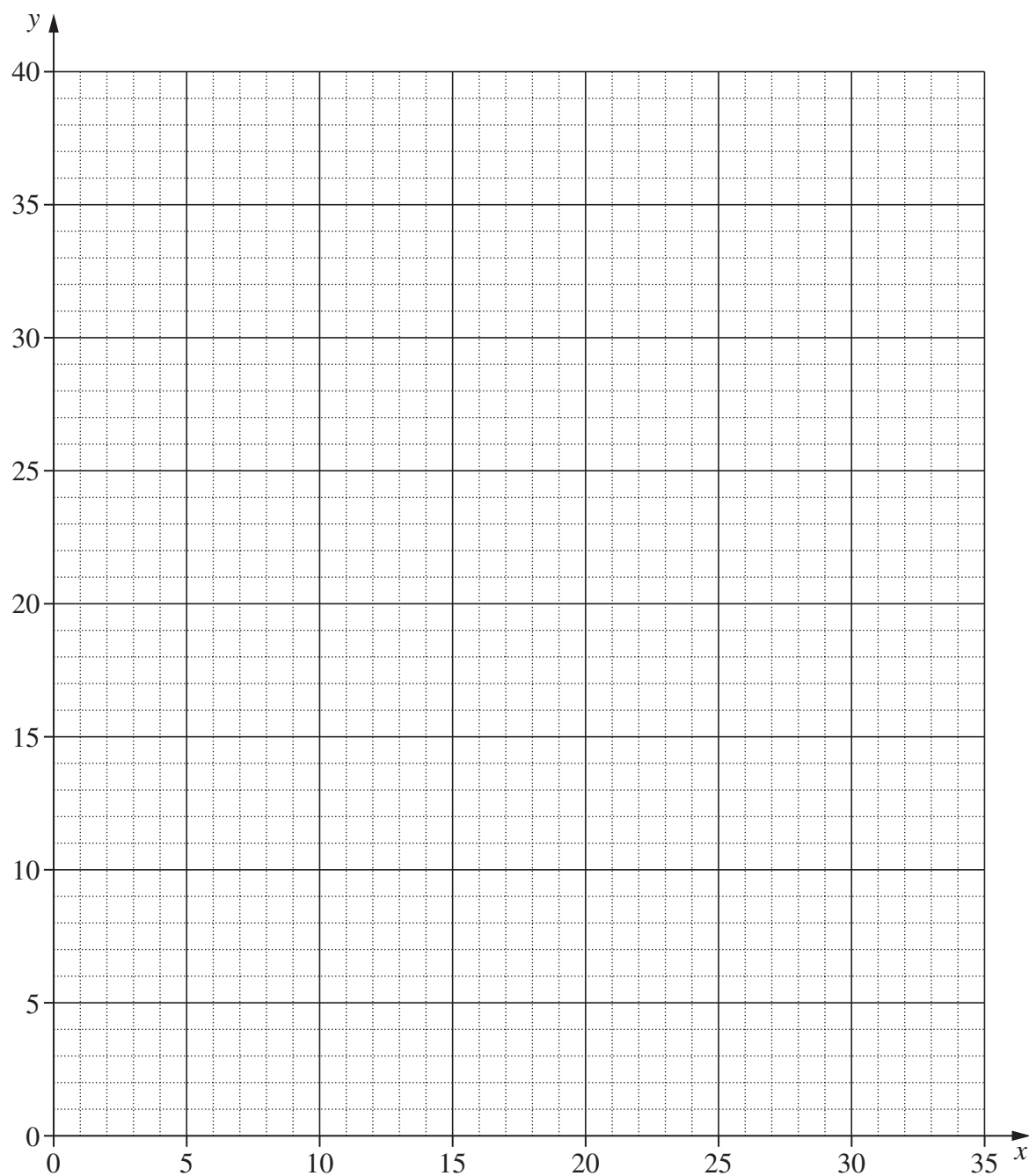
Number of goannas = ...

Number of emus = ...

**End of Question 34**

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Question 35 (3 marks)

A publisher sells a book for $10. At this price, 5000 copies of the book will be sold and the revenue raised will be $5000 \times 10 = \$50\,000$.

The publisher is considering increasing the price of the book. For every dollar the price of the book is increased, the publisher will sell 50 fewer copies of the book.

If the publisher charges $(10 + x)$ dollars for each book, a quadratic model for the revenue raised, R, from selling the books is

$$R = -50x^2 + 4500x + 50\,000.$$

A graph of this quadratic model for revenue is shown. A dashed line is used for values of x which are not relevant to the practical context of this problem.
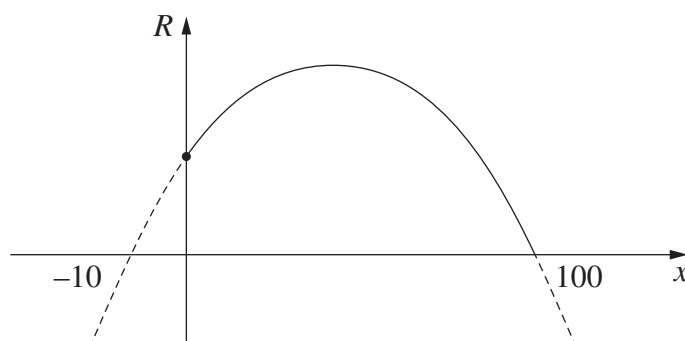
(a)   By first finding a suitable value of x, find the price the publisher should charge   **2**
for each book to maximise the revenue raised from sales of the book.

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(b)   Find the value of the intercept of the parabola with the vertical axis.   **1**

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Question 36 (4 marks)

A project requires completion of 11 tasks *A*, *B*, *C*, …, *K*.

A network diagram for the project giving the completion time for each task, in minutes, is shown.
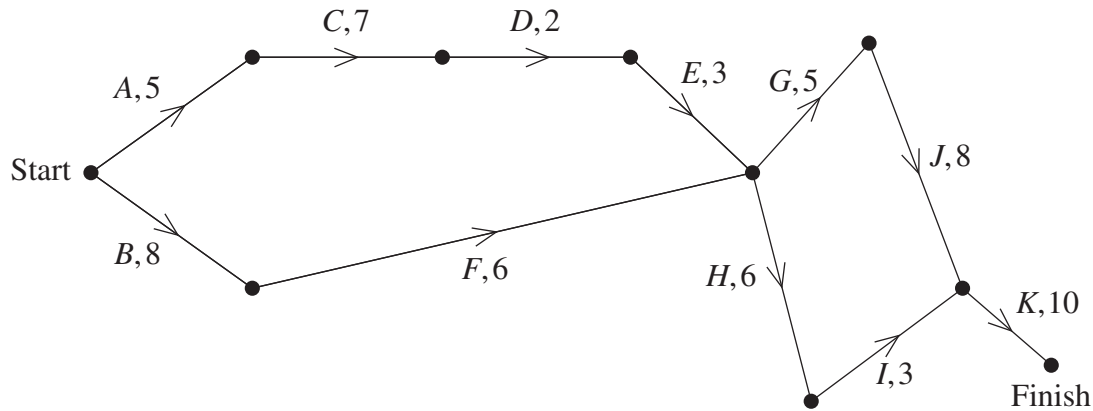
(a) Find the minimum time to complete the project. **1**

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(b) State the critical path for this project. **1**

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(c) A new task, *X*, is to be added to the project. The earliest starting time for *X* is 17 minutes, the latest starting time for *X* is 18 minutes and *X* has a completion time of 12 minutes. **2**

Add task *X* to the given network diagram above AND state the float time for this task.

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Question 37 (3 marks)

The diagram shows a triangle ABC where $AC = 25$ cm, $BC = 16$ cm, $\angle BAC = 28°$ and angle ABC is obtuse.

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A

28°

25 cm

B

16 cm

C
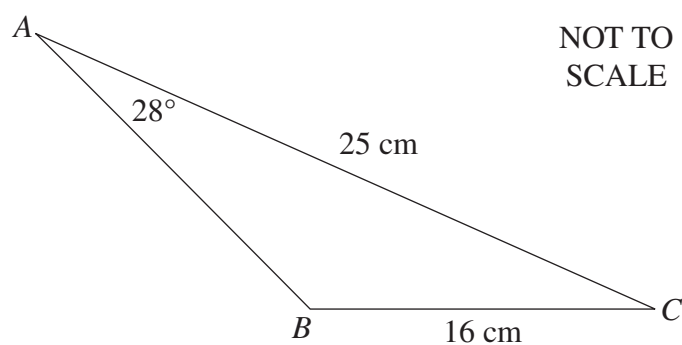
Find the size of the obtuse angle ABC correct to the nearest degree.

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Question 38 (4 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable lies between 0 and z for different values of z.

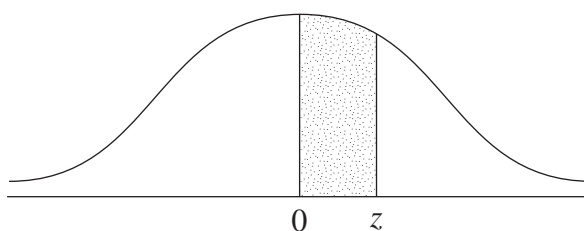| z	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.0398	0.0793	0.1179	0.1554	0.1915	0.2257

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.

(a) Using the table, show that the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1 is greater than 0.3 is equal to 0.3821. **1**

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(b) Birth weights are normally distributed with a mean of 3300 grams and a standard deviation of 570 grams. By first calculating a z-score, find how many babies, out of 1000 born, are expected to have a birth weight greater than 3471 grams. **3**

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– 39 –

Office Use Only – Do NOT write anything, or make any marks below this line.

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Question 39 (5 marks)

The diagram shows a compass radial survey of the field *ABCD*.

NOT TO SCALE

(a) Triangle *COB* has an area of 466 m². **2**

   Find the size of acute angle *COB*, correct to the nearest degree.

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Question 39 (continued)

(b) A farmer wants to put a fence around the triangle *DOC*.   **3**

Find the length of fencing required. Give your answer in metres correct to one decimal place.

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End of Question 39

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A table of future value interest factors for an annuity of $1 is shown. **3**

Table of future value interest factors

Number of periods	*Interest rate per period*				
	0.25%	0.5%	0.75%	1%	1.25%
2	2.0025	2.0050	2.0075	2.0100	2.0125
4	4.0150	4.0301	4.0452	4.0604	4.0756
6	6.0376	6.0755	6.1136	6.1520	6.1907
8	8.0704	8.1414	8.2132	8.2857	8.3589
10	10.1133	10.2280	10.3443	10.4622	10.5817

Simone deposits $1000 into a savings account at the end of each year for 8 years. The interest rate for these 8 years is 0.75% per annum, compounded annually.

After the 8th deposit, Simone stops making deposits but leaves the money in the savings account. The money in her savings account then earns interest at 1.25% per annum, compounded annually, for a further two years.

Find the amount of money in Simone's savings account at the end of ten years.

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Question 41 (4 marks)

In a particular city, the heights of adult females and the heights of adult males are each **4**
normally distributed.

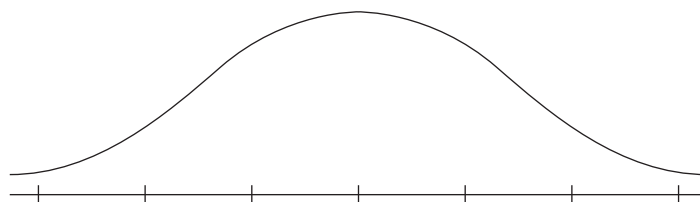Information relating to two females from that city is given in Table 1.

Table 1	*Height*	*Gender*	*Percentage of females in this city shorter than this person*
	175 cm	Female	97.5%
	160.6 cm	Female	16%

The means and standard deviations of adult females and males, in centimetres, are
given in Table 2.

Table 2		*Mean*	*Standard deviation*
	Females	μ	σ
	Males	1.05μ	1.1σ

A selected male is taller than 84% of the population of adult males in this city.

By first labelling the normal distribution curve below with the heights of the two
females given in Table 1, calculate the height of the selected male, in centimetres,
correct to two decimal places.

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Mathematics Standard 1
Mathematics Standard 2

REFERENCE SHEET

Measurement

Limits of accuracy

Absolute error $= \frac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement − absolute error

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

$$A \approx \frac{h}{2}(d_f + d_l)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV(1 + r)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$
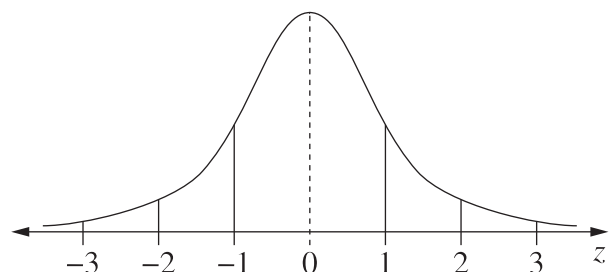
Statistical Analysis

An outlier is a score

   less than $Q_1 - 1.5 \times IQR$

   or

   more than $Q_3 + 1.5 \times IQR$

$$z = \frac{x - \mu}{\sigma}$$

Normal distribution

- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

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